Eksperiment → rezultat → statičko obilježje < KVANTITATIVNA / KVALITATIVNA

ponavljamo isti eksperiment

rezultat eksperimenta

izračunamo

$P = a \cdot b$

o obilježje ili statistička (slučajna) veličina (broj koji zavisi o rezultatu eksperimenta)

Kvantitativna: diskretno (odvojene vrijednosti, obično cijeli brojevi), kontinuirano obilježje

$P(x_i \le X \le x_i + \Delta x)$

$p_i = p(X = x_i) = \frac{\#(X = x_i \text{ povoljni})}{\#(X \text{ bilo što})}$ → frekvencija f_i za nazivnik → ∞ $g(x_i)\Delta x_i$

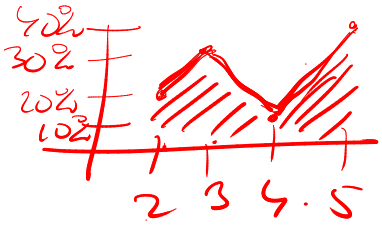
$\sum f_i$ = nazivnik

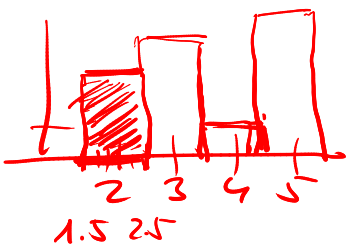
gustoća vjerojatnosti

$E[X] = \sum p_i x_i$ (srednja vrijednost)

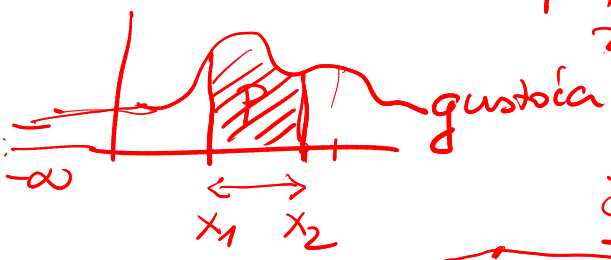
RAZDIOBA: veličine X

x_i	2	3	4	5
$p(X=x_i)$	0.2	0.3	0.1	0.4

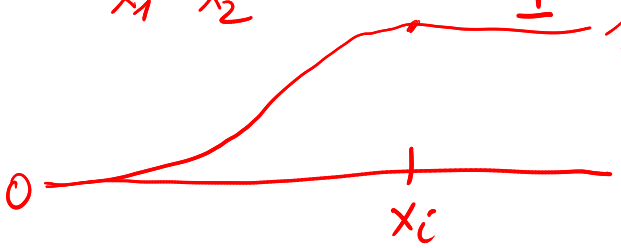




histogram



$\Phi(x) = P(X \le x_i)$



KUMULATIVNA RAZDIOBA VJEROJATNOSTI

$P(x_1 \le X \le x_2) = \Phi(x_2) - \Phi(x_1)$

$E[X] = \frac{\sum f_i x_i}{\sum f_i} = \frac{\sum \frac{f_i}{n} x_i}{\sum \frac{f_i}{n}} \to \sum p_i x_i$ $\quad \sum f_i = n \to \infty$

$p_i = P(X = x_i)$

expectation – očekivana vrijednost – matematička nada – srednja očekivana vrijednost neke veličine

$Var[X] = E[X^2] - E[X]^2 \geq 0$

|| varijanca

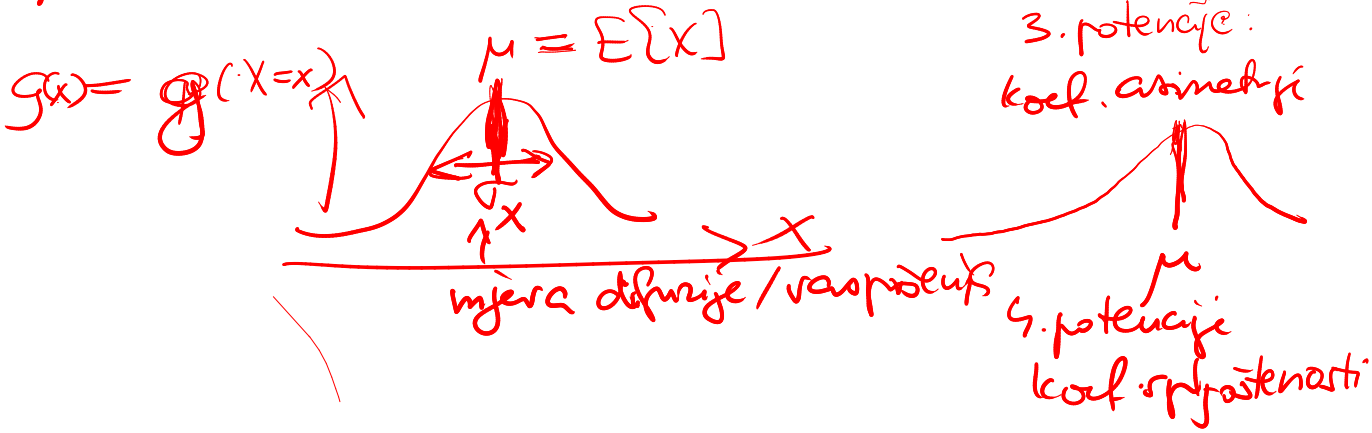
$E[(X - E[X])^2]$

odstupanje od $E[X]$

(devijacija)

$\sigma_X = \sqrt{Var[X]}$ standardna devijacija

1	$1^2 = 1$
2	$2^2 = 4$
3	$3^2 = 9$
$2^2 = 4$	$13/3 = 4.333$



$\bar{X} = E[X] = \sum p_i x_i = \langle X \rangle$

$Var[X] = \sum p_i (x_i - \bar{X})^2 = \sum p_i x_i^2 - (\sum p_i x_i)^2$

$\overline{x^2} - \bar{x}^2$

$E[X^2] - E[X]^2$

BINOMNA RAZDIOBA

Eksperiment se sastoji od n identičkih dijelova i svakom postoje dvije mogućnosti: dobra (ona koja nas zanima) i loša

Zanima nas vjerojatnost da se u tih n dijelova dobra mogućnost desi točno m puta (m je između 0 i n)

Lovac gađa glinene golubove i u prosjeku pogađa (u jednom pokušaju) u 30% slučajeva.

Kolika je vjerojatnost da u 6 pokušaja pogodi TOČNO 3 glinena goluba ?

$\mu = p \cdot n$

$E[m] = \mu = 0.7 \times 6 = 4.2$

$\sum x_i p_i = \sum m \cdot B(n,m) = np$

$n = 6 \quad m = 3$

$\binom{6}{3}$ { |x|-|-|x|x|- → 0.3 0.7 0.7 0.3 0.3 0.7

|x xx - - -|

⋮

$0.3 \times 0.7 \times 0.7 \times 0.3 \times 0.3 \times 0.7 = 0.3^3 \cdot 0.7^3$

m ; $6-3 = n-m$ (drugi nedostaje)

$0.3 \times 0.3 \times 0.3 \times 0.7 \times 0.7 \times 0.7$

$0.3^3 \times 0.7^3$; 0.3

$\binom{6}{3} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20$

$P(\#=m) = \binom{n}{m} p^m (1-p)^{n-m}$

pogodaka = $B(n,m)$

2

$B(n,m)$ binomial distribution

$P(\#=3) = B(6,3) = 20 \cdot 0.3^3 \times 0.7^{6-3}$

$= 20 \times 0.027 \times 0.343$

$= 0.54 \times 0.343$

$= 0.18522 = 18.522\%$

7×49 ; 0.54×0.343

162

216

162

0.18522

b) najmanje 3 pogotka

barem

$B(6,3) + B(6,4) + B(6,5) + B(6,6) = \dots$

c) barem 1 pogodak

$B(6,1) + B(6,2) + B(6,3) + B(6,4) + B(6,5) + B(6,6)$

$= 1 - P(0 \text{ pogodaka}) = 100\% - B(6,0) = 94.1\%$

$B(6,0) = \binom{6}{0} 0.3^0 \cdot 0.7^6 = 0.059049 \approx 5.9\%$

1 1

0.243×0.243

486

972

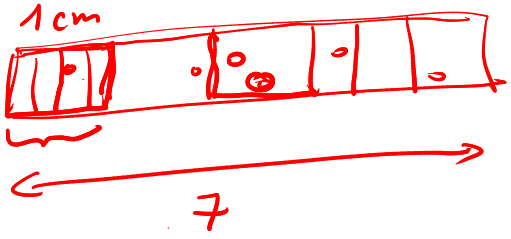
729

0.059049

POISSONOVA RAZDIOBA

– granična razdioba binomne razdiobe kad n->beskonačno, a p->0 i p*n = const

$$\lambda = \frac{\text{prosječan \# pogodaka}}{\text{željena širina prostora/duljina vremena}}$$



$p \cdot n$

$\mu = p \cdot n$

14

$$P_\lambda(m) = \frac{\lambda^m}{m!} e^{-m} = \frac{\lambda^m}{m!} \exp(-m)$$

λ ↑ broj pogodaka

prema pitanju

Ulicom prolazi u prosjeku 3 auta na 2 minute ? Kolika je vjerojatnost da će proći točno 4 auta u 3 minute ?

m

RAZRJEŠENJE

$$3 \text{ auta} : 2 \text{ min} = \lambda \text{ auta} : 3 \text{ min}$$

$$\lambda = \frac{3}{2} \cdot 3 = 4.5$$

$$\frac{\lambda^m}{m!} e^{-m}$$

$$P_\lambda(m) = \frac{4.5^4}{4!} e^{-4.5} = 0.3129$$