IN SEARCH FOR NEW OBSERVABLES

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A non–counterfactual proof of the Bell result for only two fixed positions of detectors is given. It is also shown that the only way to ascribe an individual quantum system a measured observable is to *postulate* this. A new type of observables is conjectured.

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If we disregard the *low efficiency of detectors* objection to the Bell–type experiments, only one loophole remains in the *locality conundrum* and this is a possibility that measuring arrangement causes changes in the distribution function $f(\alpha)$ of the hidden variables for composite systems for different orientations of detectors [1,2]. In order to match the latter objection I recently gave a plausibly general proof of the nonlocality of assumed hidden variables which involves joint measurements carried out on the subsystems by detectors oriented along only two fixed positions [3].

The statement proved reads as follows: "If the measured properties of the subsystems of a composite quantum system are prepared by *nature* in the same way in which *we* prepare them by our devices (polarizers, Stern–Gerlach devices, etc.), then $f(\alpha)$ must be at least singular nonlocal function (in order to enable a hidden–variable theory to give the same result as quantum mechanics) which cannot be considered affected by the measuring process."

The formulation is somewhat less general than the usual assumption about individual YES–NO predetermined, non–quantum, experimental outcomes but nevertheless boils down to the well–known con-

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clusion that, if one wanted to formulate a local hidden–variable theory which presupposes the distribution function is affected by the measuring process, one would have to start with the assumption that nature prepares quantum systems in a different way than we do.

Thus, even non–counterfactually, the quantum formalism cannot ascribe *local* observables to individual quantum subsystems. Well, this is only to be expected, but can quantum formalism ascribe *any* observable to an individual quantum system? Or only to ensemble? As shown in Ref. [4], *a priori* to neither. We have to *postulate* either one or the other option.

The only way to connect observables from the quantum formalism with properties of individual systems is by means of repeatable measurements of discrete observables. Both, continuous observables and discrete observables in the presence of a conservation law allow a value from their spectra to be a result of a measurement, but the value cannot be ascribed to a particular property of the measured system, since the measurement cannot be repeatable. As opposed to this situation, when individual systems are subjected to YES–NO measurements of a discrete observable (unrestricted by any conservation law), the eigenvalue of the measured observable (projector) *can*, though *not necessarily*, be taken to correspond to a particular property of the ensemble of individual systems. *Can* — if we *postulate* so; *not necessarily* — if we *postulate* the other way round.

Thus, for repeated YES–NO measurements of an unrestricted discrete observable, a YESevent occurs with certainty, i.e., with probability equal to unity, and we can take a view that such a YESevent *always* occurs. An individual event is then considered to possess a particular property.

Another possiblity is to assume that a YESevent with probability equal to unity need not always occur. In this case the repeatability is a bare statistical concept. It is not admitted for individual events.

To formulate a difference between the two views within the quantum formalism, in Ref. [4] an expression is constructed which is a function of the relative frequency of measured data as well as of the corresponding theoretical probability and which has a well defined physical meaning. The result achieved in the reference and generalized in [5] is that the assumption of the repeatability of the YES–NO measurements carried out on individual systems implies an actual jump in the value of a well–defined function for just one mathematical point of

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an interval. The assumption of only statistical repeatability does not imply such a jump. The latter allows the continuity of the function. The crucial point here is that we cannot have both at the same time. (Thus, a possible statistical interpretation of quantum measurements does not include the individual interpretation but is its rival.)

The result prompts the consideration of an *a priori* axiomatic calculus underlying quantum mechanics which would directly correspond neither to individual systems nor to ensembles. It could, however, serve as a basis for a more general theory which would include the Hilbert space one as a special case. Observables emerging from such a calculus need not be only the ones already known, and therefore it is important to show that quantum logic (being a calculus underlying the present quantum formalism) can be formulated without establishing any direct correspondence between its elements and measured observables, i.e., as a proper logical system. This is done in Refs. [5,6], where *unified quantum logic* is formulated as a proper implicational logical system based on the *merged* operation of implication. (The system is further simplified in [7]. It can also be proved that a quantum logic in which any two of the five possible expressions of the merged implication coincide is classical.)

The quantum axiomatic calculus obtained can then be enriched by an additional axiom (whose special case is the Arguesian law) so as to give a propositional quantum calculus of the first order. For this calculus there is a probabilistic semantics with the existence of a probability function ensured. It might be that unphysical axioms of the second order (e.g., the covering property) should follow as theorems in such a system. It can also be conjectured that the probability function (measure) impose restrictions on particular elements of a "real logic" so as to make them elements of the afore–mentioned "Arguesian" quantum logic. The latter logic need not necessarily be a logic of quantum measurements but only a calculus underlying at the same time the present quantum Hilbert–space formalism and a more general one.

POSTSCRIPT

At the present Conference, M. A. B. Whitaker and I discussed a *critique* of my distinction between individual systems and ensembles which he wrote together with D. Home [9].

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Since our discussion clarified some points of mine, I would like to stress them here.

The authors are quite right when claiming that the random bounded stochastic variable L which I used to define the afore–mentioned expression of the relative frequency and probability in [5] need not necessarily be non–zero for the probability equal to one. My point, however, is *not* that one can *prove* that there is such $0 < L < \infty$ defined on the whole [0,1] interval in addition to the open interval (0,1), i.e., that one can *prove* that there is a formal difference — *within quantum formalism*. My point is that one *can define* a function (by means of L which is bounded on (0,1) — no matter whether one can *prove* the existence of such an L for the end points of the interval or not) which would give a *formally* different description of individual system, as opposed to an exclusively statistical description of the ensemble of systems.

In other words, if one wants to have a consistent description of an individual system, one has to *postulate* this, which I have repeatedly stressed in my papers [7,8].

Therefore, I also must dismiss the claim of the authors that "I attempt to show that the relative frequency of occurence cannot be unity," i.e., in effect that a description of individual quantum system cannot be provided. On the contrary, I firmly believe that it is such a disproof which cannot be provided within the Hilbert space formalism.

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