Spin-correlated interferometry with beam splitters: preselection of spin-correlated photons

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A nonclassical feature of the fourth-order interference at a beam splitter, that genuine photon spin singlets are emitted in predetermined directions even when incident photons are unpolarized, has been used in a proposal for an experiment that imposes a quantum spin correlation on truly independent photons. In the experiment two photons from two such singlets interfere at a beam splitter, and as a result the other two photons—which nowhere interacted and whose paths nowhere crossed—exhibit a 100% correlation in polarization, even when no polarization has been measured in the first two photons. The proposed experiment permits closure of the remaining loopholes in the Bell theorem proof, reveals the quantum nonlocality as a property of selection, and pioneers an experimental procedure for the exact preparation of unequal superposition.

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1. INTRODUCTION

The fourth-order interference of photons has been given growing attention in the literature in the past few years mostly because it has provided several rather unexpected results that differ from the classical intensity-interference counterparts[1–23]: for example, downconverted induced coherence[18]; the nondependence of the interference on the relative intensity of the incoming beams[12]; a disproof[1,15] of Dirac's dictum, "Interference between two different photons never occurs"[24]; interference of photons of different colors[5]; entanglement of photons that did not in any way directly interact in the configuration space[20] or in the spin space[21,23]; and particularly successful testing of both local[7,9,13] and nonlocal[17,18] hidden-variable theories.

In this paper I close the no-enhancement and low-efficiency loopholes that weaken the Bell theorem proof, show that quantum nonlocality is essentially a property of selection, and establish a procedure for recording unequal superpositions without loss of detection counts. In accomplishing these objectives I rely on the spin features of the fourth-order interference at a beam splitter, which Summhammer and I previously used for the entanglement of two photon pairs coming from two cascade sources.[21,23] In the interference both initially polarized and initially unpolarized incident photons emerge from two different sides of the beam splitter unpolarized but correlated (Section 2). This enables me to devise an experiment in which two photons from two such singlet states interfere in the fourth order at a third beam splitter, and as a result two other companion photons from each pair turn out to be entangled and correlated in polarization, even when polarization is not measured for the first two photons at all. In Section 3 I elaborate the theory of such an entanglement, and in Section 4 I present the experiment with a realistic approach in which I discuss the spatial visibility of the correlations.

Correlated photons emerge from cascading atoms in all directions that are allowed by such a three-body process, and by registering only those pairs that reach detectors one actually selects a subset of all correlated photons; thus one can raise doubts as to whether the selected set properly represents the whole set. An affirmative assumption, known as the no-enhancement assumption, has been widely adopted since Clauser and Horne[25] first used it. Recently however, Santos[26] pointed out the problem and calculated that no experiment carried out on photons born in a cascade process can confirm the no-enhancement assumption. As opposed to this situation, photons coming from a beam splitter build spin- (polarization-) correlated pairs only in particular precisely determined directions; on the other hand, it was believed that such setups force the experimentalist to discard more than 50% of the data because detectors cannot tell one photon from two, and the experimentalist has to rely only on coincidence counts. However, I show that one can devise an experiment in which none of the data need be discarded, thus avoiding Santos's objection. I do this in Section 4 by describing a device for preselecting spin-directed correlated photons from among those photons that have not in any way directly interacted with one another. The experiment can close all the existing loopholes in disproofs of local hidden-variable theories, including the low-efficiency one, thanks to preselection of photons, and may provide a scheme for disproving nonlocal theories as well. In closing the low-efficiency loophole I show how to prepare and measure unequal superpositions exactly; this method is presented at the end of Section 4.

2. SPIN-CORRELATED INTERFEROMETRY WITH A BEAM SPLITTER

The experimental design rests on the fact that under particular conditions the fourth-order interference makes unpolarized and independent incident photons correlated in polarization (spin) and changes polarized incident photons

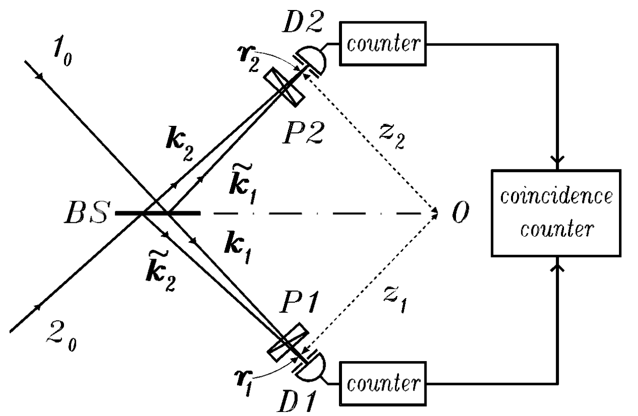

Fig. 1. Beam splitter. D's, detectors; P's, birefringent polarizers; BS, beam splitter.

into unpolarized ones. This property was recognized only recently[22] because, although the fourth-order interference in configuration space has been elaborated in detail in the literature,[1–5,7–18] the fourth-order interference lacked a detailed elaboration and apparently a proper understanding in spin space. One of the rare partial elaborations was provided by Ou *et al.*[27] for a special case of orthogonally polarized photons. They clearly recognized that orthogonally polarized photons coming into a symmetrically positioned beam splitter produce a singletlike state at a beam splitter[2,7,8,27] and that parallelly polarized photons coming into a symmetrically positioned beam splitter never appear on opposite sides of the beam splitter,[28] but it does not seem to have been recognized that the polarization of incoming photons has no effect on the correlation in polarization of the outgoing photons and that it affects only the intensity of the photons emerging from opposite sides of the beam splitter. In what follows I describe the spin of the fourth-order interference at a beam splitter by using some results obtained previously.[22]

Let two photons interfere at a beam splitter as shown in Fig. 1. First, I describe the interference of polarized and later of unpolarized photons. The state of incoming polarized photons is given by the product of two prepared linear polarization states:

$$|\Psi\rangle = (\cos\ \theta_{1_0}|1_x\rangle_{1_0} + \sin\ \theta_{1_0}|1_y\rangle_{1_0}) \otimes (\cos\ \theta_{2_0}|1_x\rangle_{2_0} + \sin\ \theta_{2_0}|1_y\rangle_{2_0}), \tag{1}$$

where $|1_x\rangle$ and $|1_y\rangle$ denote the mutually orthogonal photon states. So, e.g., $|1_x\rangle_{1_0}$ denotes the upper incoming photon polarized in direction x. If the beam splitter were removed, this photon would cause a click at detector D1 and no click at detector $D1^{\perp}$, provided that birefringent polarizer P1 is oriented along x. Here $D1^{\perp}$ means a detector counting photons coming out at the other exit of birefringent prism P1. Angles θ_{1_0} and θ_{2_0} are the angles along which incident photons are polarized with respect to a fixed direction.

I do not consider any second-order interference because the signal and the idler photons emerging from the nonlinear crystals that are used in the experiment of Section 3 have random phases relative to each other. Thus we are left with the fourth-order interference, i.e., with two interacting photons described by two corresponding electric fields. To describe the appropriate interaction of photons with the beam splitter, polarizers, and detectors I use the second quantization formalism employed, e.g., by Paul,[1] by Ou and co-workers,[4,14,15] and by Campos *et al.*[16]

Let us introduce polarization by means of the stationary-electric-field operator whose orthogonal components read as (see Fig. 1)

$$\hat{E}_j(\mathbf{r}_j, t) = \hat{a}_j(\omega_j)\exp(i\mathbf{k}_j \cdot \mathbf{r}_j - i\omega_j t). \tag{2}$$

The annihilation operators describe joint actions of the polarizers, the beam splitter, and the detectors. The operators act on the states as follows: $\hat{a}_{1x}|1_x\rangle_1 = |0_x\rangle_1$, $\hat{a}_{1x}{}^{\dagger}|0_x\rangle_1 = |1_x\rangle_1$, $\hat{a}_{1x}|0_x\rangle_1 = 0$, etc. Thus, the action of polarizers P1 and P2 and detectors D1 and D2 can be expressed as

$$\hat{a}_i = \hat{a}_{ix\,\mathrm{out}} \cos\,\theta_i + \hat{a}_{iy\,\mathrm{out}} \sin\,\theta_i\,, \tag{3}$$

where $i = 1, 2$.

The operators corresponding to the other choices of detectors are obtained accordingly. For example, the action of polarizer P2 and the corresponding detector $D2^{\perp}$ (as shown in Fig. 2) is described by

$$\hat{a}_2 = -\hat{a}_{2x\,\mathrm{out}} \sin\,\theta_2 + \hat{a}_{2y\,\mathrm{out}} \cos\,\theta_2\,. \tag{4}$$

The outgoing electric-field operators describing photons that pass through beam splitter BS and polarizers P1 and P2 and are detected by detectors D1 and D2 will thus read as

$$\hat{E}_1 = (\hat{a}_{ix}t_x \cos\,\theta_1 + \hat{a}_{1y}t_y \sin\,\theta_1)\exp(i\mathbf{k}_1 \cdot \mathbf{r}_1 - i\omega_1 t_1) + i(\hat{a}_{2x}r_x \cos\,\theta_1 + \hat{a}_{2y}r_y \sin\,\theta_1) \times \exp(i\tilde{\mathbf{k}}_2 \cdot \mathbf{r}_1 - i\omega_2 t_1)\,, \tag{5}$$

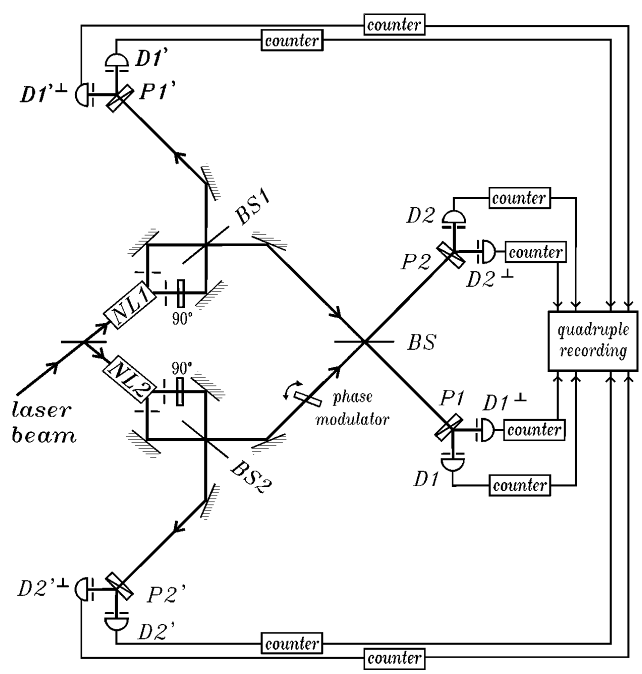

Fig. 2. Schematic of the experiment. NL1, NL2, crystals.

$$\hat{E}_2 = (\hat{a}_{2x}t_x \cos\theta_2 + \hat{a}_{2y}t_y \sin\theta_2)\exp(i\mathbf{k}_2\cdot\mathbf{r}_2 - i\omega_2 t_2) + i(\hat{a}_{1x}r_x \cos\theta_2 + \hat{a}_{1y}r_y \sin\theta_2) \times \exp(i\tilde{\mathbf{k}}_1\cdot\mathbf{r}_2 - i\omega_1 t_2)\,, \quad (6)$$

where i supplies the phase shift during reflection on the beam splitter, t_j is the time of detection of a photon by detector D_j, ω_j is the frequency of photon j, and c is the velocity of light. Here the crystal as a supposed source of (idler and signal downconverted) photons is assumed to be positioned symmetrically with respect to the beam splitter (with respect to the photon paths from the center of the crystal to the beam splitter). This is just the opposite of the elaboration in Ref. 22, where detectors were assumed to be positioned symmetrically to the beam splitter, and time delays for the sources were introduced to describe photons born in the atomic cascade processes used in Ref. 23.

The joint interaction of both photons with the beam splitter, polarizers P1 and P2, and detectors D1 and D2 is given by a projection of the wave function onto the Fock vacuum space by means of $\hat{E}_1$ and $\hat{E}_2$, which yields the following probability of the photon's being detected in coincidence[22] by D1 and D2:

$$P(\theta_{1_0}, \theta_{2_0}, \theta_1, \theta_2) = \langle\Psi|\hat{E}_2{}^\dagger\hat{E}_1{}^\dagger\hat{E}_1\hat{E}_2|\Psi\rangle = A^2 + B^2 - 2AB\cos\phi\,, \quad (7)$$

where $|\Psi\rangle$ is given by Eq. (1) and

$$\phi = (\tilde{\mathbf{k}}_2 - \mathbf{k}_1)\cdot\mathbf{r}_1 + (\tilde{\mathbf{k}}_1 - \mathbf{k}_2)\cdot\mathbf{r}_2 + (\omega_1 - \omega_2)(t_1 - t_2)\,, \quad (8)$$

$A = S_{1'1}(t)S_{2'2}(t)$, and $B = S_{1'2}(r)S_{2'1}(r)$; here

$$S_{ij} = s_x \cos\theta_i \cos\theta_j + s_y \sin\theta_i \sin\theta_j\,. \quad (9)$$

Assuming $\omega_1 = \omega_2$, we obtain (see Fig. 1) $\phi = 2\pi(z_2 - z_1)/L$, where L is the spacing of the interference fringes.[2]

For $t_x = t_y = r_x = r_y = 2^{-1/2}$ and $\cos\phi = 1$ (we can modify ϕ by moving the detectors transversely to the incident beams) the probability reads as

$$P(\theta_{1_0}, \theta_{2_0}, \theta_1, \theta_2) = (A - B)^2 = 1/4\, \sin^2(\theta_{1_0} - \theta_{2_0})\sin^2(\theta_1 - \theta_2)\,, \quad (10)$$

which with the polarizers removed yields

$$P(\theta_{1_0}, \theta_{2_0}, \infty, \infty) = 1/2\, \sin^2(\theta_{1_0} - \theta_{2_0})\,. \quad (11)$$

We can see that the probability in Eq. (10) factors (see Fig. 1) left–right (corresponding to 1_0–2_0 preparation ↔ D1–D2 detections) and not up–down (corresponding to $\frac{1_0}{2_0}$ ↕ preparation) in spite of the up–down initial independence described by the product of the upper and the lower functions in Eq. (1). We can also see that changing the relative angle between the polarization planes of the incoming photons changes only the light intensity of the photons emerging from the beam splitter at particular sides. Thus the photons either emerge on two different sides of the beam splitter, correlated according to Eq. (10), or both emerge on one side according (when we do not measure their outgoing polarization) to the following overall probability:

$$P(\theta_{1_0}, \theta_{2_0}, \infty, \infty) = 1/2[1 + \cos^2(\theta_{1_0} - \theta_{2_0})]\,, \quad (12)$$

which together with Eq. (11) adds up to 1.

We can also see that the photon beams leave the beam splitter unpolarized:

$$P(\theta_{1_0}, \theta_{2_0}, \theta_1, \infty) = 1/4\, \sin^2(\theta_{1_0} - \theta_{2_0})\,. \quad (13)$$

If both incoming photons arrive unpolarized—coming, e.g., from two simultaneously cascading independent atoms or, better, from two other beam splitters, a possibility that follows directly from Eq. (13)—then they appear[22] correlated whenever they emerge from the opposite sides of the beam splitter,

$$P(\infty, \infty, \theta_1, \theta_2) = 1/8\, \sin^2(\theta_1 - \theta_2)\,, \quad (14)$$

and partially correlated whenever they both emerge from the same side of the beam splitter,

$$P(\infty, \infty, \theta_1, \theta_2) = 1/8[1 + \cos^2(\theta_1 - \theta_2)]\,. \quad (15)$$

The latter probability can be tested experimentally with the help of an additional beam splitter in each arm, following Rarity and Tapster,[29] or by means of photons of different colors, which one can distinguish with frequency filters (prisms).[5,30,31]

In the case of nondegenerate idler and signal downconverted photons (produced by means of asymmetrically positioned pinholes), i.e., in the case of photons of different colors, we should, according to Eq. (8), obtain a space–time combination of spacelike intensity interference and timelike frequency-difference beating. The latter effect, however, cannot be measured simultaneously with observation of the intensity interference fringes because the fast photon beating would wipe out the spatial fringes. For observation of the beating itself one uses the optical-path-length-difference method, by which the coincidences are recorded.[5,30] Thus in the present notation we can simply drop the dot products in Eq. (8), and then the method consists of moving the beam splitter up or down to yield the optical path-length difference $\delta = c|t_1 - t_2|$, and thus $|\phi| = |\omega_1 - \omega_2|\delta/c$. In this way one can register beating corresponding to 30 fs by means of detectors and counters whose resolving time is 10 ns.[5] The main coincidence probability for particular polarization measurements given by Eq. (7) remains the same for the beating between photons of different frequencies as it was for the degenerate idler and signal photons. The fact that one can trace the path of each photon is not contradictory here because, first, we are dealing not with the beam intensity but with the intensity correlation, and, second, as I have already stressed, the polarization preparation of photons is erased by the beam splitter anyway.

For the experiment the most important consequence of the obtained equations is that the photons appear entangled in a singlet state whenever they appear on different sides of the beam splitter provided that the condition $\phi = 0$ is satisfied, no matter whether the incident photons were polarized, for Eqs. (10) and (14) tell us that the probability of such photons passing parallel polarizers is equal to zero.

3. THEORY OF ENTANGLEMENT IN THE EXPERIMENT

A schematic representation of the experiment is shown in Fig. 2. Two independent beam splitters, BS1 and BS2, act as two independent sources of two independent singlet pairs, which is justified by Eq. (10) as elaborated in Section 2. Two photons from each pair interfere at beam splitter BS, and as a result the other two photons, under the particular conditions elaborated below, appear to be in the singlet state, although they are completely independent and have nowhere interacted.

An ultrashort[32] laser beam (a subpicosecond one) of frequency ω_0 (split by a beam splitter) simultaneously pumps up two nonlinear crystals NL1 and NL2, producing in each of them pairs of signal and idler photons (simultaneously and with equal probability) of frequencies ω_1 and ω_2, respectively, which satisfy the following energy and momentum conservation conditions: $\omega_0 = \omega_1 + \omega_2$ and $\mathbf{k}_0 = \mathbf{k}_1 + \mathbf{k}_2$.[33] By means of the appropriately symmetrically positioned pinholes, half-frequency sidebands are selected so as to have $\omega_2 = \omega_1$. The idler and the signal photon pairs coming out from the crystals do not have definite phases[28,34] with respect to each other, and consequently one can have a second-order interference at neither BS1 nor BS2. To prevent any coherence that might be induced by the split pumping beam between the idler (or signal) photon from the first crystal and the idler (or signal) photon from the second crystal, I introduce a phase modulator (which rotates to and fro at random and destroys the second-order phase coherence); this follows Ou *et al.*[6] (I do take a correction term corresponding to the modulator into account when estimating the visibility below but do not show it in the equations for the sake of simplicity.)

Thus two sources, BS1 and BS2, both simultaneously emit two photons to the left and to the right in the singlet states given by Eq. (10). But, before beam splitters BS1 and BS2 are put into place, beam splitter BS and detectors D1, D1$^{\perp}$, D2, and D2$^{\perp}$ must be adjusted to yield $\phi = 0$. After that BS is removed, BS1 and BS2 are put into place to be adjusted, and detectors D1$'$, D1$'^{\perp}$, D2$'$, and D2$'^{\perp}$ are adjusted (while the other detectors remain fixed) to yield pure singlet states emerging from BS1 and BS2. It follows from Eq. (10) and Fig. 2 that one can do this for $\phi = 0$ by reaching the minimum of coincidences (ideally the minimum should be zero) for $\theta_{1'} = \theta_1$ for BS1 and for $\theta_{2'} = \theta_2$ for BS2. It is interesting that this step of tuning beam splitters BS1 and BS2 and detectors D1$'$, D1$'^{\perp}$, D2$'$, and D2$'^{\perp}$ is not crucial, because the four-photon entanglement is not dependent on the positions of these detectors in directions perpendicular to the photon paths; i.e., according to Eq. (21) there are interference fringes not for photons 1$'$ and 2$'$, but only for photons 1 and 2. Then beam splitter BS is put into place, and four photons form elementary quadruples of counts, which in the long run add up to the probabilities calculated below. The quadruple recording is obtained by the following preselection procedure: Whenever exactly two of the preselection detectors (D1, D1$^{\perp}$, D2, and D2$^{\perp}$) fire in coincidence (see Fig. 2), a gate for preselected counters D1$'$, D1$'^{\perp}$, D2$'$, and D2$'^{\perp}$ opens. (The gate is shown in Fig. 3 for a special case with polarizers P1 and P2 removed and is considered to be part of the quadruple recording box in Fig. 2.) When only one or none of the so preselected counters fires, the records are discarded (because they correspond to four or three photons detected by the preselection detectors). When exactly two of the four preselected counters fire, the corresponding counts contribute to our statistics. The possibility of two photons going into one arm of the beam splitter and the possibility that a detector fails to react because of its inefficiency are discusssed in Section 4.

The state of the four photons immediately after they leave BS1 and BS2 from oppposite sides is described by the product of the two superpositions corresponding to singlet pairs produced [according to Eq. (10)] at BS1 and BS2:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|1_x\rangle_{1'}|1_y\rangle_1 - |1_y\rangle_{1'}|1_x\rangle_1) \otimes \frac{1}{\sqrt{2}}(|1_x\rangle_{2'}|1_y\rangle_2 - |1_y\rangle_{2'}|1_x\rangle_2), \tag{16}$$

where $|1_x\rangle$ and $|1_y\rangle$ denote the mutually orthogonal photon states.

The annihilation of photons at detectors D1$'$ and D2$'$ after the photons pass polarizers P1$'$ and P2$'$ (oriented at angles $\theta_{1'}$ and $\theta_{2'}$) are described by the following electric-field operators:

$$\hat{E}_{1'} = (\hat{a}_{1'x}\cos\theta_{1'} + \hat{a}_{1'y}\sin\theta_{1'})\exp(-i\omega_1{}'t_{1'}), \tag{17}$$

$$\hat{E}_{2'} = (\hat{a}_{2'x}\cos\theta_{2'} + \hat{a}_{2'y}\sin\theta_{2'})\exp(-i\omega_2{}'t_{2'}). \tag{18}$$

Here phases of the photons that accumulate between beam splitters BS1 and BS2 and detectors D1$'$ and D2$'$ add the factors $\exp(-i\omega_j t_j)$, where t_j is the time of detection of a photon by detector Dj' and ω_j is the frequency of the photon. [The frequencies of photons are considered to be different for the sake of generality until we reach Eq. (21).]

The outgoing electric-field operators describing photons that pass through beam splitter BS, polarizers P1 and P2, and detectors D1 and D2 are given by Eqs. (5) and (6).

The joint interaction of all four photons with beam splitter BS, all polarizers, and detectors D1, D2, D1$'$, and D2$'$ is given by the following projection of the initial state, given by Eq. (16), onto the Fock vacuum space:

$$\hat{E}_{1'}\hat{E}_{2'}\hat{E}_1\hat{E}_2|\Psi\rangle = {}^1\!/_2(A\epsilon_{12} - B\tilde{\epsilon}_{12})\epsilon|0\rangle, \tag{19}$$

where $|\Psi\rangle$ is given by Eq. (16), $\epsilon_{12} = \exp[i(\mathbf{k}_1\cdot\mathbf{r}_1 + \mathbf{k}_2\cdot\mathbf{r}_2 - \omega_1 t_1 - \omega_2 t_2)]$, $\tilde{\epsilon}_{12} = \exp[i(\tilde{\mathbf{k}}_1\cdot\mathbf{r}_2 + \tilde{\mathbf{k}}_2\cdot\mathbf{r}_1 - \omega_1 t_2 - \omega_2 t_1)]$, $\epsilon = \exp[-i(\omega_1{}'t_{1'} + \omega_2{}'t_{2'})]$, $A = Q(t)_{1'1}Q(t)_{2'2}$, and $B = Q(r)_{1'2}Q(r)_{2'1}$; here

$$Q(q)_{ij} = q_x \sin\theta_i \cos\theta_j - q_y \cos\theta_i \sin\theta_j. \tag{20}$$

The corresponding probability of detecting all four photons with the above combination of the detectors is thus

$$P(\theta_{1'}, \theta_{2'}, \theta_1, \theta_2) = \langle\Psi|\hat{E}_{2'}{}^\dagger\hat{E}_{1'}{}^\dagger\hat{E}_2{}^\dagger\hat{E}_1{}^\dagger\hat{E}_1\hat{E}_2\hat{E}_{1'}\hat{E}_{2'}|\Psi\rangle = {}^1\!/_4(A^2 + B^2 - 2AB\cos\phi), \tag{21}$$

where

$$\phi = (\tilde{\mathbf{k}}_2 - \mathbf{k}_1)\cdot\mathbf{r}_1 + (\tilde{\mathbf{k}}_1 - \mathbf{k}_2)\cdot\mathbf{r}_2 + (\omega_1 - \omega_2)(t_1 - t_2). \tag{22}$$

For $\omega_1 = \omega_2 = \omega_1{}' = \omega_2{}'$ we obtain (see Fig. 1; this applies to BS in Fig. 2 as well) $\phi = 2\pi(z_2 - z_1)/L$, where L is the spacing of the interference fringes. ϕ can be changed by movement of the detectors transverse to the incident beams.

To make Eq. (21) clearer, without loss of generality, here I consider a 50:50 beam splitter: $t_x = t_y = r_x = r_y = 2^{-1/2}$. In Section 4 I also consider a polarized beam splitter.

For $\phi = 0$, the above probability reads as

$$P(\theta_{1'}, \theta_{2'}, \theta_1, \theta_2) = (1/4)(A - B)^2 = (1/16)\sin^2(\theta_{1'} - \theta_{2'})\sin^2(\theta_1 - \theta_2). \quad (23)$$

We can again see that the probability factorizes left–right (corresponding to D1′–D2′ ↔ D1–D2 detections; see Fig. 2) and not up–down (corresponding to $\frac{\mathrm{BS1}}{\mathrm{BS2}} \updownarrow$ preparation), as one would be tempted to conjecture from the product of the upper and the lower function in Eq. (16). With polarizers P1 and P2 removed, Eq. (23) gives

$$P(\theta_{1'}, \theta_{2'}, \infty, \infty) = {}^1\!/_8\ \sin^2(\theta_{1'} - \theta_{2'}). \quad (24)$$

The overall probability of detecting both photons in one arm of BS is given by

$$P(\theta_{1'}, \theta_{2'}, \theta_1\theta_2) = (1/16)[\cos(\theta_{1'} - \theta_1)\cos(\theta_{2'} - \theta_2) + \cos(\theta_{1'} - \theta_2)\cos(\theta_{2'} - \theta_1)]^2, \quad (25)$$

which with the polarizers removed reads as

$$P(\theta_{1'}, \theta_{2'}, \infty\infty) = {}^1\!/_8[1 + \cos^2(\theta_{1'} - \theta_{2'})]. \quad (26)$$

The latter probability is obtained by addition of all the probabilities of detecting polarizations of each photon in one arm, i.e., $P(\theta_{1'}, \theta_{2'}, \theta_1\theta_2)$ [given by Eq. (25)], $P(\theta_{1'}, \theta_{2'}, \theta_1\theta_2{}^\perp)$, etc. We can see that probabilities (24) and (26) add up to 1/4.

Probability (23) shows that, for $\phi = 0$, by removing one of the polarizers we lose any left–right (Bell-like) spin correlation completely: $P(\theta_{1'}, \infty, \theta_1, \theta_2) = (1/16)\sin^2(\theta_1 - \theta_2)$. On the other hand, for $\phi \neq 0$ we obtain a partial left–right correlation even when two polarizers, one on each side, are removed.

4. THE EXPERIMENT AND THE BELL ISSUE

The main point of the experiment is that the correlation between photons 1′ and 2′, i.e., between photons that never interacted in the past, persists even when one does not measure polarization on their companions, photons 1 and 2, at all, as follows from Eq. (24). Therefore I concentrate on the experiment without polarizers P1 and P2 behind beam splitter BS. To make my point, I present the appropriate experimental setup in the simplified and reduced scheme presented in Fig. 3. The setup deals with four photons of the same frequency and relies on (computer) time windows for coincidence detections, which compensates for the long response time of the detectors. Afterward I consider the experiment with a more realistic approach, using polarizers P1 and P2 as shown in Fig. 2.

In the idealized approach from Section 3 the probability of detecting all four photons with detectors D1, D2, D1′, and D2′ in coincidence for a 50:50 beam splitter, for $\phi = 0$, and with equal time delays (that is, for a completely symmetric position of BS) is given by Eq. (24), and the probability of detecting both photons in one of the arms by Eq. (26). We see that these two probabilities add up to 1/4. (The other 3/4 corresponds to orthogonal detections with $D^\perp$ detectors included.) The former probability, given by Eq. (24) and describing coincidence detections by D1′ and D2′, corresponds, when multiplied by 4, to the following singlet state:

$$|\Psi_s\rangle = (1/\sqrt{2})(|1_x\rangle_1|1_y\rangle_2 - |1_y\rangle_1|1_x\rangle_2). \quad (27)$$

Multiplication by 4 is for photon pairs that emerge from the same side of BS and that therefore do not belong to our statistics. Analogously, the probability of coincidental detection by D1′ and D2′$^\perp$ (which is used below),

$$P(\theta_{1'}, \theta_{2'}{}^\perp, \infty, \infty) = {}^1\!/_8\ \cos^2(\theta_{1'} - \theta_{2'}), \quad (28)$$

corresponds to the following tripletlike state:

$$|\Psi_t\rangle = (1/\sqrt{2})(|1_x\rangle_1|1_y\rangle_2 + |1_y\rangle_1|1_x\rangle_2). \quad (29)$$

Thus photons 1′ and 2′ belonging to quadruples containing photons 1 and 2, which appear at different sides of the beam splitter, exhibit quantumlike behavior, showing, according to Eq. (24), 100% relative modulation.[7] In other words, detecting the right photons on different sides of the beam splitter preselects the orthogonal individual left photons pairs (25% of all pairs) with probability 1, whereas detecting both right photons on one side of the beam splitter would (if it were experimentally possible) preselect the parallel pairs (75% of all pairs) with probability 1/3. When we compare this result with its classical formulation[22] carried out by Paul and Wegmann,[35] we

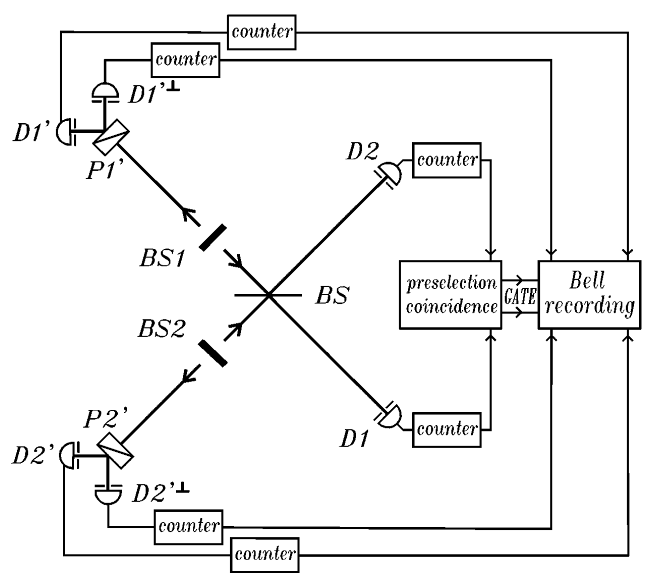

Fig. 3. Simplified schematic of the experiment.

see that the former case (photons emerging from different sides of the beam splitter) is completely nonclassical. This means that it is the nonclassical feature of the intensity correlations that makes the experiment possible.

Let us now dwell on the details of the experiment without polarizers P1 and P2 behind beam splitter BS, as shown in Fig. 3. A pair consisting of two photons $1'$ and 1 appears from BS1 simultaneously with another pair $2'$ and 2 from BS2. Photon $1'$ is directed toward detector $D1'$ or $D1'^{\perp}$, photon $2'$ is directed toward detector $D2'$ or $D2'^{\perp}$, and photons 1 and 2 are directed toward detectors D1 and D2. Of all detections registered by D1 and D2, only those counts that occur within a short enough time window (~10 ns) are fed to the preselection coincidence counter. Thanks to the ultrashort pumping beam (ω_0), which ensures an appearance of downconverted pairs of photons ($\omega = \omega_0/2$, emerging from the crystals and passing through symmetrically positioned pinholes) every 50 ns on the average, one is able effectively to control coincidences, each of which occurs (as a property of downconversion) well within our time window. In this way we overcome the problem of having a detector reaction time longer than the fourth-order correlation time and the coherence time. Thus each pair of the pulses belongs to the two photons that interfered at the BS so as to emerge at opposite sides of the beam splitter. (Realistically, as is stressed below, the visibility of this two-photon detection boils down to about 85%; I discuss the possibility below of having detected three or four photons because it is possible that both photons emerge from one side of BS1 or BS2.) Each D1–D2 time window is coupled (as calculated from the time-of-flight difference) with a computer gate for counts from detectors $D1'$, $D2'$, $D1'^{\perp}$, and $D2'^{\perp}$. If counters D1 and D2 register not coincidence counts but a only single count, then the gated $D1'$, $D2'$, $D1'^{\perp}$, and $D2'^{\perp}$ recordings are discarded. If coincidence counts are registered, then the data potentially contribute to the statistics of what I call the Bell recording in Fig. 3. Since I use birefringent polarizers, I have to have a coincidence firing of exactly two of the counters $D1'$, $D2'$, $D1'^{\perp}$, and $D2'^{\perp}$ to obtain definite data for the statistics. Firing of one or none of the counters, as well as of three or all four, eliminates the corresponding data because they do not belong to the set of quadruple events. $P(\theta_{1'}, \theta_{2'}, \infty\infty)$ of Eq. (24) is then given by the ratio between the numbers of coincidence counts,

$$f(\theta_{1'}, \theta_{2'}) = \frac{n(D1' \cap D2')}{n[(D1' \cup D1'^{\perp}) \cap (D2' \cup D2'^{\perp})]}, \qquad (30)$$

divided by 4. Division by 4 compensates for the photons that emerge from the same side of the BS and are therefore discarded from the statistics as not belonging to the considered set of events. Of course, I produce an error here because counters can remain inactive as a result of their inefficiency, but one can always use Mach–Zehnder interferometers instead of BS1 and BS2 to avoid this problem. The advantages of the interferometers would be, first, that one can adjust them so that photons almost always emerge from opposite sides of the second beam splitter and almost never from the same side and, second, that a detector resolution time that is much longer than the coherence time is no longer a problem (in contradistinction to a single beam splitter)—it is even required.[14,16] I did not use the interferometers here so as not to overcomplicate the presentation, but I comment on them in some detail below. Alternatively, one can use photons of different frequencies for each pair and rely on their beating instead on the spatial fringes as explained at the end of Section 2.

The assumed 100% visibility above is, of course, an oversimplification, since probability (21) cannot be measured at a point (see Fig. 1) but only over a detector width Δz. Therefore, to obtain a more realistic probability, following Ghosh and Mandel,[3] I integrate Eq. (21) over z_1 and z_2 over Δz to obtain

$$\begin{aligned}\mathcal{P}(\theta_{1'}, \theta_{2'}, \theta_1, \theta_2) &= 1/4 \int_{z_1-\Delta z/2}^{z_1+\Delta z/2} \int_{z_2-\Delta z/2}^{z_2+\Delta z/2} \{A^2 + B^2 - 2AB \\ &\quad \times \cos[2\pi(z_2 - z_1)/L]\} \mathrm{d}z_1 \mathrm{d}z_2 \\ &= 1/4(A^2 + B^2 - v2AB \cos\phi), \qquad (31)\end{aligned}$$

where $v = [\sin(\pi\Delta z/L)/(\pi\Delta z/L)]^2$ is the visibility of the coincidence counting. A visibility of 95% has been estimated as achievable in principle[12]; 80% and 87% were reached recently.[36,37]

Thus Eq. (24), corrected for a realistic visibility, reads as

$$P(\theta_{1'}, \theta_{2'}, \infty, \infty) = 1/8[1 - v \cos^2(\theta_{1'} - \theta_{2'})]. \qquad (32)$$

To see that the results really close all the remaining loopholes in disproving local hidden variable theories, let us end by discussing the corresponding Bell's inequality:

$$\begin{aligned}S \equiv P(\theta_{1'}, \theta_{2'}) - P(\theta_{1'}, \theta_{2'}{}') + P(\theta_{1'}{}', \theta_{2'}{}') + P(\theta_{1'}{}', \theta_{2'}) \\ - P(\theta_{1'}{}', \infty) - P(\infty, \theta_{2'}) \le 0, \qquad (33)\end{aligned}$$

where $P(\theta_{1'}, \theta_{2'}) = 4P(\theta_{1'}, \theta_{2'}, \infty, \infty)$, etc. The singlet states of photons $1'$ and $2'$ and the corresponding probabilities $1/2 \sin^2(\theta_{1'} - \theta_{2'})$ correspond to the $D1'$–$D2'$ coincidence counts preselected by D1–D2 coincidence counts. Since, ideally, none of the so preselected photons escapes detection, we have thus satisfied Santos's demand.[26] To be more specific, $P(\theta_{1'}, \theta_{2'})$ is not obtained as a coincidence-count rate as in the previous experiments[2,7,8] but as the ratio (frequency) $f(\theta_{1'}, \theta_{2'})$ given by Eq. (30), where the total number of counts in the denominator can actually be recorded.

Ideally, for a violation of Bell's inequality and hence for a possible exclusion of hidden-variable theories, v must be[8] larger than $2^{-1/2}$. If we also take into account the overall efficiency of detectors η defined by $P(\theta_{1'}, \theta_{2'}) = \eta f(\theta_{1'}, \theta_{2'})$ for the case of equal superposition given by Eq. (16), inequality (33) can be violated only if[38,39]

$$\eta(1 + v\sqrt{2}) > 2. \qquad (34)$$

So, for the visibility $v = 1$, we must have $\eta > 83\%$. For the recently achieved visibilities $v = 0.8$ (Ref. 37) and $v = 0.87$ (Ref. 36), according to Eq. (34) this implies $\eta > 0.94$ and $\eta > 0.9$, which was already announced as achievable.[40,41] Thus the experiment in the presented setup is just about to become feasible. However, using my most recent result one can adjust it so as to be comfortably over this threshold and conclusively feasible with the present technology. Let me elaborate this in some detail.

As forerunners of the singlet states selected among photons whose paths nowhere crossed in the experimental space, several simpler setups involving only two photons interfering at beam splitters were reported. In particular, the Mach–Zehnder interferometer was recognized as a possible source of 100% correlated photons (i.e., without both photons emerging from the same side of the second beam splitter).[14,16] However, until the results of Kwiat *et al.*[41] and my results[22] it was not recognized that these photons appear correlated in polarization and automatically satisfy Santos's demand up to the efficiency of the detectors. Kwiat *et al.*[41] carried out an explicit calculation for the single Mach–Zehnder interferometer and immediately addressed detector efficiency limitations and focused on a recent result obtained by Eberhard[42] as a possible remedy. (It should be stressed here that with regard to detector efficiencies Hardy's[43] proposal cannot be considered an answer to Santos's objection because the available visibility in his proposal is 30%.) Eberhard has tried to show that if one uses unequal superpositions,

$$|\Psi_r\rangle = \frac{1}{\sqrt{1+r^2}}(|1_x\rangle_1|1_y\rangle_2 + r|1_y\rangle_1|1_x\rangle_2)\,, \qquad (35)$$

instead of equal ones (given by $r = 1$), then one would be able to lower the required efficiency of detectors to 67%. (It can be shown that the efficiency minimum cannot be so low. However, it is still lower than 83%.) The problem is how to prepare $|\Psi_r\rangle$. Eberhard himself connected the effect with the background noise, and the drawbacks of this definition are, first, that one can hardly specify the background and, second, that one loses counts. I have, however, found the following way to use Eberhard's result without any losses and without invoking any background noise.

From Eqs. (21) and (4) it follows that the probability of having coincidence counts by detectors D1′ and $D2'^{\perp}$ after a selection by (see Fig. 2) detectors D1 and D2 with the polarizer orientations $\theta_2 = 0$ and $\theta_1 = \pi/2$ and with $t_y = r_y = 2^{-1/2}$ is given by

$$P(\theta_{1'}, \theta_{2'}{}^{\perp}) = \frac{1}{1+r^2}(\cos\,\theta_{1'}\,\cos\,\theta_{2'} + r\,\sin\,\theta_{1'}\,\sin\,\theta_{2'})^2\,, \qquad (36)$$

where $r = r_x/t_x$ and where counts registered by $D1'^{\perp}$ and D2′ are also taken into account to yield the proper probability. Since it can easily be shown that the detected photons are in the state described by Eq. (35), Eberhard's term r is recognized as the ratio between the reflection and the transmission coefficients of the polarized beam splitter. On the other hand, Eq. (36) establishes an experimental procedure for measuring unequal superposition without loss of detection counts, since the probability $P(\theta_{1'}, \theta_{2'}{}^{\perp})$ can be obtained as the frequency

$$f(\theta_{1'}, \theta_{2'}{}^{\perp}) = \frac{n(\mathrm{D1'} \cap \mathrm{D2'^{\perp}})}{n[(\mathrm{D1'} \cup \mathrm{D1'^{\perp}}) \cap (\mathrm{D2'} \cup \mathrm{D2'^{\perp}})]}\,, \qquad (37)$$

where both $n(\mathrm{D1'} \cap \mathrm{D2'^{\perp}})$ and $n[(\mathrm{D1'} \cup \mathrm{D1'^{\perp}}) \cap (\mathrm{D2'} \cup \mathrm{D2'^{\perp}})]$ can be recorded with equal accuracy.

This should[44] close all the remaining loopholes in the Bell's proof and constitutes a most discriminating test of Bell's inequality.

5. CONCLUSION

The proposed experiment is an application of a polarization correlation between two independent and unpolarized photons. The experiment is based on a newly discovered nonclassical effect in the fourth-order interference at a beam splitter according to which two unpolarized incident photons emerge from a beam splitter correlated in polarization, as follows from Eqs. (7) and (14). The essential new element of the experiment is that it puts together two photons from two singlets formed at two beam splitters and makes them interfere at a third beam splitter, and as a result one finds polarization correlations between the other two photons, which nowhere interacted and whose paths nowhere crossed, even when no polarization measurement was carried out on the former two photons, as follows from Eqs. (21) and (23). As for the latter two photons that nowhere interacted, one of their subsets turns out to contain only photons in the singlet state, and since one is able to extract these photons with a probability of 1, one can consider them preselected by their pair-companion photons that interfered at the beam splitter. By using birefringent prisms, one can in principle detect all the photons from the subset and obtain probability (24) as a proper frequency (a ratio of counts) given by Eq. (30). In this way we close the enhancement loophole of the Bell theorem proof. On the other hand, the experiment shows that it is not a direct interaction between photons or their common origin that entangles them in a polarization singlet state but particular correlations, which one can preselect without resorting to polarization measurement at all. We conclude that nonlocality is essentially a property of selection. This conclusion might exclude all nonlocal hidden-variable theories that rely on some kind of a physical entanglement by means of a common origin.

The realistic estimation of the experiment carried out in Section 4 for the equal superposition given by Eq. (27) shows that such a setup is just about to become feasible within the so-called (see Section 4) 83% limit, thus narrowing the second, efficiency loophole in the Bell theorem proof. It should be stressed here that a helpful feature of the considered effect is that the entanglement, quadruple firing of preselection detectors D1 and D2 behind the beam splitter and two of the second group of detectors, catching the other two free photons, is independent of the positions of the second group of detectors and also of the moment of their firing, as follows from Eqs. (21) and (31). In other words, the visibility of the whole entanglement and the visibility of the two-photon coincidence at BS practically do not differ.

To narrow the efficiency loophole, I resort to polarization measurement and unequal superposition [given by Eq. (35)], whereby one can make the experiment with an efficiency that is less than 83% by recognizing Eberhard's r term not as a measure of a background noise but as the ratio of the reflection to the transmission coefficient in one of the measured polarization directions (at a polarized beam splitter). At the same time this approach establishes what is to my knowledge the first experimental procedure for exact preparation and measurement of unequal superposition without loss of detection counts. In other words, when, with the help of partially polarized beam splitter BS and all detectors D1, $D1^{\perp}$, D2, and $D2^{\perp}$, photons are preselected in the subset of photons in the unequal tripletlike state given by Eq. (35), one does not lose counts because the detectors, by means of birefringent polarizers $P1'$ and $P2'$, register all counts, so that one can form a proper frequency, given by Eq. (37), to verify the corresponding probability, given by Eq. (36). This closes the efficiency loophole in the Bell theorem proof.

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