Two-loop calculation of the dispersion relation for soft static fermions beyond the HTL approximation

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- Motivation
- The Hard Thermal Loop (HTL) approximation
- The problem of the gauge-dependence of the dispersion relation
- A method for calculating two-loop diagrams at finite temperature in the imaginary time formalism
- Some “results”, questions and plans

Collaborators: E. Mottola (Los Alamos), M. Carrington (Brandon University)

Heavy quarks as a probe of a new state of matter
(research proposal at Los Alamos)

properties of the matter created at RHIC studied using charm and beauty heavy quarks as clean probes $\longrightarrow$ silicon micro-vertex detector with very good spatial resolution required

heavy quarks live much longer than the duration of the QGP and travel macroscopic distances away from the creation point

lifetime of charm and beauty is ~ 1ps and their decay in muons detected by the PHENIX muon detector

longer lifetime of beauty particles and higher momentum of muons from their decays $\longrightarrow$ separation of charm and beauty possible

Experimental setting used to answer pressing questions like:

– are the interactions in the plasma so strong that heavy quarks are quickly equilibrated and exhibit hydrodynamic flow?

– do heavy quark bound states dissociate in QGP at extreme pressure and temperature?

– what is the mechanism of energy loss for heavy quarks in the plasma?

The importance of the soft physics

existing hydrodynamic models assume that the fireball, which is initially far from thermodynamic equilibrium reach perfect thermalization within 1fermi/c, some 5 - 7 times faster than expected by estimates based on hard scattering processes
$\longrightarrow$ softer collective processes are necessary to account for the apparent success of the hydro models

the rapid thermalization time and short mean free path for the quasi-particle excitations indicate that the new state of matter at RHIC behaves more like a liquid rather than a gas of quarks and gluons
$\rightarrow$ computation of the transport coefficients (viscosity, conductivity and diffusion) is important
$\rightarrow$ the goal is to include non-perfect fluid therms into hydro models and compare with the experimental data on elliptic flow

particles become "dressed" when they propagate in a medium $\Longrightarrow$ studying the quasi-particles by looking at the analytical structure of the corresponding thermal propagators one can tell something about the properties of the medium

theoretically, one can try to calculate analytically properties of bound states (mass, width, decay constant) versus the temperature and compare them to those determined by a lattice QCD simulation from the spectral functions

The problem of gauge independence

this is about, "how to extract physical quantities from the expectation values of gauge variant operators" using perturbation theory

R. Kobes, G. Kunstatter, A. Rebhan, NPB355, 1 (1991)

in abelian gauge theory
– photon self-energy is purely transverse and gauge independent to all orders
– the fermion self-energy is gauge-fixing dependent

in non-Abelian theories the gauge boson propagator is also gauge dependent

However, it is possible to obtain gauge-independent physical information from a gauge dependent quantity R. Kobes, G. Kunstatter, A. Rebhan, PRL25, 2992 (1990)

it has been shown from general principles (functional techniques + DS equation) that in a consistent resummation the singularity structure of certain components of the gauge propagators (zeros of the determinants, that is the dispersion relations) are gauge-independent

How to find a consistent resummation?

Hard thermal loop resummation

E. Braaten and R. D. Pisarski, NPB 337, 569 (1990)

triggered by the problem of finding a gauge independent soft gluon damping rate

explicit one-loop level calculations at finite T performed by many people in different gauges have shown that both the sign and the magnitude of the soft gluon damping rate are gauge dependent

Pisarski and Braaten have shown both in covariant and in Coulomb gauges that the complete resummation of (hard thermal) loop effects results in gauge-fixing-independent dispersion relations

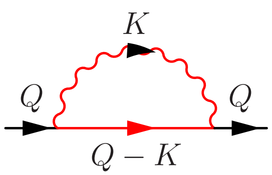
They not only gave the correct expression of the gluon damping rate but found all those one-loop n-point functions which are the same order as the tree-level ones and constructed the generating functional for them: the so-called HTL action.

The soft fermion dispersion relation at 1-loop level

the high-T approximation of the weakly coupled QED is characterized by
- large mean field path
- big separation of scales (perturbation theory accurate) hard: $\mathcal{O}(T)$, soft: $\mathcal{O}(eT)$

for static fermion ($\mathbf{q}=0$) one has to solve $q_0=\Sigma(q_0,\mathbf{q}=0)$ with $q_0=M-i\gamma$

1. LO calculation of Σ

$\Sigma(Q) = -$ 

$\approx -e^2\int\frac{d^4k}{(2\pi)^4}\gamma_\mu(\not{K}+\not{Q})\gamma_\mu\Delta(K)\tilde{\Delta}(Q+K)$

$$\approx 2e^2\gamma_4\int_{\mathbf{k}}\frac{i}{4E_2}\left[(1+f_1-\tilde{f}_2)\left(\frac{1}{i\omega-E_1-E_2}+\frac{1}{i\omega+E_1+E_2}\right)-(f_1+\tilde{f}_2)\left(\frac{1}{i\omega+E_1-E_2}+\frac{1}{i\omega-E_1+E_2}\right)\right]$$
$$+2e^2\gamma_i\int_{\mathbf{k}}\frac{\hat{\mathbf{k}}_i}{4E_2}\left[(1+f_1-\tilde{f}_2)\left(\frac{1}{i\omega-E_1-E_2}-\frac{1}{i\omega+E_1+E_2}\right)+(f_1+\tilde{f}_2)\left(\frac{1}{i\omega+E_1-E_2}-\frac{1}{i\omega-E_1+E_2}\right)\right]$$

$$E_1{=}k,\ E_2{=}|\mathbf{q}{-}\mathbf{k}|\approx k{-}q\cos\theta,\ \tilde{f}_2\approx\tilde{f}(k),\ i\omega+E_1-E_2\approx i\omega+q\cos\theta=Q\cdot\hat{K},\quad \hat{K}=(-i,\hat{\mathbf{k}})$$

$$\Sigma(Q)\big|_{\mathrm{LO}}=\frac{e^2}{2\pi^2}\underbrace{\int_0^\infty dk\,k(f(k)+\tilde{f}(k))}_{\pi^2T^2/4}\int\frac{d\Omega}{4\pi}\frac{-\gamma_4 i+\gamma_i\hat{k}_i}{Q\cdot\hat{K}},\quad d\Omega=d(\cos)\theta d\phi$$

gauge-independent result :
$$\boxed{\Sigma(Q)\big|_{\mathrm{LO}}=\frac{e^2T^2}{8}\int\frac{d\Omega}{4\pi}\frac{\not{K}}{Q\cdot\hat{K}}}$$

eff. action giving the correct electron propagator $\tilde{S} = \bar{\psi}\Sigma\psi = \frac{e^2T^2}{8}\int\frac{d\Omega}{4\pi}\bar{\psi}\frac{\not{K}}{\partial\cdot\hat{K}}\psi$

with $\partial_\mu \to D_\mu = \partial_\mu - ieA_\mu \Longrightarrow \tilde{S}_{\mathrm{HTL}} = \frac{e^2T^2}{8}\int\frac{d\Omega}{4\pi}\bar{\psi}\frac{\not{K}}{D\cdot\hat{K}}\psi =: \sum_{N=2}^{\infty}\bar{\psi}\delta\tilde{\Gamma}^N A^{N-2}\psi$

$\tilde{S}_{\mathrm{HTL}}$ is the generator of the hard thermal loops $\delta\tilde{\Gamma}^N$ between an electron pair and any number of photons

solution of the dispersion relation:

$$\Sigma_{\mathrm{LO}}(\omega,0) = \lim_{q\to 0}\left[\frac{e^2T^2}{16q}\ln\left|\frac{\omega+q}{\omega-q}\right| - i\frac{\pi}{2}\Theta(q^2-\omega^2)\right] \longrightarrow M_{\mathrm{LO}} = \frac{eT}{\sqrt{8}},\ \gamma_{\mathrm{LO}} = 0$$

1. NLO calculation for ξ=1 everything kept and done for $\mathcal{R}\Sigma$ term $\sim$ with -$i\gamma_4$

$$\Sigma^0\Big|_{\mathrm{NLO}}=2e^2\int_{\mathbf{k}}\frac{1}{4E_1}\left[(1+f_1-\tilde{f}_2)\left(\frac{1}{i\omega-E_1-E_2}+\frac{1}{i\omega+E_1+E_2}\right)+(f_1+\tilde{f}_2)\left(\frac{1}{i\omega+E_1-E_2}+\frac{1}{i\omega-E_1+E_2}\right)\right]$$

at $\mathbf{q}=0$ one has $E_1 = E_2 = k$ for the real part $i\omega \to \omega$

$$\to \frac{e^2}{2\pi^2\omega}\underbrace{\int_0^\infty k(f(k)+\tilde{f}(k))}_{\pi^2T^2/4} - \frac{e^2\omega}{8\pi^2}\underbrace{\mathcal{P}\int_0^\infty dk\frac{k}{k^2-p_0^2/4}\underbrace{(f(k)-\tilde{f}(k))}_{2f(2k)}}_{-\ln(T/p_0)}$$

$$= \frac{e^2T^2}{8\omega} + \frac{e^2\omega}{8\pi^2}\ln\frac{T}{\omega}$$

The complete 1-loop $\xi, \mathbf{q}$-dependent calculation at NLO in the HTE done in

I. Mitra, PRD62 (2000) 045023 real part

S.-Y. Wang, PRD70 (2004) 065011 real and imaginary parts

$$\mathcal{R}\Sigma_{\mathrm{NLO}}(\omega,0) = \frac{e^2\omega\xi}{8\pi^2}\ln\frac{T}{\omega}$$
$$\mathcal{I}\Sigma_{\mathrm{NLO}}(\omega,0) = -\frac{e^2 T}{16\pi}(3\xi-1)$$

$$\longrightarrow$$

$$M_{\mathrm{NLO}}^{\mathrm{1loop}} = \frac{e^3 T\xi}{8^{\frac{3}{2}}\pi^2}\ln\frac{1}{e}$$
$$\gamma_{\mathrm{NLO}}^{\mathrm{1loop}} = \frac{e^2 T}{16\pi}(3\xi-1)$$

the result in a covariant R_ξ gauge is gauge-parameter dependent $\rightarrow$ some sort of resummation is needed

What to resum?

What is the lowest set of graphs which gives contribution of the same order?

A first conjecture made on dimensional grounds by E. Mottola

The guessed/predicted form of the self-energy for soft, static electrons

$$\Sigma(q_0) = \underbrace{\frac{eT^2}{8q_0^2}}_{\text{LO HTL}} + \underbrace{c_1 e^2 q_0 \ln\frac{T}{q_0} - i d_1 e^2 T}_{\text{1-loop NLO HTL}} + \underbrace{c_2 \frac{e^4T^2}{q_0} \ln\frac{T}{q_0} - i d_2 \frac{e^4 T^3}{q_0^2}}_{\text{2-loop NLO HTL}}$$

at NLO HTL one solves the equation for the dispersion relation iteratively using the LO result $M_{\text{LO}} = \frac{eT}{\sqrt{8}}, \gamma_{\text{LO}} = 0$, to obtain

$$M_{\text{NLO}} = \frac{e^3 T}{\sqrt{8}}(c_1 + 8c_2)\ln\frac{\sqrt{8}}{e}, \qquad \gamma_{\text{NLO}} = (d_1 + 8d_2)e^2 T.$$

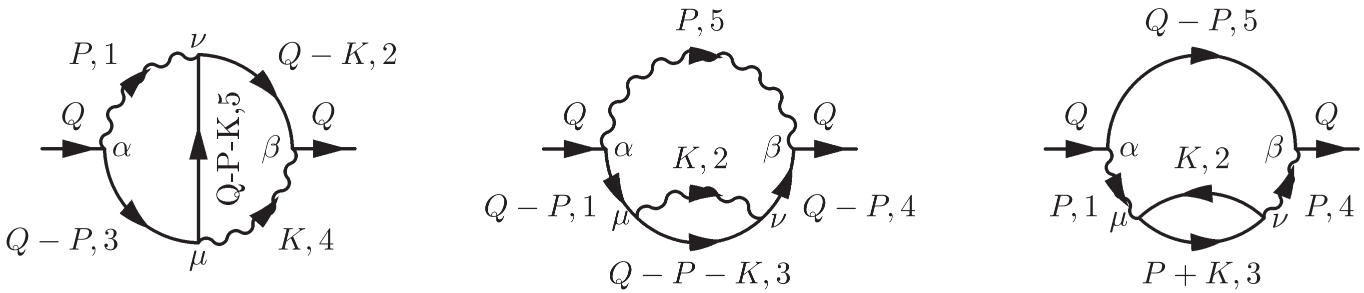
Expectation: ξ-dependence of M and γ cancels with the inclusion of the 2-loop NLO HTL terms

Conclusion of a 2-loop calculation done in RTF was presented at the SEWM06 $c_2 = \frac{1-\xi}{64\pi^2} \longrightarrow$ at least the fermion mass is gauge invariant beyond LO in the HTL approximation

Carrington & Mottola, NPA785, 142c (2007)

but a scrutiny of the calculation reveals a severe problem of the result

2-loop finite temperature calculation



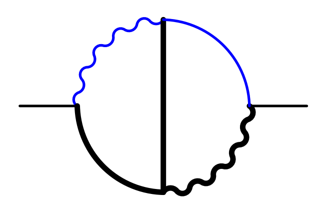
Method: - imaginary time formalism
- spectral representation for the propagators $\Delta(i\omega_n, \mathbf{k}) = \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} \frac{\rho(k_0, \mathbf{k})}{k_0 - i\omega_n}$
- Gaudin's method for performing Matsubara sums

M. Gaudin, Nuov. Cim.**38** (1965) 84, (see also J.-P. Blaizot, et al. NPA**764** (2006) 393)

Advantages of the method – no need for 7 3-point, 15 4-point vertices as in RTF
– no need for contour integration, instead:

- specify the orientation of lines, affect to each line a label k and a number $\tau_k = k\theta$
- decompose the graph in trees: $\mathcal{T}$ is a connected set of lines of the original graph which joins all the vertices and contains no loop

sample tree



$$\prod_{i=1}^{N} \frac{e^{i\omega_i \tau_i}}{p_i^0 - i\omega_i} = e^{i\omega T_e} \sum_{\mathcal{T}} \prod_{j\in\mathcal{T}} \frac{1}{p_j^0 - i\Omega_j(i\omega, p_l^0)} \prod_{l\in\bar{\mathcal{T}}} \frac{e^{i\omega_l T_l}}{p_l^0 - i\omega_l}$$

the complement of the tree, $\bar{\mathcal{T}}$ carries the independent frequencies ω_l
the frequency of each line of $\mathcal{T}$, Ω_j is expressed in terms of ω_l and external ω

- let $\theta \to 0$ and for each tree do the summation over indep. frequencies ω_l using

$$T\sum_n \frac{e^{i\omega_l T_l}}{p_0 - i\omega_l} = \pm\epsilon(T_l) f_\pm(\epsilon(T_l)p_0) e^{p_0 T_l}$$

$+:$ ω_l bosonic $(2\pi n T)$ $-:$ ω_l fermionic

The decomposition formula is a generalization of the partial fractioning

bubble $\omega = \omega_n + \omega_m$

$$\frac{e^{i\omega_n\tau_n}}{p_0 - i\omega_n}\frac{e^{i\omega_m\tau_m}}{q_0 - i\omega_m} = \frac{1}{p_0 + q_0 - i\omega}\left[\frac{e^{i\omega\tau_m}e^{i\omega_n(\tau_n - \tau_m)}}{p_0 - i\omega_n} + \frac{e^{i\omega\tau_n}e^{i\omega_n(\tau_m - \tau_n)}}{q_0 - i\omega_m}\right]$$

setting sun $\omega_n + \omega_m + \omega_r = 0$

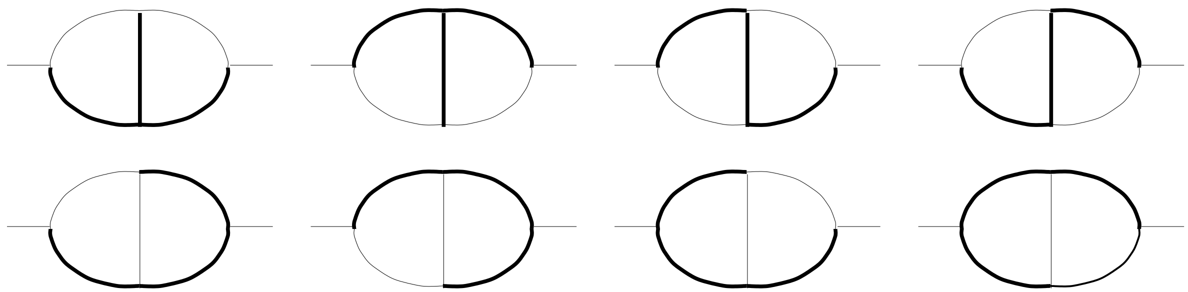
$$\frac{e^{i\omega_n\tau_n}}{p_0 - i\omega_n}\frac{e^{i\omega_m\tau_m}}{q_0 - i\omega_m}\frac{e^{i\omega_r\tau_r}}{r_0 - i\omega_r} = \frac{1}{p_0 + q_0 + r_0}\left[\frac{e^{i\omega_n(\tau_n - \tau_r)}e^{i\omega_m(\tau_m - \tau_r)}e^{i\omega\tau_r}}{(p_0 - i\omega_n)(q_0 - i\omega_m)}\right.$$
$$\left. + \frac{e^{i\omega_n(\tau_n - \tau_m)}e^{i\omega_r(\tau_r - \tau_m)}e^{i\omega\tau_m}}{(p_0 - i\omega_n)(r_0 - i\omega_r)} + \frac{e^{i\omega_m(\tau_m - \tau_n)}e^{i\omega_r(\tau_r - \tau_n)}e^{i\omega\tau_n}}{(q_0 - i\omega_m)(r_0 - i\omega_r)}\right]$$

Performing the Matsubara sums for the crossed-rainbow diagram

$$\Sigma_{\rm cr}(i\omega,\mathbf{q}) = -e^4\int_{\mathbf{k},\mathbf{p}}\left(\prod_{i=1}^{5}\int\frac{dp_i^0}{2\pi}\right)\gamma_\alpha\rho_F(p_3)\gamma_\mu\rho_F(p_5)\gamma_\nu\rho^{\nu\alpha}(p_1)\rho_F(p_2)\gamma_\beta\rho^{\beta\mu}(p_4)T^2\sum_{n,m}\prod_{i=1}^{5}\frac{1}{p_i^0-i\omega_i}.$$

$$p_1 = p, p_2 = q-k, p_3 = q-p, p_4 = k, p_5 = q-p-k$$

Trees:



$$T^2\sum_{n,m}\prod_{i=1}^{5}\frac{1}{p_i^0-i\omega_i} =$$

$$\frac{(-f(-p_1^0))(-\tilde{f}(p_2^0))}{(p_3^0+p_1^0-i\omega)(p_4^0+p_2^0-i\omega)(p_5^0+p_1^0-p_2^0)} + \frac{(-\tilde{f}(p_3^0))(-f(-p_4^0))}{(p_1^0+p_3^0-i\omega)(p_2^0+p_4^0-i\omega)(p_5^0+p_4^0-p_3^0)} +$$

$$\frac{(-\tilde{f}(p_2^0))(-\tilde{f}(p_3^0))}{(p_1^0+p_3^0-i\omega)(p_4^0+p_2^0-i\omega)(p_5^0-p_2^0-p_3^0+i\omega)} + \frac{(-f(-p_1^0))(-f(-p_4^0))}{(p_2^0+p_4^0-i\omega)(p_3^0+p_1^0-i\omega)(p_5^0+p_1^0+p_4^0-i\omega)} +$$

$$\frac{(-f(-p_1^0))(-\tilde{f}(p_5^0))}{(p_2^0-p_1^0-p_5^0)(p_3^0+p_1^0-i\omega)(p_4^0+p_1^0+p_5^0-i\omega)} + \frac{(-\tilde{f}(p_3^0))(-\tilde{f}(p_5^0))}{(p_1^0+p_3^0-i\omega)(p_2^0+p_3^0-p_5^0-i\omega)(p_4^0+p_5^0-p_3^0)} +$$

$$\frac{(-\tilde{f}(p_2^0))(-\tilde{f}(p_5^0))}{(p_1^0+p_5^0-p_2^0)(p_3^0+p_2^0-p_5^0-i\omega)(p_4^0+p_2^0-i\omega)} + \frac{(f(p_4^0))(-\tilde{f}(p_5^0))}{(p_1^0+p_4^0+p_5^0-i\omega)(p_2^0+p_4^0-i\omega)(p_3^0-p_4^0-p_5^0)}$$

Findings

– soft-hard pattern: same order contribution as at NLO in the 1-loop case occurs when one loop momentum is soft and the other is hard

$$\mathcal{R}\Sigma_{\rm cr}(\omega,\mathbf{q}=0)\big|_{\sf LO} = -\frac{e^4}{\omega\pi^4}\underbrace{\int_0^\infty dk\,k(f(k)+\tilde{f}(k))}_{\text{hard},\quad \pi^2T^2/4}\underbrace{\int_0^\infty dp(f(p)-\tilde{f}(p))\frac{p}{\omega^2-4p^2}}_{\text{soft},\quad 4^{-1}\ln(T/\omega)}$$

$$= -\frac{e^4T^2}{16\pi^2\omega}\ln\frac{T}{\omega}$$

$$\mathcal{I}\Sigma^{sc}_{\rm cr}(\omega,\mathbf{q}=0)\big|_{\sf LO} = \frac{e^4}{8\pi^3\omega}\underbrace{\int_0^\infty dk k(f(k)+\tilde{f}(k))}_{\text{hard},\quad \pi^2T^2/4}\underbrace{\left[f\left(\frac{\omega}{2}\right)-\tilde{f}\left(\frac{\omega}{2}\right)\right]}_{\text{soft},\quad 2f(\omega)}$$

$$= \frac{e^4T^2}{16\pi\omega}f(\omega)\approx\frac{e^4T^3}{16\pi\omega^2}$$

$$\mathcal{R}\Sigma_{\rm r}(\omega,\mathbf{q}=0)\big|_{\sf LO}=0, \qquad \mathcal{I}\Sigma_{\rm r}(\omega,\mathbf{q}=0)\big|_{\sf LO}=-\frac{e^4T^3}{32\pi\omega^2}$$

– the collinear divergences ($\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}=\pm1$) are either subleading in the HTE or miraculously cancel in the real and imaginary part of the crossed-rainbow and rainbow diagrams

– IR and collinear divergences in the real part of the bubble diagram

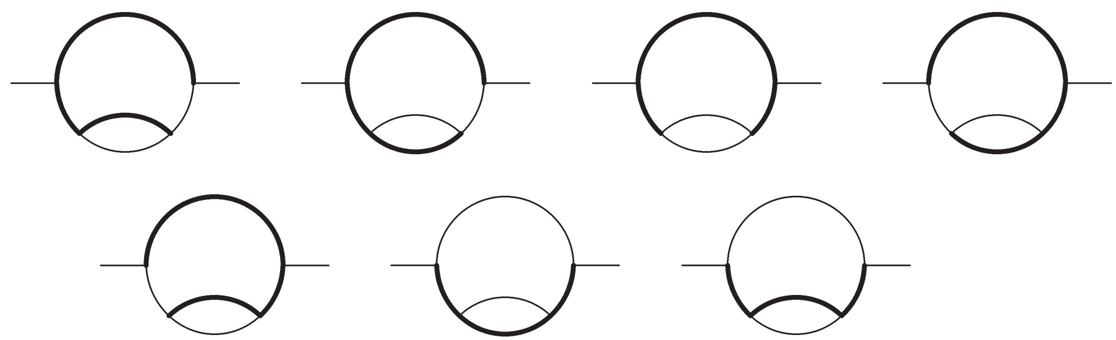
$\mathcal{R}\Sigma_{\rm b}(\omega, \mathbf{q}=0)\big|_{\rm LO} \sim \frac{e^4T^2}{\omega}\frac{\omega^2}{m^2}\ln\frac{T}{\omega}$ order is changed when the IR regulator $m \sim eT$

IR divergence indicates that resummation of the photon propagator is necessary

– collinear divergences in the imaginary part of the bubble diagram

$\mathcal{I}\Sigma_{\rm b}(\omega, \mathbf{q}=0)\big|_{\rm LO} \sim \frac{e^4T^3}{\omega^2}\frac{\omega^2}{m^2}$ the order is not changed with $m \sim eT$

How does m enter the game?
As a regulator in the problem of double poles.



indep. freq.$\rightarrow$thin line$\rightarrow$thermal prop.$\rightarrow\delta(k_0^2$-$\mathbf{k}^2)$ $\Longrightarrow \mathcal{R}\frac{\delta(k_0^2-\mathbf{k}^2)}{k_0^2-\mathbf{k}^2\pm i\epsilon}$ is singular

dep. freq.$\rightarrow$thick line$\rightarrow$T=0 prop.$\rightarrow\frac{1}{k_0^2-\mathbf{k}^2\pm i\epsilon}$

Solution: representation $\delta(x)$=$\frac{\epsilon}{\pi}\frac{1}{x^2+\epsilon^2}$ coming from $2\pi i\theta(x)$= $\ln(\text{-}x+i\epsilon)$- $\ln(\text{-}x\text{-}i\epsilon)$

one can derive: $\frac{\delta(k_0^2-\mathbf{k}^2)}{k_0^2-\mathbf{k}^2\pm i\epsilon} = -\frac{1}{2}\frac{\partial}{\partial k_0^2}\delta(k_0^2-\mathbf{k}^2) \mp i\pi[\delta(k_0^2-\mathbf{k}^2)]^2.$

better to use the prescription: $\frac{\partial^2}{\partial k_0^2}\delta(k_0^2\text{-}\mathbf{k}^2)$= $\lim_{m^2\to 0}\frac{\partial}{\partial k_0^2}\delta(k_0^2\text{-}\mathbf{k}^2\text{-}m^2)$=- $\lim_{m^2\to 0}\frac{\partial}{\partial m^2}\delta(K^2\text{-}m^2)$

Conjecture for resummation based on the calculation at $\xi = 1$

the lowest set of diagrams to be resummed contains
– a chain of 1-loop HTL fermion bubble insertions in the photon propagator
– a chain of 1-loop HTL photon-electron bubble insertions in the electron prop.
– could have HTL corrected photon-electron vertex

that is, one has to calculate an 1-loop diagram with HTL corrected propagators and vertices

Check: cancellation of the ξ-dependence

calculation done by M. E. Carrington in PRD **75**, 045019 (2007)
but without explaining why this is the path one has to take after SEWM06 (which led to a wrong conclusion)

we have now a better understanding of why this is the needed resummation

What is the relevance of this calculation?

still debated by the authors

clarifies some issues and explicitly shows which are the NLO corrections in the HTE which comes by evaluating singular integrals and cannot be guessed by simple power counting (e. g. no NLO from a 2-loop diagram with two hard momenta as stated in Le Bellac's book, because of the IR and collinear divergences one cannot even guess the correct order of the diagram)

Dreams

- generalization to the $\mathbf{q} \neq 0$ case
- identification of the quasi-particles - needed to understand the equation of state
- construction of the effective action - path to non-equilibrium studies
- calculation of conductivity, viscosity (Kubo formula) at NLO HTL
- generalization of the procedure to QCD