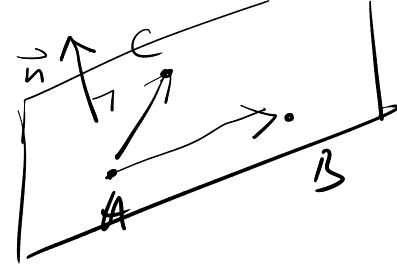


Nadji jednadžbu ravnine koja sadrži točke A(2, 3, 1), B(1, 1, 1), C(3, 2, 9).



Implicitna?

Parametarska? $(u, v) \mapsto$

$$\bar{A}x + \bar{B}y + \bar{C}z + \bar{D} = 0$$

$$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} \quad (\text{dakle})$$

$$(-1, -2, 0) \times (1, -1, 8)$$

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -2 & 0 \\ 1 & -1 & 8 \end{vmatrix} = -16\vec{i} - (-8)\vec{j} + 3\vec{k} = -16\vec{i} + 8\vec{j} + 3\vec{k}$$

$$\begin{aligned} a &= \|\vec{a}\| \\ b &= \|\vec{b}\| \\ \|\vec{a} \times \vec{b}\|^2 &= (a \cdot b)^2 \\ &= a^2 b^2 \end{aligned}$$

$$\begin{aligned} \text{počes} \\ -1+2 &= 1 \\ (256+64+5)+1 &= 5 \cdot 66 \\ 329+1 &= 330 \text{ OK} \end{aligned}$$

$$-16x + 8y + 3z + \bar{D} = 0 \quad B(1, 1, 1)$$

$$-16 \cdot 1 + 8 \cdot 1 + 3 \cdot 1 + \bar{D} = 0$$

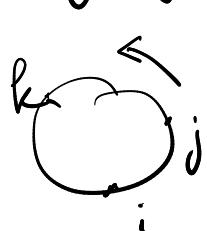
$$-16 + 8 + 3 = -\bar{D} \quad \bar{D} = 16 - 11 = 5$$

$$-16x + 8y + 3z - 5 = 0$$

$$(u, v) \mapsto A + u \cdot \underbrace{\overrightarrow{AB}}_{\vec{a}} + v \cdot \underbrace{\overrightarrow{AC}}_{\vec{b}}$$

$$\vec{r}(u, v) = \begin{pmatrix} 2 & -1 & u + 1 & v \\ 3 & -2 & u - 1 & v \\ 1 & -0 & u + 8 & v \end{pmatrix} = \begin{pmatrix} 2-u+v \\ 3-2u-v \\ 1+8v \end{pmatrix}$$

$$2 \cdot \binom{3+i}{j} \cdot \binom{3+k}{j-k} \cdot \binom{3+i}{k} = \binom{3+3k+i-j}{j+i} \cdot \binom{3i-1}{-k-j}$$



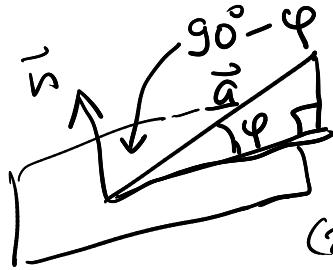
$$a+bi+cj+d\ell = \begin{pmatrix} 3+i-j+3k & -1+3i \\ i+j & -j-k \end{pmatrix}$$

5. Nadji kut izmedju pravca parametarski zadano $\vec{r}(t) = (2t, t-1, 2t) = (0, -1, 0) + t \underbrace{(2, 1, 2)}_{\vec{a}}$
 $(t) = (2t, t-1, 1)$

i ravnine $x + y + 4z = 5$.

$$\underbrace{\vec{n}}_{\vec{n} (1, 1, 4)}$$

$$\begin{aligned}\vec{a} \cdot \vec{n} &= \|\vec{a}\| \|\vec{n}\| \cos \varphi (\vec{a}, \vec{n}) \\ &= \|\vec{a}\| \|\vec{n}\| \cos (90^\circ - \varphi) \\ &= \|\vec{a}\| \|\vec{n}\| \sin \varphi\end{aligned}$$



$$(2,1,2) \quad (1,1,1)$$

$$\varphi = \arcsin \frac{\vec{a} \cdot \vec{n}}{\|\vec{a}\| \|\vec{n}\|} = \arcsin \frac{2+1+8}{\sqrt{9} \sqrt{18}}$$

$$\varphi = \arcsin \frac{11}{\sqrt{81} \sqrt{2}} = \arcsin \frac{11}{9\sqrt{2}} \underset{< 1}{<}$$

$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \leftarrow$ stand. baza od \mathbb{R}^3

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

ko za linearni operator $g : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ vrijedi $g(\vec{e}_1) = -\vec{e}_1 + 3\vec{e}_2$, $g(\vec{e}_2) = 2\vec{e}_1 - 3\vec{e}_2$, koliko je $g(\vec{e}_1 - 4\vec{e}_2)$? (u bazi e_1, e_2 ; koristi linearnost od g ! Rezultat napiši i u standardnoj bazi)

$$\begin{aligned}g(\vec{e}_1) &= -\vec{e}_1 + 3\vec{e}_2 \\ g(\vec{e}_2) &= 2\vec{e}_1 - 3\vec{e}_2\end{aligned}$$

$$1) g(\vec{e}_1 - 4\vec{e}_2) \stackrel{\text{lin.}}{=} g(\vec{e}_1) - 4g(\vec{e}_2)$$

$$= -\vec{e}_1 + 3\vec{e}_2 - 4(2\vec{e}_1 - 3\vec{e}_2) = -9\vec{e}_1 + 15\vec{e}_2$$

$$2) -9\vec{e}_1 + 15\vec{e}_2 = -9 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + 15 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -36-15 \\ -18+45 \\ 63 \end{pmatrix} = \begin{pmatrix} -51 \\ 27 \\ 63 \end{pmatrix}$$

7. Nadji volumen trostrane kose prizme s vrhovima ABC u osnovici dolje i EFG u osnovici gore (bridovi E do A, F do B i G do C) gdje su A(1, 0, 3), B(1, 0, 1), C(2, 0, 2), E(4, 1, 0). Nadji i koordinate preostala dva vrha.

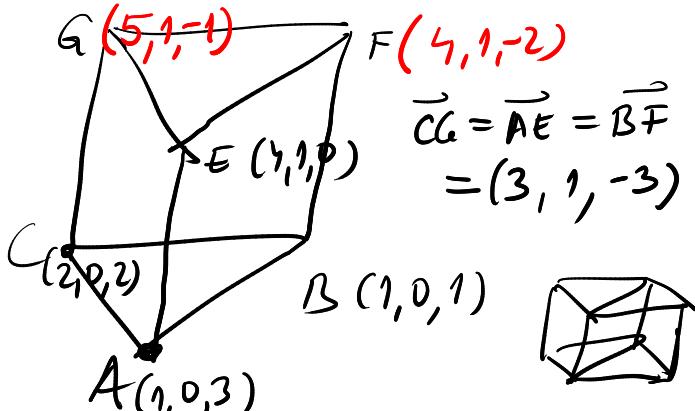
$$F = t \vec{BF} (B)$$

$$= \underbrace{B}_{B} + \underbrace{\vec{BF}}_{B}$$

$$= (1, 0, 1) + (3, 1, -3)$$

$$= (4, 1, -2)$$

$$G = (2, 0, 2) + (3, 1, -3) = (5, 1, -1)$$



$$V = \frac{1}{2} |(\vec{AB} \times \vec{AC}) \cdot \vec{AE}| = \frac{1}{2} \begin{vmatrix} 0 & 0 & -2 \\ 1 & 0 & -1 \\ 3 & 1 & -3 \end{vmatrix} = \frac{1}{2} (-2) \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix} = +1$$

Izračunaj kompoziciju permutacija skupa {A, B, C, D} (gornji red je početno, a donji red u svakom stupcu završno stanje, kao i obično) i odredi je li parna ili neparna

$$\begin{array}{l}
 (\text{ABC}D \\
 \text{BADC} \\
) \\
 \cdot \\
 (\text{ABCD} \\
 \text{DCBA} \\
) \\
 \cdot \\
 (\text{ABCD} \\
 \text{CDBA}
 \end{array}
 \quad
 \begin{pmatrix} \text{ABC}D \\ \text{BADC} \\ \text{CDBA} \end{pmatrix} \circ \begin{pmatrix} \text{ABC}D \\ \text{DCBA} \\ \text{CDBA} \end{pmatrix} = \begin{pmatrix} \text{ABC}D \\ \text{A} \text{BD} \text{C} \\ \text{A} \text{BD} \text{C} \end{pmatrix}$$

invertir = +1
paritet +1

9. Nadji udaljenost od $A(1, 0, 0)$ do pravca parametarski zadatog s $\vec{r}(t) = (2+t, 1-t, t)$.

$$\begin{aligned}
 & \text{Nadji udaljenost od } A(1, 0, 0) \text{ do pravca parametarski zadatog s } \vec{r}(t) = \\
 & (2+t, 1-t, t).
 \end{aligned}$$

$$\begin{aligned}
 & \text{V} = \frac{\vec{P}}{\|\vec{a}\|} = \frac{\|\vec{a} \times \vec{b}\|}{\|\vec{a}\|} = \frac{\|\vec{a} \times \vec{b}'\|}{\|\vec{a}\|} \\
 & \|\vec{a}\| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}
 \end{aligned}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ -1 & -1 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{vmatrix} \vec{i} = \vec{i} - \vec{j} - 2\vec{k}$$

$$\|\vec{a} \times \vec{b}\| = \sqrt{1^2 + (-1)^2 + (-2)^2} = \sqrt{1+1+4} = \sqrt{6}$$

$$v = d = \frac{\sqrt{6}}{\sqrt{3}} = \sqrt{\frac{6}{3}} = \sqrt{2}$$

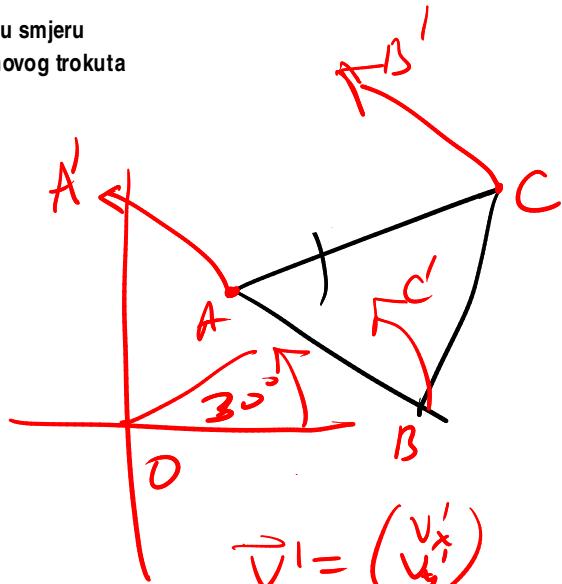
10. Trokut A(1, 2), B(3, 4), C(2, 3) rotiramo oko ishodišta za 30° u smjeru protivnog kazaljki na satu. Nadji koordinate vrhova A', B', C' novog trokuta

$$\overrightarrow{OA} \xrightarrow{30^\circ} \overrightarrow{OA'}$$

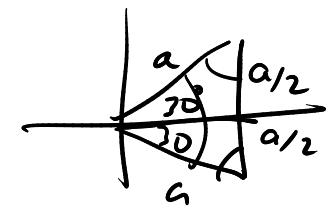
$$\overrightarrow{OA} (v_x, v_y)$$

$$\overrightarrow{OB} (3, 4)$$

$$\overrightarrow{OC} (2, 3)$$



$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

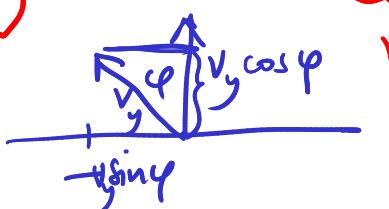


$$\sin 30^\circ = \frac{1}{2}$$

$$\vec{v}' = (v'_x, v'_y)$$

$$v'_x = v_x \cos \varphi - v_y \sin \varphi \quad \left. \right\} = R(\varphi) \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

$$v'_y = +v_x \sin \varphi + v_y \cos \varphi$$



$$\overrightarrow{OA'} \left(\frac{\sqrt{3}}{2} - 2 \frac{1}{2}, 1 \cdot \frac{1}{2} + 2 \frac{\sqrt{3}}{2} \right)$$

$$= A' \left(\frac{\sqrt{3}}{2} - 1, \frac{1}{2} + \sqrt{3} \right)$$

$$\overrightarrow{OC'} \left(2 \frac{\sqrt{3}}{2} - 3 \frac{1}{2}, 2 \frac{1}{2} + 3 \frac{\sqrt{3}}{2} \right) = C' \left(\frac{3\sqrt{3}}{2} - \frac{3}{2}, \frac{3}{2} + \frac{3\sqrt{3}}{2} \right)$$

$$\overrightarrow{OB'} = (3 \cos \varphi - 4 \sin \varphi, 3 \sin \varphi + 4 \cos \varphi)$$

$$R(\varphi) = \begin{pmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{pmatrix}$$