Constructing Quantum Reality

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It is argued that the recent success in the foundational quantum mechanical research relies on free tailoring of quantum subsystems with the goal of engineering their desired properties and on a corresponding manipulation of parts of standard quantum and classical mathematical formalisms describing composite systems. We give three examples: interaction-free measurements as a kind of patching classical and quantum photon theory by experimental quantum principles, entanglement and teleportation as a kind of chopping the standard Hilbert space description, and an automated finding of arbitrary Kochen-Specker vectors as a kind of reducing equational problems to their graphical representation.

Keywords: Quantum mechanics, interaction-free measurements, entanglement, teleportation, Kochen-Specker problem

A. Introduction

When we speak about *the role of mathematics in physics* (the title of this conference) we usually start from the assumption that a particular physical problem is supported by a well defined model. We then also assume that a consistent theory is formulated for the model and that such a theory serves us to predict relevant experimental outcomes. So, at the first glance, the role of mathematics (under which we understand the mathematics of a physical model or theory) would be to guide us to the expected experimental outcomes and even to the new experiments and results. In this paper, we argue that the role is not one-directional and that it immediately invokes the question of the role of physics in the mathematics used for a formulation of a theory.

We limit ourselves to the quantum theory. Quantum theory is considered to be one of the most reliable, consistent, and complete theory and it has certainly withstood all the trails so far. Our argument does not, therefore, challenge any of these features of the quantum theory but merely underlines the fact that experimental feedbacks and experimental considerations provide us with guidelines on how to apply the mathematical formalism. In today's physics we are more and more concerned with the systems we construct as opposed to systems that already exist in Nature. Such a construction is supported by a *construction* of a mathematical description which by itself does not follow from some general underlying theory, e.g., the Hilbert space theory in the case of quantum mechanics. In a word, manipulation of systems requires manipulation of formalism. We illustrate and explain the claim by three short case studies from the field of quantum mechanics and its foundational experiments. Technical details are given for the sake of completeness but should be disregarded by nonspecialists.

B. Interaction-Free Experiments

Quantum interference of individual systems has recently been found capable of detecting objects without transferring energy to them. In 1986 I formulated this in the following way. "Consider a photon experiment shown in Fig. 1 which results in an interference in the region D provided we do not know whether it arrived to the region by path s_1 or by path s_2. As it is well-known, experimental facts are: If we, after a photon passed the beam splitter B and before it could reach the point C, suddenly introduce a detector in the path s_2 in the point C and do *not* detect *anything*, then it follows that the photon must have taken the path s_1—and, really, one can detect it in the region D but it does *not* produce interference there. Quantum mechanically, if we registered the interference in the region D, we could not find an experimental procedure to directly either prove or disprove that the photon uses both paths simultaneously. The fact that by detecting *nothing* in point C we destroy the interference implies that the photon *somehow* knows of the other path when it takes the first one." (Ref. [1], pp. 31, 32)

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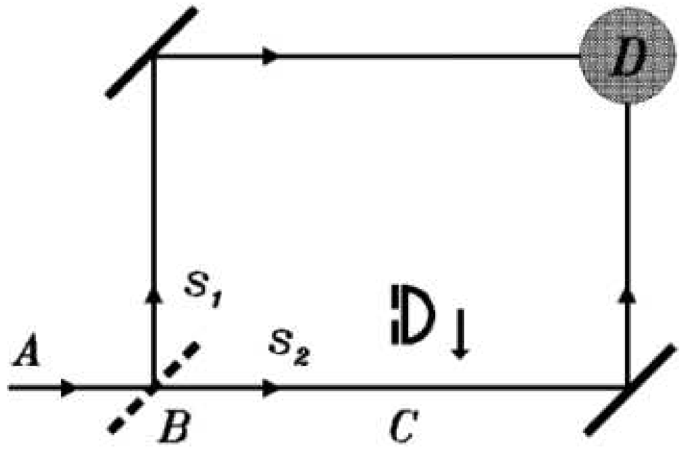

FIG. 1: Figure taken from Pavičić (1986): "By detecting *nothing* in the point C we destroy the interference [in the region D]." [1, p.31]; [2]

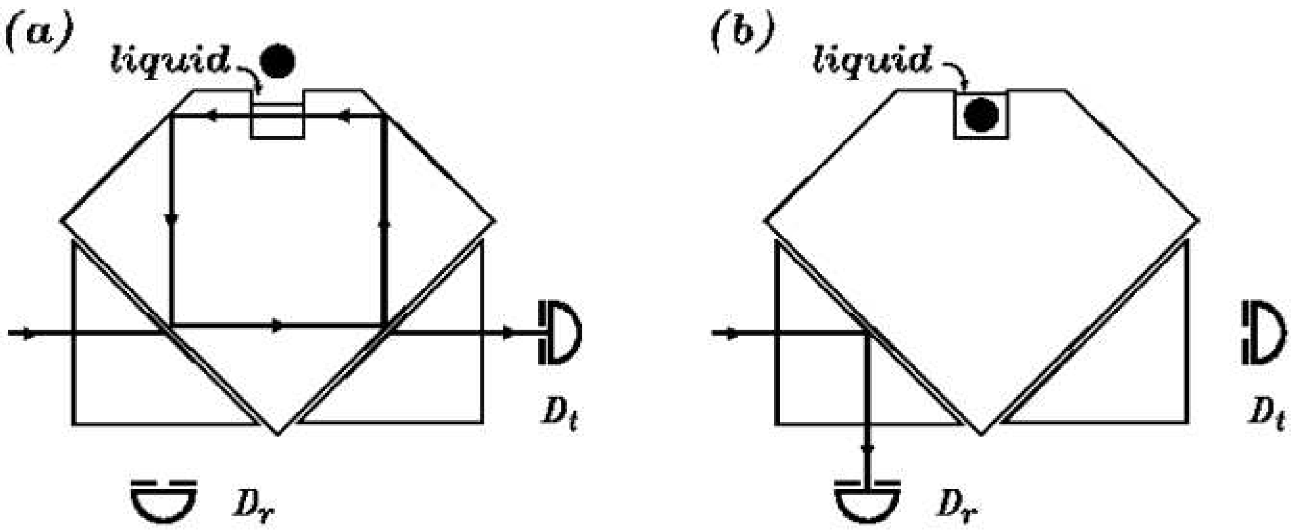

FIG. 2: Figure taken from Paul and Pavičić (1997): "Lay-out of the proposed interaction-free experiment; (a) In the shown free round–trips the intensity of the reflected beam is approaching 0 for R approaching 1, i.e., detector D_r does not react; (b) However, when an absorbing object is immersed in the liquid (whose refractive index is the same as the one of the crystal in order to prevent losses of the free round–trips), for $R = 0.999$, 99.9% of the incoming beam reflect into D_r, 0.0001% go into D_t, and 0.0999% hit the object." [5]

This "photons's knowledge" is not explicitly described by the quantum mechanical formalism. The interference is described by the probability waves, so, the afore-mentioned consideration of the paths of the energy carriers, i.e., of the photons themselves was not foreign to physicists of the time but was considered useless. However, a decade ago Elitzur and Vaidman [3] realized that path considerations of the waves can be used for realistic measurements and several such experiments have been carried out since. [4] There were several designs of the experiments but we will keep to the one which reveals our point the best.

The lay–out of the proposed experiment introduced in Paul and Pavicic (1996,7) [5–7] is shown in Fig. 2. The outcomes has been confirmed by a real experiment carried out by Tsegaye, Goobar, Karlsson, Björk, Loh, and Lim (1998) [8]. Our experimental proposal uses an uncoated monolithic total–internal–reflection resonator (MOTIRR) coupled to two triangular prisms by the frustrated total internal reflection (FTIR). A squared MOTIRR requires a relative refractive index with respect to the surrounding medium $n > 1.41$ in order to confine a beam to the resonator (the angle of incidence being 45°). If, however, another medium (in our case the right triangular prism in Fig. 2) is brought within a distance of the order of the wavelength, the total reflection within the resonator will be *frustrated* and a fraction of the beam will "tunnel out" from the resonator. Depending on the dimension of the gap and the polarisation of the incidence beam one can well define reflectivity R within the range from 10^{-5} to 0.99995. The main advantage of such a coupling—in comparison with coated resonators—is that the losses are extremely small: down to 0.3%. In the same way a beam can "tunnel into" the resonator through the left triangular prism in Fig. 2, provided the condition $n > 1.41$ is fulfilled for the prism too. The incident laser beam is chosen to be polarised perpendicularly to the incident plane so as to give a unique reflectivity for each photon. The faces of the resonator are polished spherically to give a large focusing factor. A round trip path for the beam is created in the resonator as shown in Fig. 2. A cavity is cut in the resonator and filled with an index–matching fluid to reduce losses. (Before we carry out the measurement we have to wait until the fluid comes to a standstill to avoid a possible destabilization of the phase during round trips of the beam.) Now, if there is an object in the cavity in the round trip path of the beam in the resonator, the incident beam will be almost totally reflected (into D_r) and if there is no object, the beam will be almost totally transmitted (into D_t).

To understand this result we sum up the contributions originating from round trips in the resonator, to the reflected wave. The portion of the incoming beam of amplitude $A(\omega)$ reflected into plane determined by δ is described by the amplitude $B_0(\omega) = -A(\omega)\sqrt{R}$, where $R = |r|^2$ is reflectivity. The transmitted part will travel around the resonator guided by one frustrated total internal reflection (at the face next to the right prism) and by two proper total internal reflections. After a full round trip the following portion of this beam joins the directly reflected portion of the beam by tunnelling into the left prism: $B_1(\omega) = A(\omega)\sqrt{1-R}\sqrt{R}\sqrt{1-R}\,e^{i\psi}$. $B_2(\omega)$ contains three frustrated total internal reflections and so on; each subsequent round trip contributes to a geometric progression which gives the reflected amplitude

$$B_n(\omega) = A(\omega)\sqrt{R}\{-1 + (1-R)e^{i\psi}[1 + Re^{i\psi} + (Re^{i\psi})^2 + \ldots]\} = \sum_{i=0}^{n} B_i(\omega)\,, \tag{1}$$

where $\psi = (\omega - \omega_{res})T$ is the phase added by each round trip. Here ω is the frequency of the incoming beam, T is the round trip time, and ω_{res} is the selection frequency corresponding to a wavelength which satisfies $\lambda = L/k$, where L is the round trip length of the cavity and k is an integer. At summing up the round–trip contributions we have taken into account that (because of the above condition imposed on the total phase shift ϕ) all the contributions must lie in the reflected–wave plane and that their amplitudes must carry the opposite sign (to that of the reflected wave, $-A(\omega)\sqrt{R}$) so as to cancel out at resonance $\psi = 0$.

For an insight into the physics of the experiment it is sufficient to consider plane waves [$A(\omega) = A_0$]. The limit of $B_n(\omega)$ yields the total amplitude of the reflected beam:

$$B_r(\omega) = \lim_{n\to\infty} B_n(\omega) = -A_0\sqrt{R}\frac{1-e^{i\psi}}{1-R\,e^{i\psi}}\,. \tag{2}$$

We see that for any $R < 1$ and $\omega = \omega_{res}$, i.e., if nothing obstructs the round trip of the beam, we get no reflection at all [i.e., no response from D_r (see Fig. 2)]. When an object blocks the round trip and R is close to one, then we get almost a total reflection. In terms of single photons (which we can obtain by attenuating the intensity of a laser until the chance of having more than one photon at a time becomes negligible) the probability of detector D_r reacting when there is no object in the system is zero. A response from D_r means an interaction–free detection of an object in the system. The probability of the response is R, the probability of a photon hitting the object is $R(1-R)$, and the probability of a photon exiting into detector D_t is $(1-R)^2$. These results have been confirmed by several recent experiments. [8]

A more realistic experimental approach we achieve by looking at two possible sources of individual photons: a cw laser and a pulse laser and by using wave packets instead of plane wave as we did in [5, 7]. However, since all results as well as physics of the experiments remain the same under such more realistic approach we will not enter into its details here.

What comes out from the above elaboration is that a description by means of essentially classical wave fields—taken over directly from classical optics and the theory of electromagnetic waves—matches the behaviour of photons. But waves in quantum mechanics, being "waves of probability," are considered virtual, not real. Does this bring us to a contradiction? No, only to another—non-classical—meaning of reality. For, it turns out that "switching off" the destructive interference of the waves does not spread within a finite interval of time by the speed of light but establish propagation conditions for photons instantaneously. Let us look at the following thought experiment shown in Fig. 3 and proposed by Fearn, Cook, and Milonni.[9]

"Emission of an excited atom in a cavity is inhibited and the question is being addressed of whether a sudden replacement of one of the cavity mirrors by a detector can result in a photon count immediately or only after

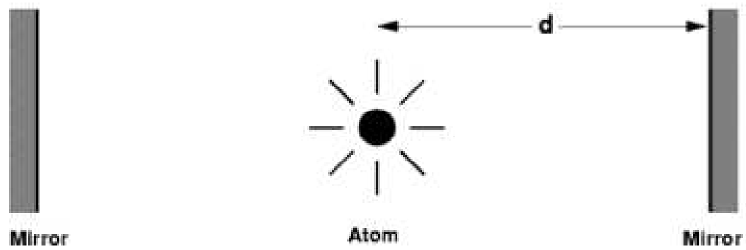

FIG. 3: Figure made following Fearn, Cook, and Milonni (1995) [9]: Inhibited photon emission of an excited atom in a cavity.

some retardation time. It is argued in the paper that it is possible to count a photon immediately following the substitution of photodetector for a mirror." [9] This has recently been confirmed by a real experiment with an inhibited downconversion of photons from a crystal. [10]

"Two plausible explanations, leading to different answers, have been proposed. According to one argument, the inhibited atom cannot "know" the mirror has been removed until the time $t = T + d/c$, where d is the atom-mirror distance, and the atom can begin to radiate after this time. Since the propagation time to the detector is d/c, a photon can be detected only after a time $t + d/c = T + 2d/c$, i.e., after a time $2d/c$ following the mirror switchout.

The second viewpoint holds that, as in the case of a classical dipole radiator in a cavity, there are always fields (or, more precisely, probability amplitudes) propagating from the atom to the removable mirror and back to the atom, and that the inhibition of a spontaneous emission implies a destructive interference of the two counterpropagating fields. The sudden removal of the mirror allows that part of the field propagating toward the mirror to escape from the cavity, so that a photon can be counted immediately following the switchout of the mirror.

In the absence of detailed calculations or an experiment objections can be raised against either prediction." [9] So, Fearn, Cook, and Milonni offered a "detailed calculations" from which it followed "that the non-vanishing photon counting rate at $t = T$ occurs not at the expense of the atom, but rather as a depletion of *field* energy, i.e., a depletion of the energy associated with the backward-propagating field and the interference of the counterpropagating fields." [9]

However, the conclusion cannot "follow" from a calculation made for probability amplitudes because they are not real and they are not carriers of energy; the photons are the energy carriers but the photons are absent in the offered picture till the removal of the mirror. It is actually, the other way round: the calculation is made for the offered physical picture which is correct and ingenious but not directly derivable in either classical or quantum theory of light. There is no general thumb rule to derive photon behaviour in the considered example: "Sometimes the classical model is best, and sometimes the quantum one offers more understanding." [11] Actually the combination of the two supported by the experimental results can give us a (tentative) theory.

To see the meaning of such an approach and the interplay of experiment and theory let us look at the following two modifications of the afore presented interaction-free resonator experiment.

The first one is shown in Fig. 4. The proposed experiment uses a combination of atom interferometer with ultracold metastable atoms and the resonance interaction-free path detection by means of a movable MOTIRR (of course, without liquid what only slightly reduces the efficiency). To increase the probability of an atom being hit by the round tripping beam, the incoming laser beam should be split into many beams by multiple beam splitters, each beam containing in average one photon in the chosen time window, so as to feed MOTIRR through many optical fibres. The atom source in the atom interferometer is a magneto–optical trap containing $1s_5$ neon metastable atoms which are then excited to the $2p_5$ state by a 598–nm laser beam. Of all the states to which $2p_5$ decays we follow only $1s_3$ atoms whose trajectory are determined only by the initial velocity and gravity (free fall from the trap). Now the atoms fall with different velocities but each velocity group forms interference fringes calculated as for the optical case and only corrected by a factor which arises from the acceleration by the gravity during the fall. MOTIRR is mounted on a device which follows (with acceleration) one velocity group from the double slit to microchannel plate detector (MCP). (Atoms from other groups move with respect to MOTIRR and therefore cannot decohere MOTIRR). The laser is tuned to a frequency equal to the $1s_3$ resonance frequency which in effect increase the cross section of the atoms so as to make efficiently "visible" to (virtual) photons. The source is attenuated so much that there is in average only one atom in a velocity group. The whole process repeats every 0.4 s. Assuming that we have 10 ns recovery time for the photon detectors and 300 optical fibres we arrive at about 10^7 counts which all go into one detector D_t when no atom obstructs a round trip. (For reflectivity $R = 0.999$ the probability of D_r being activated is $2 \cdot 10^{-9}$.) As soon as D_r detector fires we know which slit the observed atom passed through. After 10^3 repeating of such successful detections we have enough data to see whether the interference fringes are destroyed significantly with respect to unmonitored reference samples or not.

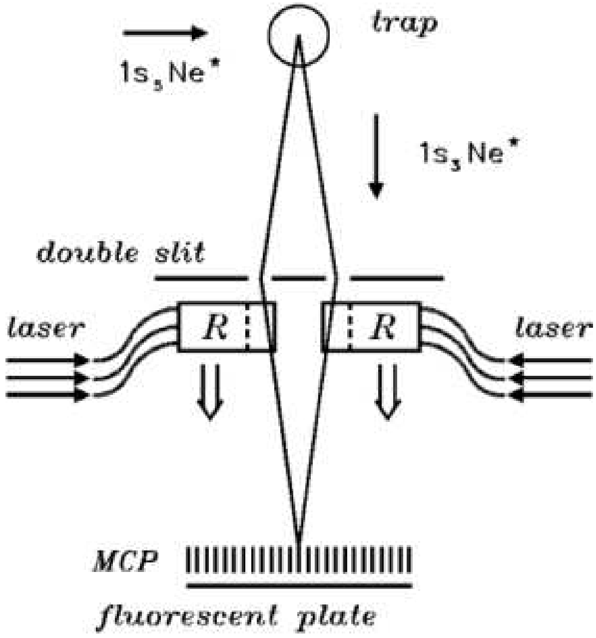

FIG. 4: Figure taken from Pavičić (1996): Proposal for a *welcher Weg* experiment with ultracold atoms. MOTIRR resonators R, see Fig. 2, here shown sideways, move together with the falling atoms which sit in their openings." [7]

Figuratively, one could call the just described device a "Heisenberg microscope without a kick." The Heisenberg microscope reasoning for a *welcher Weg* experiment traditionally rest on the Heisenberg uncertainty relations. Uncertainty relations always refer to the mean values of the operators and that means—even when the operators are projectors—statistics obtained by recording an interaction, i.e., by a reduction of the wave packet. In our *"interaction-free microscope"* measurement we do not attach any value to any operator in the Hilbert space description of the observed systems and therefore, *no* uncertainty relation is involved. As for the *welcher Weg* experiment it has recently been shown that "it is possible to obtain *welcher Weg* information without exposing the interfering beam to uncontrollable scattering events... That is to say, it is simply the information contained in a functioning measuring apparatus that changes the outcome of the experiment and not uncontrollable alterations of the spatial wave function, resulting from the action of the measuring apparatus on the system under observation." [12] There is, however, an essential difference between our proposal and the one by Scully, Englert, and Walther [12] (microwave cavity proposal). In the latter one there is slight exchange of energy which does not significantly disturb the spatial wave function of the system taking part in the interference but does disturb its phase. In our proposal we have no exchange of energy between atoms and photons.

Hence the result does not follow by means of any existing thumb rule within either classical or quantum formalism. We arrived at it following the indistiguishability principle which tells us that we would have the atom interference fringes only if we did not know which slit an atom passed through. Then we can construct a quantum mechanical description which the result would fit into as carried out, e.g., by Karlsson, Björk, and Forsberg (1998) [13].

The second modification of the original interaction-free resonator experiment which is even more intricate than the previous one is shown in Fig. 5. "We tune in our FTIR–MOTIRR system so as to have as big a gap between the coupling prisms and the crystal as possible (e.g., corresponding to $R = 0.9999$). The Rochon prism p is rotated so as to fully match the phase shift as its O–wave. Therefore, when the Pockels cell is *off* the round–trip path is not influenced at all. When the Pockels cell is *on* the path is redirected through Rochon prism p (as E–wave) into detector D_p. We switch on a cw laser and let it feed the system. When the Pockels cell is *on* detector D_r should fire with the probability approaching 1. When it is *off* detector should D_t fire with the probability approaching 1." [5]

"We carry out two kinds of measurement. The first kind of measurement is switching the Pockels cell *on* and monitoring D_r immediately afterwards. We accommodate the intensity of the laser beam so as to have one photon in 0.1 ns in average. The fastest Pockels cells have reaction time down to 0.1 ns. The time an information travelling at the speed of light needs to spread from the Pockels cell to the incoming gap can be made as high as 4 ns by choosing the biggest available crystals. The fastest detectors have reaction time of under 1 ns. Before we switch on the Pockels cell almost only detector D_t fires. After we switch on the Pockels cell we monitor detector D_r and see whether it reacts instantaneously or after 4 ns. (Of course we cannot have a source of photons "on demand" and we

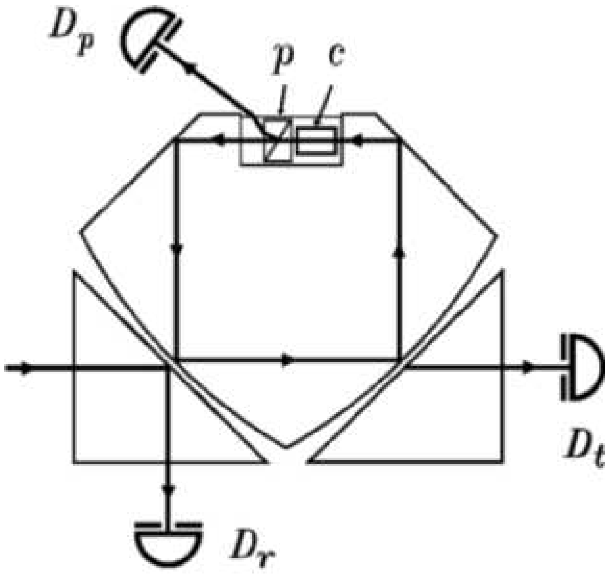

FIG. 5: Figure taken from Paul and Pavičić (1997): "Lay–out of the proposed virtual–or–real–path experiment. When the Pockels cell c is *on* it redirects the round–trip path through Rochon prism p into detector D_p and therefore almost only detector D_r fires. When the Pockels cell c is *off* there is no influence on the round–trip path and almost only detector D_t fires. [5]

only determine whether the statistics change.) In accordance with the above reasoning of Fearn, Cook, and Milonni and the corresponding experiment the detector should react instantaneously." [5]

"The second kind of measurement is switching the Pockels cell from *on* to *off* and monitoring detector D_r immediately afterwards. We lower down the intensity of the laser beam so as to have one photon in 10 ns in average. We calculated that for $R = 0.9999$ the resonance fully establishes after 100 ns, i.e., after that time D_r cannot fire (almost) at all. We monitor D_r within this 100 ns and see whether detector D_r stops firing immediately or only after several firing within the first 100 ns. We have chosen 10 ns in the incoming beam so as to make sure that after switching off the Pockels cell only an "empty" wave is coming to the system. A variety of the experiment would be to lower down the intensity of the laser beam further down to under one photon in 100 ns." [5] Of course we again only monitor the statistics of clicks. Here, the reasoning of Fearn, Cook, and Milonni seems to require that the resonator has to charge first, i.e., that the electric field wave (probability amplitude) has to wind up —at least 100 round trips (1 μs) in the classical approach [6]. Still there is a viewpoint that a photon "sees" the cavity immediately and that one could use a very narrow time window with downconverted photons (signal and idler photons appear within the order of femtoseconds and a Pockels cell can provide a time window of under 1 ns) to carry out the measurement. An experiment here would be useful.

C. Teleportation and Entanglement

Entanglement is taken to be "one of the most interesting and puzzling ideas associated with composite quantum systems." [14, p. 95] "Consider the two qubit state

$$|\psi >= \frac{|00\rangle + |11\rangle}{\sqrt{2}} \tag{3}$$

This state has a remarkable property that there are no single qubit states $|a\rangle$ and $|b\rangle$ such that $|\psi\rangle = |a\rangle|b\rangle$. We say that a state of composite system having this property (that it can't be written as a product state of its component systems) is an *entangled* state. For reasons which nobody fully understands, entangled states play a crucial role in quantum computation, quantum information, and quantum teleportation." [14, p. 96]

In this section we argue that entanglement is a typical example of our manipulation of systems and their formalism with the aim of constructing quantum technological reality. Let us have a look at the following experimental proposal of Pavičić and Summhammer (1993,1994) [15, 16] as given in Fig. 6.

Two independent sources, S_I and S_{II}, both simultaneously emit two photons correlated in polarisation to the left and right. On the left photons we measure polarisations by the polarisation filters P1 and P2 and on the right photons

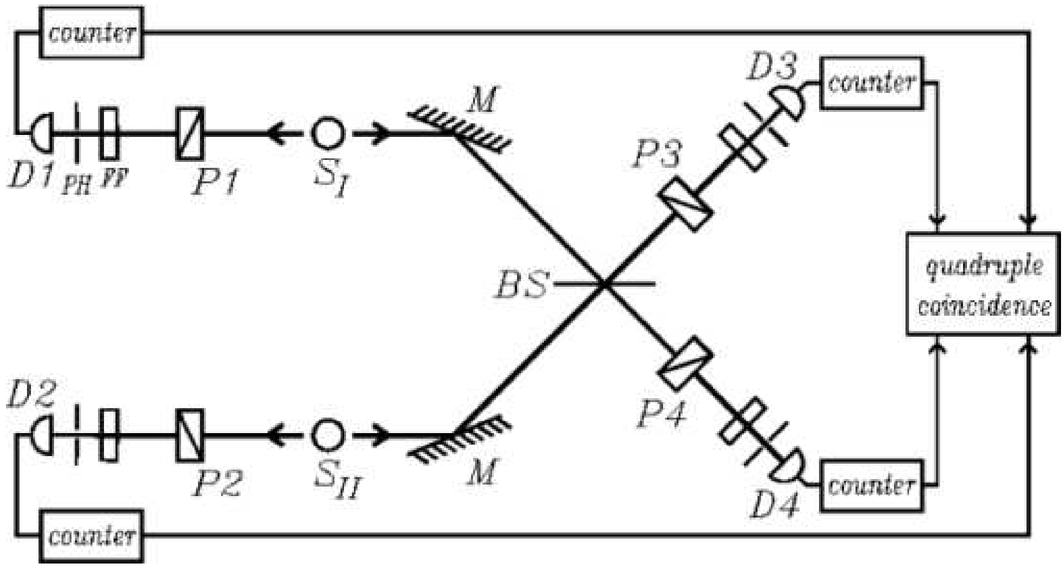

FIG. 6: Figure taken from Pavičić and Summhammer (1994): "Two photons from different unpolarised sources each pass through a polariser to a detector. Although their trajectories never mix or cross they exhibit 4th-order-interference-like correlations when the other two photons interfere on a beam splitter even when the latter two do not pass any polarisers at all." [5]

by P3 and P4. To point out that we get photons really unprepared photons we like to stress that the sources can in principle be atoms exhibiting cascade emission. But if want a feasible experiment we shall of course use downconverted photons.

The state of the four photons immediately after leaving the sources is described by the product of two entangled states:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|1_x\rangle_1|1_x\rangle_3 \ + \ |1_y\rangle_1|1_y\rangle_3\right)\otimes\frac{1}{\sqrt{2}}\left(|1_x\rangle_2|1_x\rangle_4 \ + \ |1_y\rangle_2|1_y\rangle_4\right) \tag{4}$$

Here, $|_x\rangle$ and $|_y\rangle$ denote the mutually orthogonal photon states. So, e.g., $|1_x\rangle_1$ means the state of photon 1 leaving the source S_I to the left polarised in direction x. In the following we use the annihilation operator formalism, often employed in quantum optical analysis.The operator describing the polarisation at P1 oriented along the x-axis and the subsequent detection at D1 acts as follows: $\hat{a}_{1x}|1_x\rangle_1 = |0_x\rangle_1$, $\hat{a}^\dagger_{1x}|0_x\rangle_1 = |1_x\rangle_1$, $\hat{a}_{1x}|0_x\rangle_1 = 0$, etc. When P1 is oriented at some angle θ_1 polarisation and detection are represented by $\hat{a}_1 = \hat{a}_{1x}\cos\theta_1 + \hat{a}_{1y}\sin\theta_1$. The phase the photon accumulates between the source S_I and the detector D1 adds the factor $e^{i\omega_1(r_1/c+t_0^I-t_1)}$, where ω_1 is the frequency of photon 1, r_1 is the path length from S_I to D1, c is the velocity of light, t_0^I is the time of emission of a pair of photons at S_I, and t_1 is the time of detection at D1. Hence the annihilation of a photon at detector D1 means application of the operator $\hat{E}_1 = (\hat{a}_{1x}\cos\theta_1 + \hat{a}_{1y}\sin\theta 1)e^{i\omega_1(r_1/c+t_0^I-t_1)}$ onto the initial state of Eq. (4). Similarly, detection of photon 2 at D2 means application of $\hat{E}_2 = (\hat{a}_{2x}\cos\theta_2 + \hat{a}_{2y}\sin\theta_2)e^{i\omega_2(r_2/c+t_0^{II}-t_2)}$, where the symbols are defined by analogy. On the right side of the sources, a detection at D3 can be caused by photon 3 emitted by source S_I or by photon 4 emitted by source S_{II}. The beamsplitter BS may have polarisation dependent transmission and reflection coefficients, denoted by T_x, T_y, and R_x, R_y, respectively. The angle of the polariser P3 is given by θ_3.

$$\hat{E}_3 = \Big(\hat{a}_{4x}\sqrt{T_x}\cos\theta_3 + \hat{a}_{4y}\sqrt{T_y}\sin\theta_3\Big)e^{i\,\omega_4(\frac{r_{II}+r_3}{c}+t_0^{II}-t_3)} + i\,\Big(\hat{a}_{3x}\sqrt{R_x}\cos\theta_3 + \hat{a}_{3y}\sqrt{R_y}\sin\theta_3\Big)\,e^{i\,\omega_3(\frac{r_I+r_3}{c}+t_0^{I}-t_3)}$$

For D4 one defines $\hat{E}_4$ analogously."[16]

Till this point in calculation everything follows from the standard quantum mechanics and there is no entanglement. Entanglement comes to stage when we want to make some measurements on some subsystem and *not* some other measurements on some other subsystems. For example, the coincidence probability for all four photons detected by detectors D1,D2,D3 and D4 (see Fig. fig:josab-95) only (which means not by detectors D1$^\perp$,D2$^\perp$,D3$^\perp$,D4$^\perp$ or not by D1,D2 and two photons 3,4 by one detector D3, etc.) reads: [16]

$$P(\theta_1,\theta_2,\theta_3,\theta_4) = \langle\Psi|\hat{E}_1^\dagger\hat{E}_2^\dagger\hat{E}_3^\dagger\hat{E}_4^\dagger\hat{E}_4\hat{E}_3\hat{E}_2\hat{E}_1|\Psi\rangle = \tfrac{1}{16}\sin^2(\theta_1-\theta_2)\sin^2(\theta_3-\theta_4) \tag{5}$$

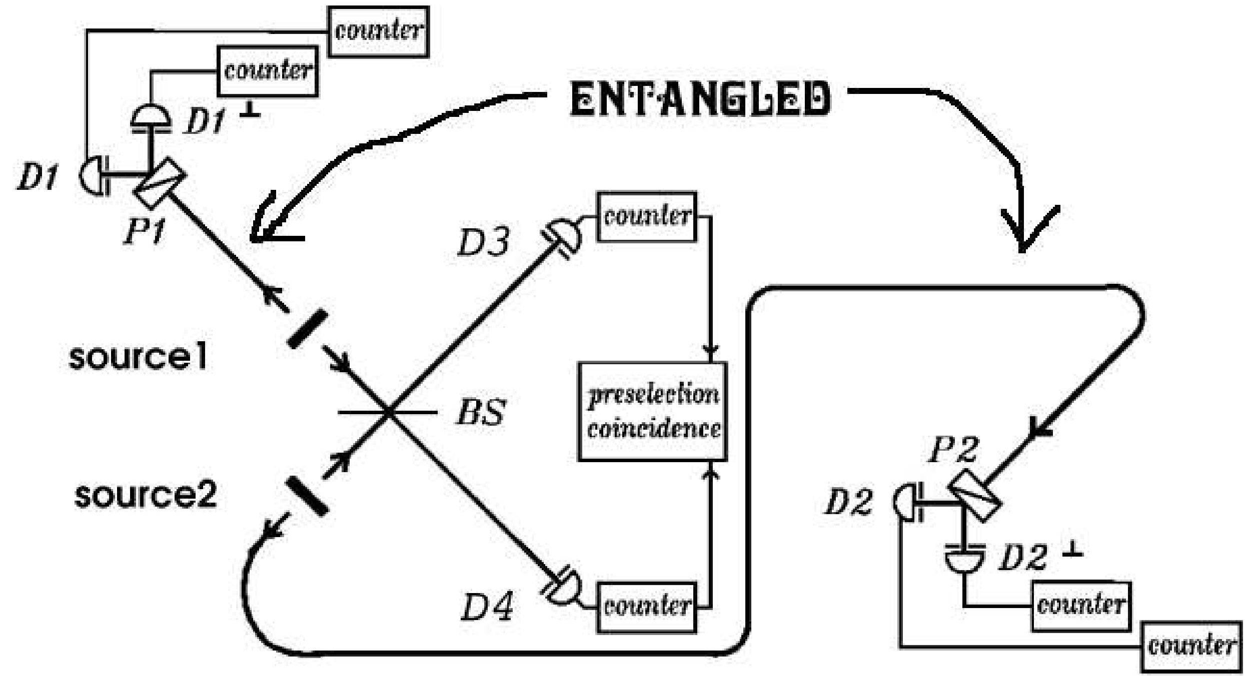

FIG. 7: Figure taken from Pavičić (1995): “In the experiment two photons from two singlets interfere at a beam splitter, and as a result the other two photons—which nowhere interacted and whose paths nowhere crossed—exhibit a 100% correlation in polarisation, even when no polarisation has been measured in the first two photons.” [17]

and this is what we called entanglement of photons 1 and 2: “two photons from two singlets interfere at a beam splitter, and as a result the other two photons—which nowhere interacted and whose paths nowhere crossed—exhibit a 100% correlation in polarisation, even when no polarisation has been measured in the first two photons: [17]

$$P(\theta_1,\theta_2,\infty,\infty) = \frac{1}{8}\sin^2(\theta_1-\theta_2) \tag{6}$$

The latter result is presented in Fig. 7 and verified experimentally [18].

“The probability given by Eq. (5) and describing coincidence detections by D1 and D2 corresponds — when multiplied by 4 — to the following singlet state:

$$|\Psi_s\rangle = \frac{1}{\sqrt{2}}(|1_x\rangle_1|1_y\rangle_2 - |1_y\rangle_1\,|1_x\rangle_2)\,. \tag{7}$$

Multiplication by 4 is for photons that emerge from the same side of BS and which therefore do not belong to our statistics. Analogously, the probability of coincidental detection by D1 and D2$^\perp$:

$$P(\theta_1,\theta_2^\perp,\infty,\infty) = \frac{1}{8}\cos^2(\theta_1-\theta_2)\,. \tag{8}$$

corresponds to the following triplet–like state:

$$|\Psi_t\rangle = \frac{1}{\sqrt{2}}(|1_x\rangle_1|1_x\rangle_2 + |1_y\rangle_1\,|1_y\rangle_2)\,. \tag{9}$$

Thus, photons 1,2 belonging to quadruples containing photons 3,4 which appear at different sides of the beam splitter behave *quantum–like* showing — according to Eq. (7) — 100% *relative modulation*. In other words, by detecting the photons 3 and 4 on different sides of the beam splitter we *preselect* the orthogonal individual photon 1 and 2 pairs (25% of all pairs) with probability one, while by detecting both photons 3 and 4 on one side of the beam splitter we preselect the parallel pairs (75% of all pairs) with probability 1/3.”[17]

We see that Eq. (9) is actually Eq. (3) with $|1_x\rangle_1|1_x\rangle_2 = |0\rangle|0\rangle$ and $|1_y\rangle_1|1_y\rangle_2 = |1\rangle|1\rangle$. Note that to obtain Eq. (9) we had to multiply the corresponding substate of the overall system by 4 to get it out of the statistics of the whole system given by Eq. (8). And while for measurements corresponding to Eqs. (7) and (9) we do have entanglement, for other measurements in the considered set-up we do not have entanglements. E.g., the overall probability of detecting both photons 3,4 in one arm of BS and detecting photons 1,2 by D1 and D2 is given by:

$$P(\theta_1,\theta_2,\theta_3\times\theta_4) = \langle\Psi|\hat{E}_1^\dagger\hat{E}_2^\dagger\hat{E}_3^\dagger\hat{E}_3^\dagger\hat{E}_3\hat{E}_3\hat{E}_2\hat{E}_1|\Psi\rangle = \frac{1}{16}\left[\cos(\theta_1-\theta_3)\cos(\theta_2-\theta_3)+\cos(\theta_1-\theta_4)\cos(\theta_2-\theta_4)\right]^2.$$

which for removed polarisers P3 and P4 reads:

$$P(\theta_1,\theta_2,\infty\times\infty) = \frac{1}{8}[1+\cos^2(\theta_1-\theta_2)]\,.$$

We can also see that by removing one of the polarisers P1 and P2, say P2, we lose any left–right (Bell–like) spin correlation completely: $P(\theta_1,\infty,\theta_3,\theta_4)=\frac{1}{8}\sin^2(\theta_3-\theta_4)$, $P(\theta_1,\infty,\infty,\infty)=\frac{1}{4}$, $P(\theta_1,\infty,\infty\times\infty)=\frac{1}{4}$. [17]

Hence, the entanglement is just a property of some subsystems of the whole composite system under a particular measurement arrangement. This entaglement is also almost synonymous to *teleportation*. To see this let us look at the source 1 in Figs. 6 and 7. (Sources 1 and 2 are simultaneously triggered by a common pumping laser beam.) Photons coming out of it are in the singlet state and therefore their polarisations are completely unprepared but correlated. With such an unprepared polarisation, one of the photons from the source 1 come to the beam splitter, interferes at it with another photon coming from the source 2, loses its polarisation and "teleports" it to the second photon from the source 2, i.e., to the photon 2. What does this mean? It means that by measuring polarisation of the photon 2 we recover the polarisation of the photon coming from the source 1 to the beam splitter. How do we know this? By measuring polarisation of photon 1 by detector D1. (Since the photons coming out of the source 1 are in the singlet state, measuring of the polarisation of the photon 1 answers the question which polarisation the other photon coming out of the source 1 should have had if it had been directly measured.) This has also been verified experimentally. [19] The experiment actually confirms Eq. (6). So, both entanglement and teleportation are about engineering particular subsystems with particular properties corresponding to just some parts of a complete mathematical description of the complete composite system.

D. Kochen-Specker Vectors

In entanglement we make a tensor product state for the whole composite system and then extract just a part of the state. In a similar way we can approach other problems. For example, the problem of finding arbitrary vectors satisfying the Kochen-Specker theorem. It says that one can find such arrangement of either Hilbert subspace vectors or of measuring experimental devices that would detect outputs of quantum systems but would not of classical. The problem boils down to the orthogonality of the one dimensional subspaces. This means that the relevant vectors are included in the span of the other one dimensional subspaces of the space, i.e., that the vectors orthogonal to each other build a whole Hilbert space in such a way that its classical interpretation is not possible. Under "classical interpretation" we mean ascribing the vectors some definite values, e.g., 0 and 1. We have chosen the Kochen-Specker problem because of recently proposed experimental tests of Kochen-Specker theorem [20, 21] and because of recent disputes on feasibility of such experiments [22–26].

The original Kochen-Specker theorem [27] produced a set of 117 3-dimensional Hilbert space vectors for which there is no way to assign 1's and 0's to their states and therefore no way to provide quantum space with a classical Boolean model. The proof was tedious and subsequent attempts to reduce the number of vectors gave the following minimal results: 33 [28] and 31 [29, p. 114] 3-dim vectors, 18 [30] and 14 [31] 4-dim vectors, 29, 31, and 34 5-dim, 6-dim, and 7-dim vectors, respectively [32], 36 8-dim vectors [33], etc. Reducing the number of vectors turn out to be important because a direct connection between such vectors and an experimental setup can be established. [32] However, no general method for constructing sets of Kochen-Specker vectors has been proposed so far. Recently we found one.

The main idea of our approach is to first show that for particular set of orthogonal Hilbert space vectors one can impose no 0-1 state on the vectors. However, we do that using the Hilbert space orthogonality: $a\le b\cup c\cup\ldots$, not the standard one: $(a,b)=0$, $(a,c)=0$, …, which boils down to a non-linear system: $a_1b_1+a_2b_2+a_3b_3+\ldots=0$, $a_1c_1+a_2c_2+a_3c_3+\ldots=0$, … But even this Hilbert orthogonality we do not "calculate"—it is "built in" the MMP diagrams defined below by its generation algorithm. We only check whether one can or cannot impose classical 0-1 state on the diagrams. We then only have to find the one which does not allow such a state and this is done by a simple program which follows the definition of the classical state.

MMP diagrams are diagrams which are organised as connected blocks of mutually orthogonal vectors. MMP diagrams are defined as follows:

1. Every vertex (i.e., atom when a diagram corresponds to a lattice) belongs to at least one block;

2. If there are at least two vertices then every block is at least 2-element;
3. Every block which intersects with another block is at least 3-element;

and then generated by the the isomorph-free generation procedure according to the following algorithm [34]:

procedure scan (D: diagram; β: integer)

  if D has exactly β blocks **then**
    output D
  else
    for each equivalence class of extensions $D + e$ **do**
      if $e \in m(D + e)$ **then** scan($D + e$,β)

end procedure

Without the latter algorithm MMP diagrams would remind us of Greechie diagrams [35] with one of the conditions dropped. The isomorph-free generation procedure is what make them very different. Greechie diagrams are a handy way to draw Hasse diagrams but Hasse diagrams get more and more intrinsically complicated when we enlarge the number of atoms. E.g., a four-atom Greechie block has 16 elements, a five-atom Greechie block has 32 elements, and an n-atom Greechie block has 2^n elements, so they soon become intractable. MMP diagrams are however just strings. A five vertex block has 5 elements, an n vertex block has n elements.

Depending on parameters we use in their generation (parameters appear as options in our programs) MMP diagrams can be represented as lattices, but also as partially ordered sets, or as vectors from a Hilbert space which do not form a lattice; they can even be used for representing relations between vectors, planes, and subspaces of any n-dim space in classical physics. Which diagram will be appropriate for which purpose is determined by a selection procedure we use once they are generated.

So, the 3 simple aforementioned conditions imposed on diagrams gives us all we need to get all finite subspaces of a Hilbert space of arbitrary complexity: we just eliminate diagrams in which Hilbert lattice properties do not hold. We currently use programs which generate and use lattices with up to 100 atoms but for all results we have obtained so far, 15 to 28 atoms suffice. The results we obtained in dealing with Kochen-Specker vectors we are going to present in forthcoming publications.

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