New Classes of Kochen-Specker Contextual Sets

Norman D. Megill[†] and Mladen Pavičić[‡]

[†] *Boston Information Group, Lexington MA 02420, U.S.A.*
[‡] *Center of Excellence for Advanced Materials and Sensors (CEMS)*, Research Unit *Photonics and Quantum Optics*, *Institute Ruđer Bošković (IRB)*, Zagreb, Croatia.

[‡] *Nano Optics, Department of Physics, Humboldt University (HU)*, Berlin, Germany.,

MIPRO 2017 - 40th International Convention,May 22-26, 2017,
DC VIS - Distributed Computing, Visualization..., Opatija, Croatia.

MIPRO DC VIS 2017

Quantum Computation Perspectives

NEWS IN FOCUS

426 | NATURE | VOL 532 | 28 APRIL 2016

A €1-billion (US$1.1-billion) European flagship project could advance the state of quantum computing.

FUNDING

Billion-euro boost for quantum tech

Quantum Computation Magic

QUANTUM COMPUTING

Powered by magic

What gives quantum computers that extra oomph over their classical digital counterparts? An intrinsic, measurable aspect of quantum mechanics called contextuality, it now emerges. SEE ARTICLE P.351

ARTICLE 19 JUNE 2014 | VOL 510 | NATURE | 351
doi:10.1038/nature13460

Contextuality supplies the 'magic' for quantum computation

Mark Howard[1,2], Joel Wallman[2], Victor Veitch[2,3] & Joseph Emerson[2]

The Definition of the Kochen-Specker Contextual Sets

Non-contextuality: In classical theories, observables always assume a specific value, even if we may not know this value.

Quantum contextuality: Kochen and Specker showed that for some sets it is impossible to assign a value to all observables simultaneously.

KS theorem. In $\mathcal{H}^n$, $n \geq 3$, there exist sets of n-tuples of mutually orthogonal vectors, called *KS sets*, to which it is impossible to assign 1's and 0's in such a way that:

The Definition of the Kochen-Specker Contextual Sets

Non-contextuality: In classical theories, observables always assume a specific value, even if we may not know this value.

Quantum contextuality: Kochen and Specker showed that for some sets it is impossible to assign a value to all observables simultaneously.

KS theorem. In $\mathcal{H}^n$, $n \geq 3$, there exist sets of n-tuples of mutually orthogonal vectors, called *KS sets*, to which it is impossible to assign 1's and 0's in such a way that:

(i) No two orthogonal vectors are both assigned the value 1;

The Definition of the Kochen-Specker Contextual Sets

Non-contextuality: In classical theories, observables always assume a specific value, even if we may not know this value.

Quantum contextuality: Kochen and Specker showed that for some sets it is impossible to assign a value to all observables simultaneously.

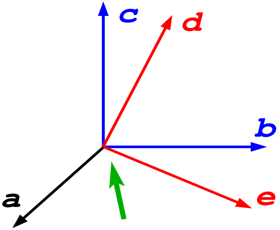
KS theorem. In $\mathcal{H}^n$, $n \geq 3$, there exist sets of n-tuples of mutually orthogonal vectors, called *KS sets*, to which it is impossible to assign 1's and 0's in such a way that:

(i) No two orthogonal vectors are both assigned the value 1;

(ii) Not all of any mutually orthogonal vectors are assigned the value 0.

Orthogonality; 3-Dim Example

vectors



orthogonalities

system of nonlinear equations

$$\mathbf{a}\cdot\mathbf{b} = a_x b_x + a_y b_y + a_z b_z = 0$$

$$\mathbf{a}\cdot\mathbf{c} = a_x c_x + a_y c_y + a_z c_z = 0$$

$$\mathbf{b}\cdot\mathbf{c} = b_x c_x + b_y c_y + b_z c_z = 0$$

$$\mathbf{a}\cdot\mathbf{d} = a_x d_x + a_y d_y + a_z d_z = 0$$

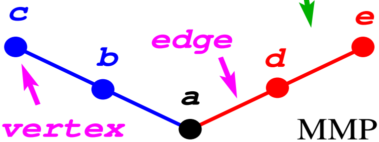
$$\mathbf{a}\cdot\mathbf{e} = a_x e_x + a_y e_y + a_z e_z = 0$$

$$\mathbf{d}\cdot\mathbf{e} = d_x e_x + d_y e_y + d_z e_z = 0$$

coordinatization

exponential complexity

MMP hypergraph



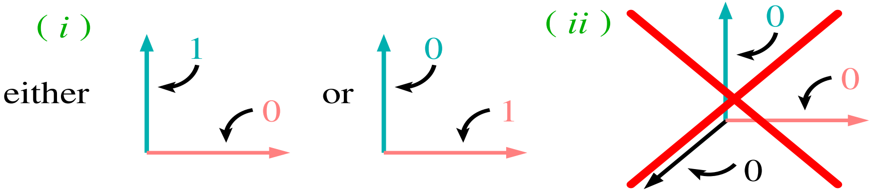
statistically linear complexity

MMP hypergraph string: **cba,ade.**

KS Conditions $\longleftrightarrow$ Our Algorithms and Programs

KS sets are constructive proofs of the the KS theorem.

KS sets violate the following conditions (from KS th.):



Our algorithms and our program "states01" only check whether these conditions are violated. Coordinatization (vector assignment) is dropped from hypergraphs.

It is added to them when verified on KS, later on.

Parity Proof

p–def: a (p–)hypergraph in an even–dim space in which each vertex shares an even number of edges.

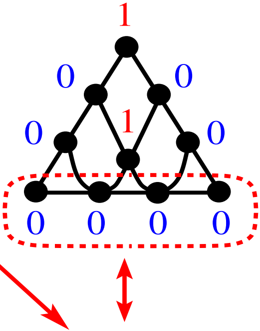
Parity Proof

Consider now a p–hypergraph with an odd number of edges.

p–def ➔ ***even No of edges with 1s***

Odd number of edges ➔ each edge should contain one 1 per KS–def

➔ ***odd No of edges with 1s*** ⟶ **contradiction!**



However, this triangle lacks a coordinatization!

Missions (Im)Possible

Problem: Find a coordinatization = Solve systems of non-linear equations = Exponentially complex task = **Mission impossible**

Solution: Start with a big KS set—call it a *master set*—and strip away edges down to *critical* KS sets, i.e., those KS sets that do not properly contain any KS subset. (Experimentally distinguishable are only critical sets.) = **Mission possible**

Options: (i) Parity proofs—with coordinatization;

(ii) MMP hypergraphs—without coordinatization; it is added later on, if needed;

(iii) combination of parity proofs and MMP hypergraphs —*both without coordinatization*; it is added later on, if needed;

Possible Option

Problems with options:

(i) (a) KS sets with even number of edges cannot have parity proofs —per definition;

(b) The majority of KS sets with odd number of edges turn out not to have parity proofs, either;

(c) In one third of the KS classes less than 0.1% or even none of the sets have parity proofs;

(ii) MMP hypergraph processing are slower than parity proof ones, for some classes—much slower;

(iii) None! Optimal approach.

McKay-Megill-Pavičić (MMP) Hypergraphs

We obtain McKay-Megill-Pavičić (MMP) hypergraphs from diagrams we defined previously for generating algebraic sets within Hilbert spaces.

Definition 2. We define MMP hypergraphs as follows

(i) Every vertex belongs to at least one edge;

(ii) Every edge contains at least 3 vertices;

(iii) Edges that intersect each other in $n-2$ vertices contain at least n vertices.

We encode MMP hypergraphs by means of ASCII characters.

McKay-Megill-Pavičić (MMP) Hypergraphs

We obtain McKay-Megill-Pavičić (MMP) hypergraphs from diagrams we defined previously for generating algebraic sets within Hilbert spaces.

Definition 2. We define MMP hypergraphs as follows

(i) Every vertex belongs to at least one edge;

(ii) Every edge contains at least 3 vertices;

(iii) Edges that intersect each other in $n-2$ vertices contain at least n vertices.

We encode MMP hypergraphs by means of ASCII characters.

MMP hypergraphs together with subsequently developed algorithms and programs developed by Brendan D. McKay, Norman D. Megill, Jean-Pierre Merlet, and Mladen Pavičić (2005-2016) enable us to generate KS sets arbitrary exhaustively (in principle).

Bibliography

Pavičić, M., J.-P. Merlet, B.D. McKay, and N.D. Megill, Kochen-Specker Vectors, *J. Phys.* A, **38**, 1577-1592 (2005).

Pavičić, M., N.D. Megill, and J.-P. Merlet, New Kochen-Specker Sets in Four Dimensions. *Phys. Lett.* A, **374**, 2122-2128 (2010);

Pavičić, M., B. McKay, N. Megill, and K. Fresl, Graph Approach to Quantum Systems, *J. Math. Phys.*, **51**, 102103-1-31 (2010);

Pavičić, M., N. Megill, P. K. Aravind, and M. Waegell, New class of 4-dim Kochen-Specker sets, *J. Math. Phys.*, **52**, 022104-1-9 (2011);

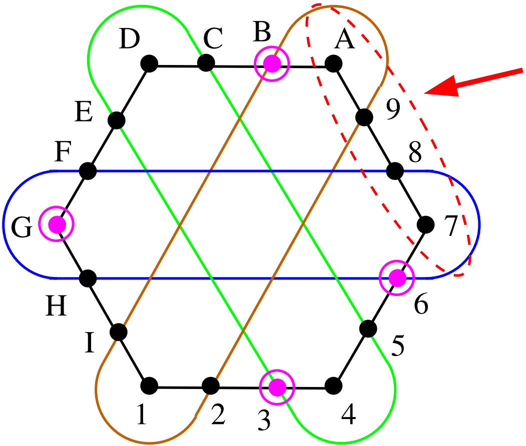
Waegell, M., P. K. Aravind, N. Megill, and M. Pavičić, Parity Proofs of the BKS Theorem Based on the 600-cell, *Found. Physics*, **41**, 883-904 (2011);

Megill, N., K. Fresl, M. Waegell, P. K. Aravind, and M. Pavičić, Probabilistic Generation of Quantum Contextual Sets. *Phys. Lett.* A, **375**, 3419-3424 (2011); Supplementary Material;

Pavičić, M., *Companion to Quantum Computation and Communication*, Wiley-VCH (2013); Sec. 1.17;

Pavičić, M., Arbitrarily Exhaustive Hypergraph Generation of 4-, 6-, 8-, 16-, and 32-Dimensional Quantum Contextual Sets, *Phys. Rev. A*, [to appear] (2017).

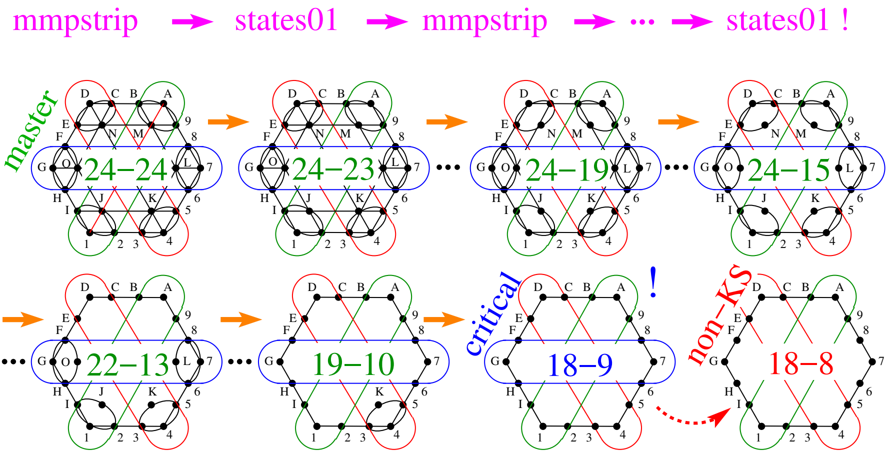
An example—Contextuality Visualisation; KS set 18-9



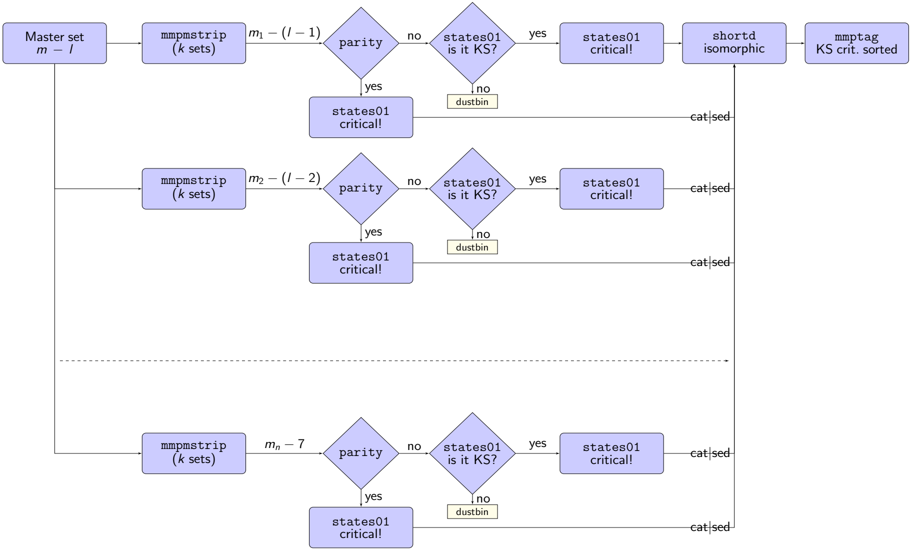
1234,4567,789A,ABCD,DEFG,GHI1,29BI,35CE,68FH.

1={0,0,0,1}, ... A={0,1,1,0}, ... C={1,1,-1,-1}, ... I={0,1,0,0}.

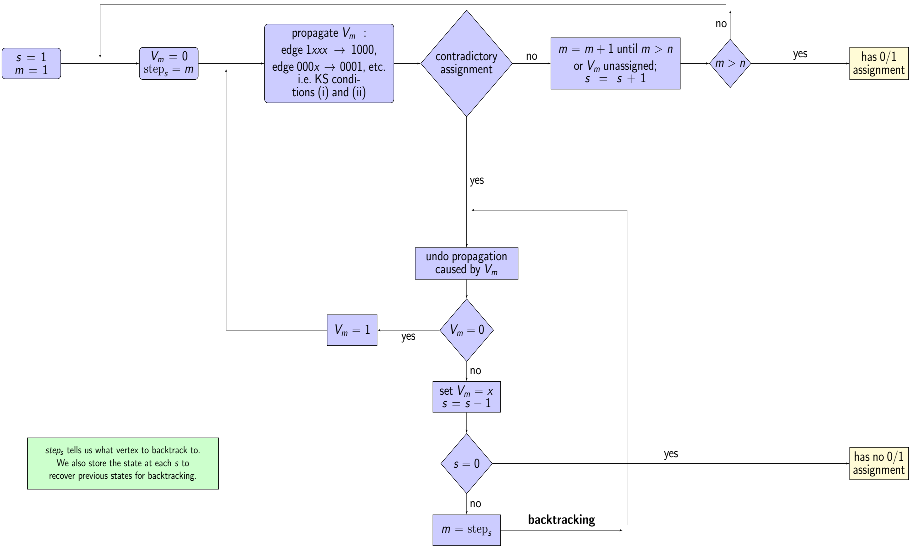
Example of Stripping and Filtering



Hypergraph program flowcharts

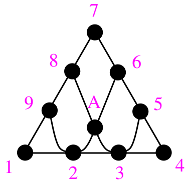


states01 Algorithm ($x =$"unassigned;" vertices $V_1 \ldots V_n$)



Full run of states01 flowchart for MMP 1234,4567,7891,92A6,8A35.
s in the flowchart is step number below; V_1=1... V_n=A where n=10.

1234,4567,7891,92A6,8A35	Actions causing result
0... ....0	assign (value) 0 to (vertex) 1
00.. ....0 .0......	assign 0 to 2
0001 1000 0..0 .0.0 ..00	assign 0 to 3; propagate 4,5,6,7 → 1,0,0,0
0001 1000 0010 10?0 0?00	assign 0 to 8; 9 → 1; but A = ?
0001 1000 0100 00?0 1?00	backtrack; assign 1 to 8; 9 → 0; but A = ?
0010 00.. .0.0 .00. 0010	backtrack; assign 1 to 3; 4,8,A,5 → 0,0,0,0
0010 0001 10?0 ?000 0010	assign 0 to 6; 7 → 1; but 9 = ?
0100 0.0. ..00 0100 .00.	backtrack; assign 1 to 2; 3,4,A,6,9 → 0,0,0,0,0
0100 0001 1?00 0100 ?000	assign 0 to 5; 7 → 1; but 8 = ?
1000 0..0 0001 00.. 0.0.	backtr.; ass. 1 to 1; 2,3,4,7,8,9 → 0,0,0,0,0,0
1000 0010 0001 00?1 0?00	assign 0 to 5; 6 → 1; but A = ?
1000 0100 0001 00?0 0?01	backtrack; assign 1 to 5; 6 → 0; but A = ?



Backtracking exhausted; no 0/1 assignment possible

Breakdown of **Step 3** of full run (previous slide):

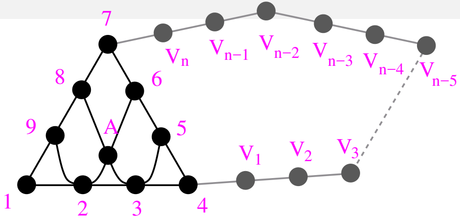
1234,4567,7891,92A6,8A35	Actions causing result
00. . . . . .0 .0	**Step 2** left us in this state (for reference)
000. . . . . .0 .0 . . . . .0.	Assign value 0 to vertex 3 (3 is first unassigned vertex)
0001 1. . . . . . .0 .0 . . . . .0.	Since we have 3 0s on edge 1234, vertex 4 must be 1 by KS condition (ii)
0001 1000 0. .0 .0 .0 . .00	Since vertex 4 is 1 on edge 4567, 5,6,7 must be 0 by KS condition (i)

Breakdown of **step 5** of full run:

1234,4567,7891,92A6,8A35	Actions causing result
0001 1000 0010 10?0 0?00	**Step 4** left us in this state (for reference)
0001 1000 0..0 .0.0 ..00	Backtracking removes the changes made in **step 4** i.e. all the colored entries above
0001 1000 01.0 .0.0 1.00	8 is first unassigned vertex above. Assignment of 0 to 8 in **step 4** failed, so assign 1 to 8
0001 1000 0100 00.0 1.00	Since 8 is 1 on edge 7891, 9 must be 0 by KS condition (i)
0001 1000 0100 00?0 1?00	KS cond. (ii) requires 1st A = 1, but KS cond. (i) requires 2nd A = 0 (contradiction)

Thus the assignment of 1 to vertex 8 failed, so we will backtrack in **step 6** of previous slide (in that case, all the way back to before **step 3**; see “backtracking” loop in flowchart)

Clustering Algorithm

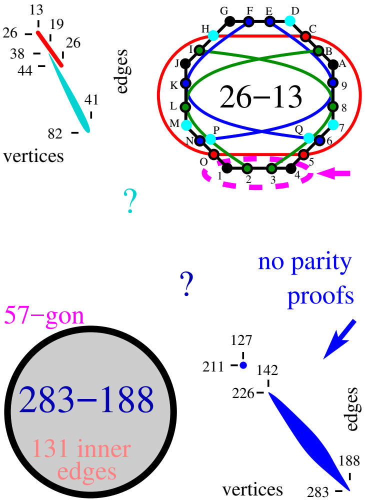


Naive algorithm: trial assignments in the order 1, 2, 3, 4, V_1, V_2, ... , V_n, 5, 6, 7, 8, 9, A.

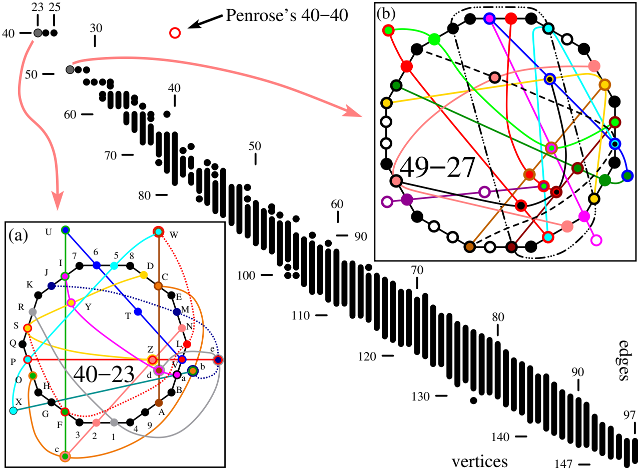
Problem: The backtracking algorithm successfully assigns 1, 2, 3, 4, V_1, . . . , V_n. When a conflict is found in 5, . . . , A, it does $\sim 2^n$ backtracks before exhausting assignments.

Clustering algorithm: do trial assignments on edges with the most connections to other edges first. Contradiction will be found in vertices 1, 2, . . . , 9, A before even trying $V_1, \ldots, V_n$. Total backtracks are reduced from 2^n to 5.

What Parity-Proof Algorithms Can Do Only Partly



What Parity-Proof Algorithms Cannot Do at All



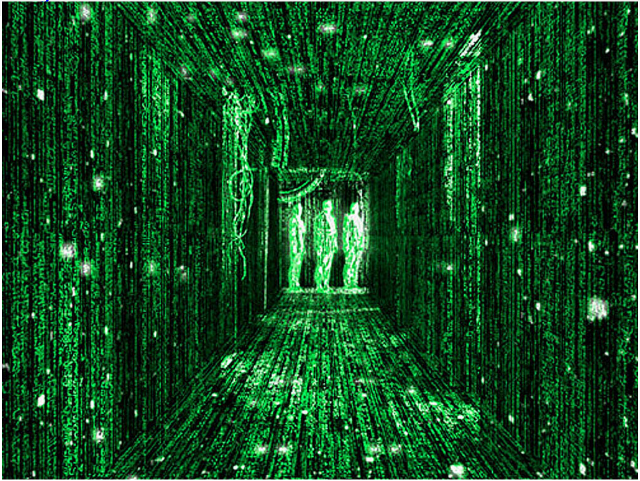
Acknowledgements 😎

The work presented here is supported by the *Croatian Science Foundation* through project IP-2014-09-7515 and CEMS funding by the *Ministry of Science, Education and Sports of Croatia*.

Computational support was provided by the cluster *Isabella* of the *University Computing Centre* of the *University of Zagreb* and by the *Croatian National Grid Infrastructure*.

Travelling expenses were covered through the project IP-2014-09-7515.

Thanks for your attention 😎



http://cems.irb.hr/en/research-units/photonics-and-quantum-optics/
http://www.irb.hr/users/mpavicic/
http://goo.gl/xbx8U2