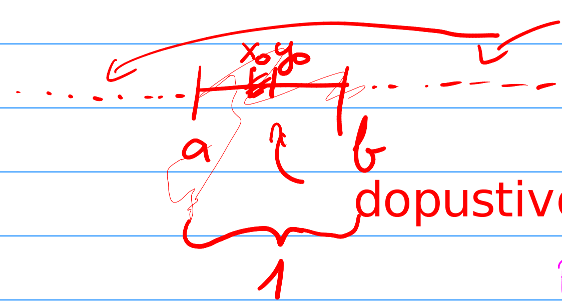
statistika 30. ožujka 2021.

uniformna = jednolika = ravnomjerna

UNIFORMNA RAZDIOBA - kontinuirana razdioba vjerojatnosti u kojoj razlikujemo neko konačno područje koje je dopustivo i u kojem je vjerojatnost razmjerna veličini područja koje promatramo



nedopustivo

pravac

dopustivo

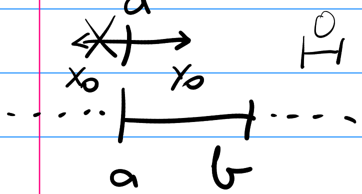
$P \propto y - x$

$$\rho = \frac{P(a \le x \le b)}{b-a} = \frac{1}{b-a}$$

gustoća vjerojatnosti

$$P(x_0 < x < y_0) = (y_0 - x_0)\cdot \rho$$

$$= (y_0 - x_0)\cdot \frac{1}{b-a} = \frac{y_0 - x_0}{b-a}$$



$$\frac{y_0 - a}{b-a} \begin{cases} a < y_0 < b \\ x_0 < a \end{cases}$$

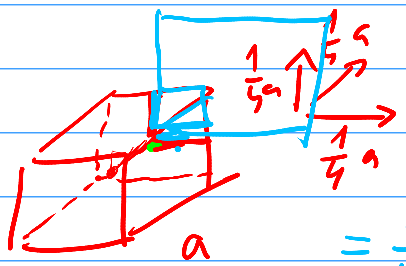
BUNAR U RAVNINI

$$\text{Površina} = \left(\frac{d}{2}\right)^2 \pi$$

$$\rho = \frac{1}{\left(\frac{d}{2}\right)^2 \pi}$$

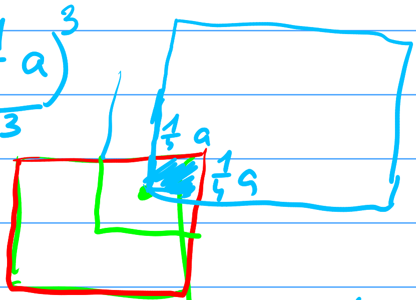
Kolika je vjerojatnost da slučajna toča po ravomjernor razdiobi bude unutar pola polumjera od središta

$$\left(\frac{d}{4}\right)^2 \pi \cdot \rho = \frac{(d/4)^2 \cancel{\pi}}{(d/2)^2 \cancel{\pi}} = \frac{1}{4}$$



$$\frac{\left(\frac{1}{4}a\right)^3}{a^3}$$

$$= \frac{1}{4^3} = \frac{1}{64}$$



$$P = \frac{1}{64} \sim 0.015625 \sim 1.56\,\%$$

Bacamo iglu na papir s kvadratnom mrežom

$d(\text{igla}) < a$

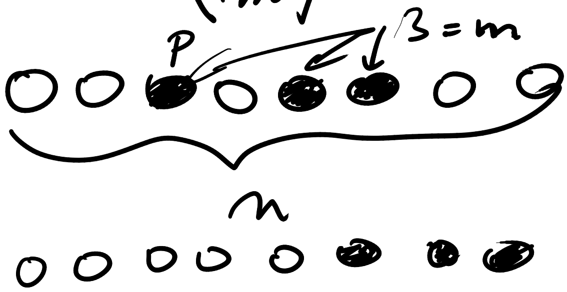
Hoće li igla presjeći neku stranicu?

binomna razdioba i Poissonova su diskretne P(m)

binomna: ponavljamo neki eksperiment n puta, i svaki od n eksperimenata je nezavisan
i vjerojatnost da se nešto desi u jednom eksperimentu je p, da se ne desi je 1-p = q
B(n,p)(m) je vjerojatnost da se to nešto desi u m eksperimenata

$$B(n,p)(m) = \binom{n}{m} p^m \cdot (1-p)^{n-m}$$

$$\binom{5}{3} \left\{ \begin{array}{l} p^3 q^5 \\ + \\ p^3 q^5 \\ + \end{array} \right.$$



$$\left. \right\} \binom{n}{m} p^m (1-p)^{n-m}$$

$$\binom{n}{m} = \frac{n!}{m!(n-m)!} \qquad \binom{5}{3} = \binom{5}{2} \qquad \frac{5\cdot 4\cdot 3\cdot \cancel{2\cdot 1}}{3\cdot 2\cdot 1\cdot \cancel{2\cdot 1}}$$

$$\binom{n}{n-m} = \frac{n!}{(n-m)!\,\underbrace{(n-(n-m))}_{m}!}$$

$$5-(5-3) = 5-2 = 3$$

$$\frac{5\cdot 4\cdot 3}{3\cdot 2\cdot 1} \qquad \frac{5\cdot 4}{3\cdot 2}$$

p = 0.2, i provodimo 4 eksperimenta. Kolika je vjerojatnost da se desi taj događaj u barem 3 eksperimenta

$$B(4,3) + B(4,4) = \qquad \geq 3 \text{ znači } 3 \text{ ili } 4$$

$$\binom{4}{3} p^3 q^1 + \binom{4}{4} p^4 q^0 = 4 \cdot 0.2^3 \cdot 0.8 + 1 \cdot 0.2^4 \cdot 1$$

$$= 0.0256 + 0.016 = 0.0272$$

Ako p = q $\quad p = 1 - p = 0.5$

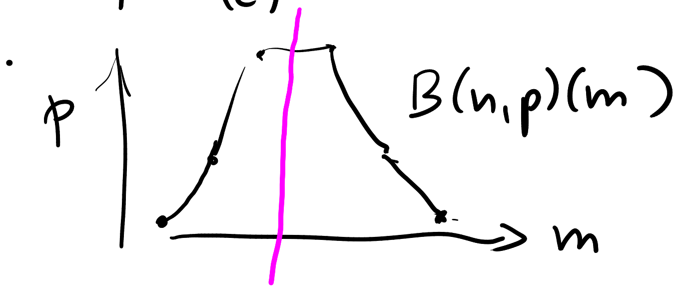
$n = 5, \quad m = 0 \quad \binom{5}{0} = 1$

$\binom{5}{0} \; \binom{5}{1} \; \binom{5}{2} \; \binom{5}{3} \; \binom{5}{4} \; \binom{5}{5}$

$1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1$

$p^m q^{n-m}$

$p^n = \left(\frac{1}{2}\right)^5 = 0.03125$



Pascalov trokut

$\binom{0}{0}$

$\binom{1}{0} \; \binom{1}{1}$

$\binom{2}{0} \; \binom{2}{1} \; \binom{2}{2}$

1

1 1

1 2 1

1 3 3 1

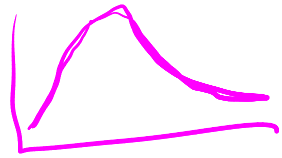
1 4 6 4 1

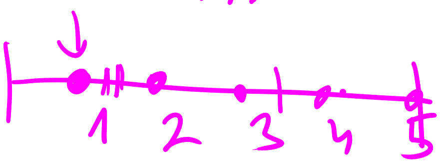
1 5 10 10 5 1

$p > q$

$q < p$

$$P(m) = \frac{\lambda^m}{m!} e^{-\lambda}$$





n jako velik

p jako malen

$n \cdot p = \lambda = \frac{\text{vjer}}{\text{duljini}}$ za 1x

Lovac gađa glinene golubove i puca 6 puta. Ako je za svaki pucanj šansa da pogodi 0.3 kolika je vjerojatnost da pogodi točno tri glinena goluba.

$p = 0.3, \quad n = 6, \quad m = 3$

$\binom{6}{3} \times 0.3^3 \times 0.3^3 = \frac{\cancel{6} \cdot 5 \cdot 4}{\cancel{3 \cdot 2} \cdot 1} \cdot 0.00729 = 0.01458$

$\sim 1.458\%$

27×27

189

54

729

Auto ide prerijom i u prosjeku pogodi 3 komarca na minutu.
Koja je vjerojatnost da u 100 sekundi pogodi 4 komarca ?

prosjek 3 kom / minutu

u 100 sekundi je to koliko

$$3 : \underline{\underline{1\,\text{min}}} = x : 100\,\underline{\underline{\text{sek.}}}$$

$$3 : 60\,\text{sek.} = x : 100\,\text{sek}$$

$$x = \frac{3}{60} \cdot 100 = 5$$

$\lambda = 5$ kom / 100 sek.

očekivani broj u intervalu koji nas zanima

$m = 4$ komarca

$$P(4) = \frac{\lambda^m}{m!} e^{-\lambda} = \frac{5^4}{4!} e^{-5} = 0.175467$$

$$\approx \underline{17.5\%}$$

$e = 2.7182818\ldots$ EULEROV BROJ

baza prirodnog logaritma

$$b^{\log_b x} = x \qquad e^{\ln(x)} = x$$

ln x

log naturalis

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

$n = 10$ $\left(\frac{11}{10}\right)^{10} = 2.5937$ $\left(\frac{1001}{1000}\right)^{1000} = 2.7169$

$n = 100$ $\left(\frac{101}{100}\right)^{100} = 2.7048$ $\left(\frac{1000001}{1000000}\right)^{1000000} = 2.718280$

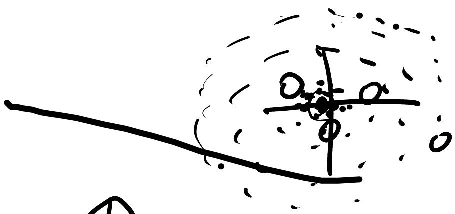
očekivanje $E[X]=\sum_{x} X\cdot P(X=x)=\sum_{i} x_i p_i$

za Poissonovu razdiobu je $E[X]=\lambda$

za binomnu razdiobu je $E[X]=p\cdot n$

$$\sum_{m=0}^{n} m\, B(n,m)=\sum_{m=0}^{n} m\binom{n}{m}p^m(1-p)^{n-m}=p\cdot n$$

normalna ili Gaussova razdioba = zbroj vrijednosti mnogo malih nezavisnih doprinosa koji se zbivaju slučajno s nekom zakonitošću sa srednjom vrijednosti μ



2dim Gaussova razdioba

1dim Gaussova razdioba

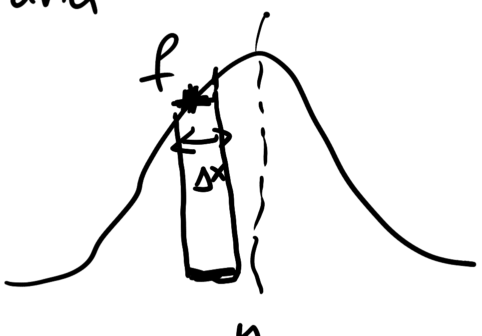
$$\frac{1}{\sigma\sqrt{2\pi}}=\frac{1}{\sqrt{2\pi\sigma^2}}$$

P zavisi o μ i σ

$$f(X=x)=\frac{1}{\sigma\sqrt{2\pi}}\,e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

σ je standardna devijacija

$$f=\frac{P(x<X<x+\Delta x)}{\Delta x}$$

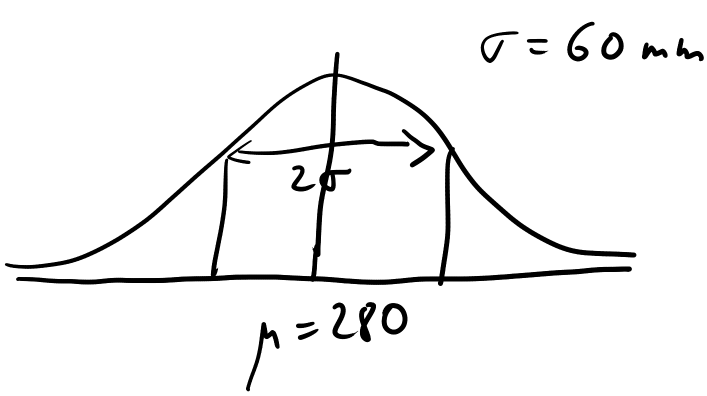


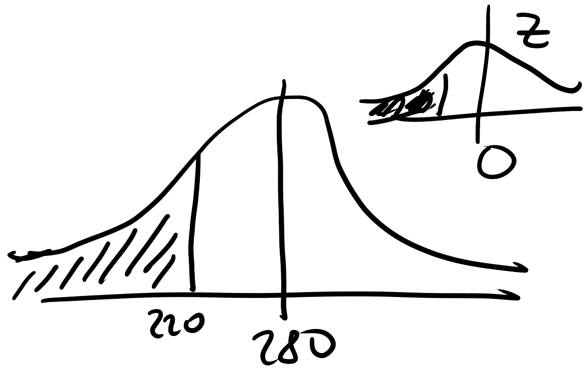
Z-value

vrijednost jedinične normalne varijable

$\mu=0,\ \sigma=1$

$$f(X=x)=\frac{1}{\sqrt{2\pi}}\,e^{-\frac{1}{2}z^2}$$

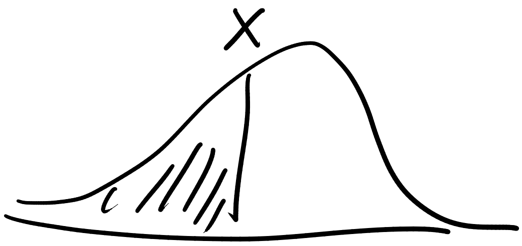


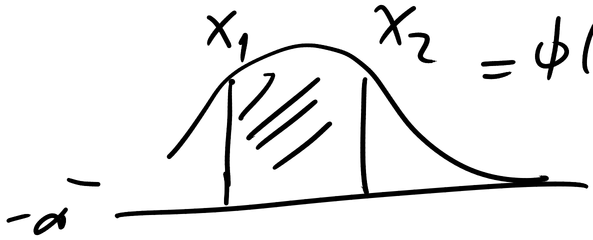


$$Z = \frac{X-\mu}{\sigma} = \frac{220-280}{60} = -1$$

$$P(X < 220) = P(Z < -1) = \Phi(-1) = 0.1587 = 15.87\%$$

Domaća zadaća: prvi zadatak na stranici https://ncatlab.org/zoranskoda/show/stat-21-test-drugi-dio



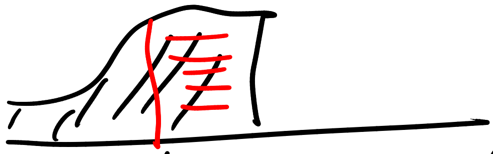


$= \phi(z_2) - \phi(z_1)$

$$P(x < X) = \phi(z)$$

$$z = \frac{x-\mu}{\sigma}$$

$$P(x_1 \le X < x_2) = \phi\Big(\underbrace{\tfrac{x_2-\mu}{\sigma}}_{z_2}\Big) - \phi\Big(\underbrace{\tfrac{x_1-\mu}{\sigma}}_{z_1}\Big)$$



$$\phi(z_2) = \phi\left(\frac{x_2-\mu}{\sigma}\right)$$

$-$  $\phi(z_1)$