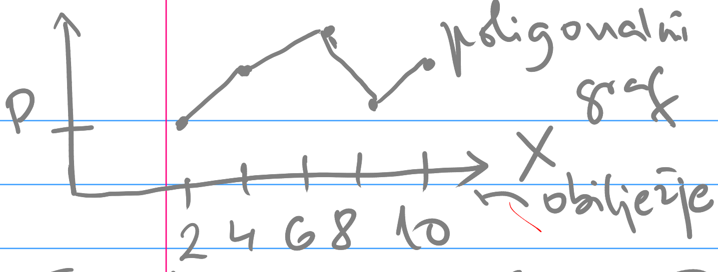
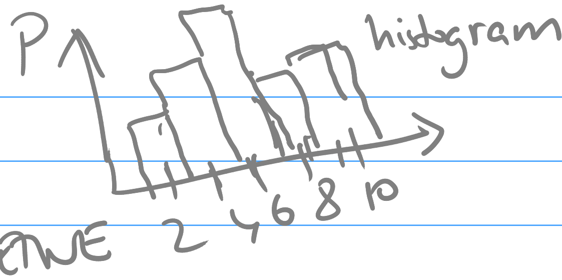
Još o razdiobama, obilježjima i očekivanju, 23-03-21



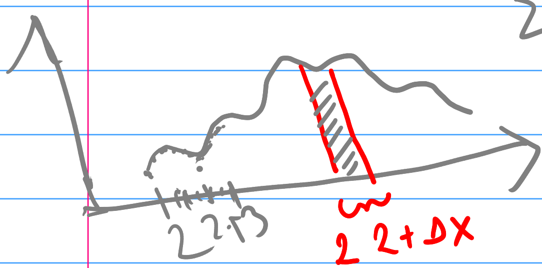


$\sum P(x_i) = 1$ ← nema 3.5

DISKRETNE

$\sum P(\cancel{x}) = \infty$

NEPREKIDNE



$P(X = 2)$ točno ?

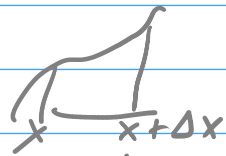
$= 0$ 2.0000000

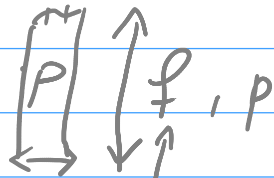
nikad ne vrijedi !

Δx prirast varijable X (obilježja)

0.02 2 do 2.02

$$\underbrace{P(x < X < x+\Delta x)}_{2 \quad 2+0.02} \neq 0$$



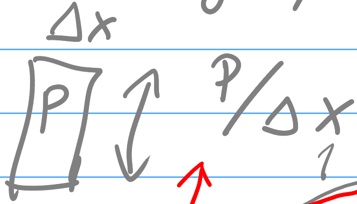


površina stupca $P = f \cdot \Delta x$

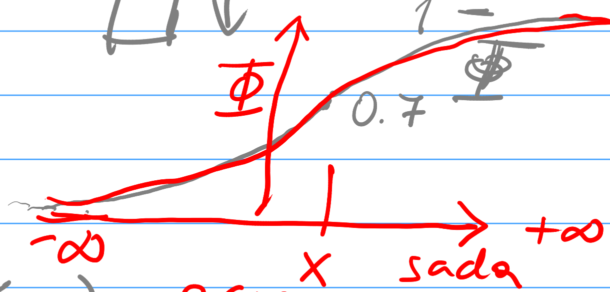
gustoća vjerojatnosti

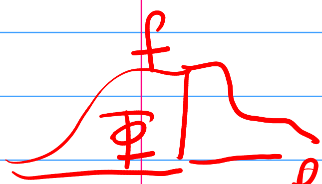
$$f(x) = \frac{P(\text{interval})}{\Delta x} \quad \Delta x \text{ mali}$$

$$= \frac{P(x < X < x+\Delta x)}{\Delta x}$$

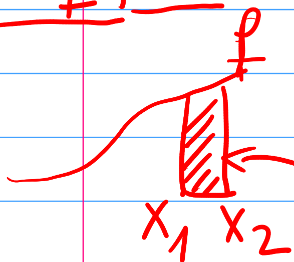


$$= \frac{d\Phi}{dx}$$ derivacija

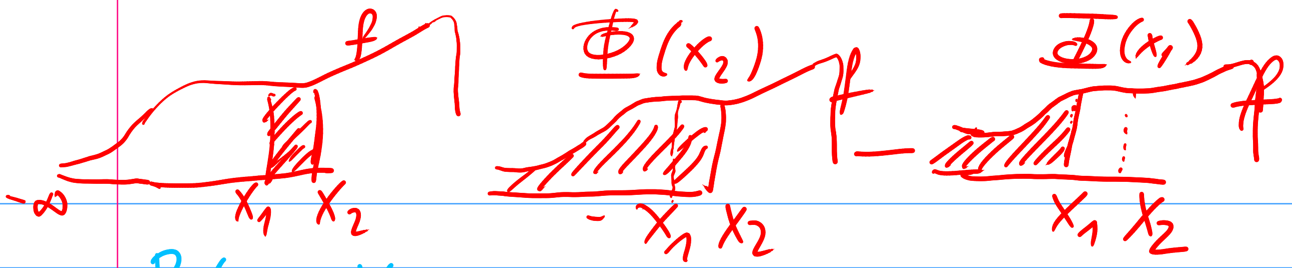




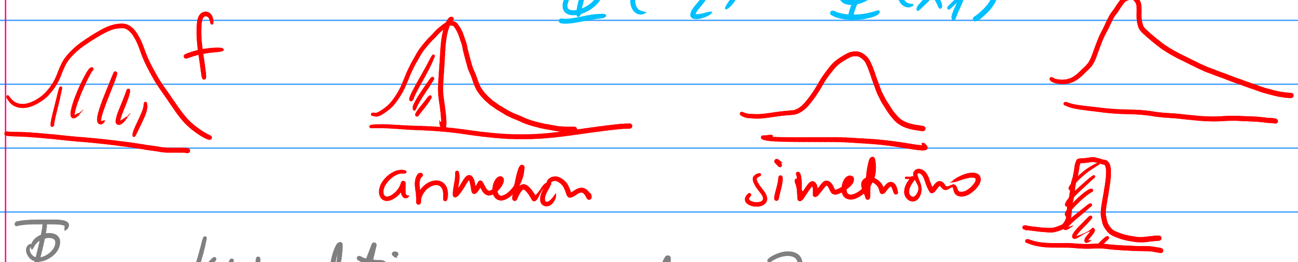
$$\Phi = \underbrace{P(-\infty < X < x)}_{\text{kumulativna vjerojatnost}} = P(X < x)$$



$$P(x_1 < X < x_2) = P(X < x_2) - P(X < x_1)$$
$$= \Phi(x_2) - \Phi(x_1)$$



$$P(x_1 < X < x_2) = P(X < x_2) - P(X < x_1)$$
$$= \Phi(x_2) - \Phi(x_1)$$



Φ kumulativna vjerojatnost

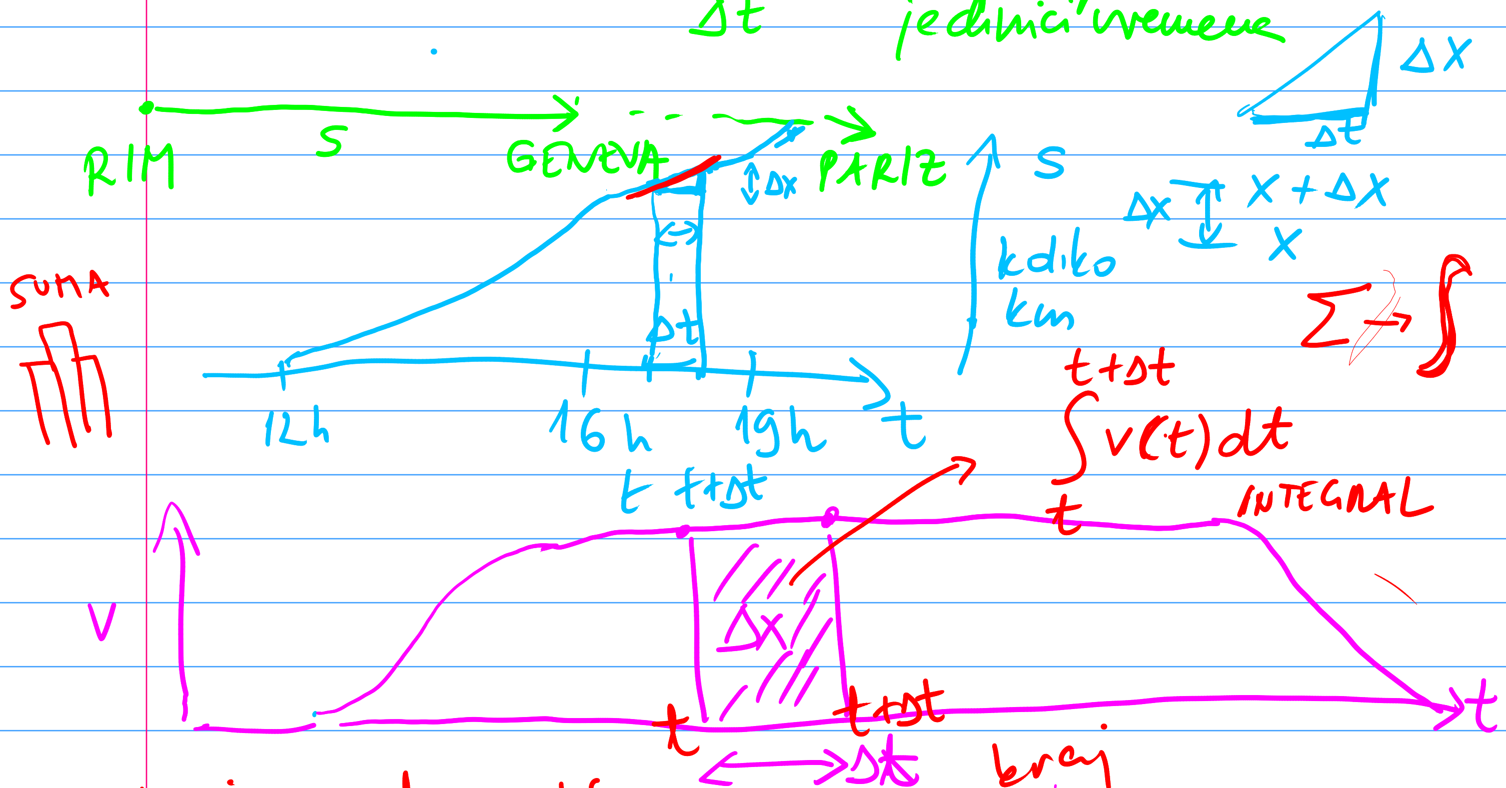
$f = p$ gustoća vjerojatnosti

} neprekidna statistička obilježja

ANALOGIJA U FIZICI

put $s = x$

$v =$ brzina $= \dfrac{\Delta x}{\Delta t}$ prevaljeni put u jedinici vremena



ako je funkcija gustoće
onde je ispod masa

kraj
$\int$
početak

Diskretne razdiobe, primjeri

U dva bacanja kocke zbroj vrijednosti kocke

$X = X_1 + X_2$ ← broj kod drugog bacanja

X_1: broj kod prvog bacanja

36 mogućnosti: 1+1, 1+2, 2+1, 1+3, 3+1 ... , 6+5, 6+6

X	2	3	4	5	6	7	8	9	10	11	12	Σ
f	1	2	3	4	5	6	5	4	3	2	1	36
p	1/36	2/36	3/36									1

Cestom prolaze auti, u prosjeku 1 auto na minutu, ali stohastički.
Može biti jedna minuta 3 auta ili 3 minute niti jedan.

Vjerojatnost da auto prođe u intervalu t
je jednaka $r t$

r je prosječan broj auta u nekoj jedinici vremena, a t u onoj koja nas zanima

$\lambda = r t$

$r = \frac{\#\text{auta}}{\text{jed. vremena}}$

rate

frekvencija pojavljivanja

Kolika je vjerojatnost da u λ minuta prođe m auta ?

INTERVAL U KOJEM PROĐE 1 AUTO

$$P(X=m) = \frac{\lambda^m}{m!} e^{-\lambda}$$

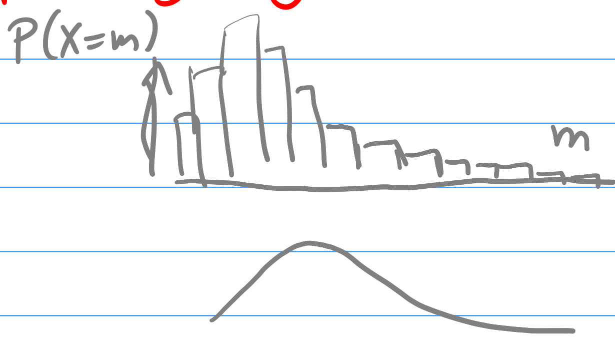
$\exp(-\lambda)$

$m = 0, 1, 2, 3, \ldots$

$e = 2.7182818\ldots$ Eulerov broj
baza prirodnog logaritma

3 minute 2 auta
$\lambda = 3,\ m = 2$

$$\frac{3^2}{2!} e^{-3}$$



POISSONOVA RAZDIOBA

Sa stabla jabuke padnu 3 jabuke u prosjeku svake 2 minute.
Kolika je vjerojatnost da u određenih 5 minuta padnu točno 4 jabuke ?

$3:2 = \lambda : 5$

zanima

$\frac{3\ JAB}{2\ MIN}$

$r = \frac{3}{2}$ jab/min

$t = 5$ min

$\lambda = \frac{3}{2} \cdot 5 = \frac{15}{2} = E[m]$

OČEKIVANI BROJ JABUKA

$$P(m=4) = \frac{\lambda^m}{m!} e^{-m} = \frac{(15/2)^4}{4!} \exp(-4)$$

$$\frac{7.5^4}{24} \cdot 2.71828^{-4}$$

$$m \mapsto P(m) = \frac{\lambda^m}{m!} e^{-m}$$

to je Poissonova razdioba

BINOMNA RAZDIOBA

broj pokušaja

$$P(m) = B(n,m) = \binom{n}{m} p^m \overbrace{(1-p)}^{q}{}^{n-m}$$

p = 0.2
u 20% pokušaja sreća
u 80% nije sreća

q = 1-p = 0.8

Vjerojatnost da u n pokušaja
m puta imamo „sretan" pogodak

$\circ\ \circ\ \times\ \circ\ \times\ \circ \quad \binom{6}{2} p^2 (1-p)^{6-2}$

p je vjerojatnost da u jednom pokušaju dobijemo „sretan" pogodak

Binomna i Poissonova razdioba su povezane,
ako gledamo granični slučaj Poissonove razdiobe kad je
p jako mali, a n jako veliki onda su one približno iste
pod uvjetom da je umnožak p puta n jednak lambda

$\lambda = n \cdot p = \text{const.}$

$n \to \infty$

$p \to 0$

$$P(m) = \binom{n}{m} p^m (1-p)^{n-m}$$

$\downarrow \lim$

$$\frac{\lambda^m}{m!} e^{-m}$$