

$$A\vec{x} = \vec{b}$$

kuadraticna matica

$$\begin{cases} 2x + 3y = 7 \\ 3x - y = 6 \end{cases}$$

$$\begin{bmatrix} 2 & 3 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

$$\det \begin{bmatrix} 2 & 3 \\ 3 & -1 \end{bmatrix} = \begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix} = 2 \cdot (-1) - 3 \cdot 3 = -11$$

$$\det \begin{bmatrix} 7 & 3 \\ 6 & -1 \end{bmatrix} = -7 - 18 = -25$$

$$\det A_x = \begin{vmatrix} 2 & 7 \\ 3 & 6 \end{vmatrix} = 2 \cdot 6 - 7 \cdot 3 = 12 - 21 = -9$$

$$x = \frac{\det A_x}{\det A} = \frac{-25}{-11} = \frac{25}{11} \quad y = \frac{\det A_y}{\det A} = \frac{-9}{-11} = \frac{9}{11}$$

$$\begin{cases} 2x + 3y = 7 \\ 3x - y = 6 \end{cases}$$

$$2 \cdot \frac{25}{11} + 3 \cdot \frac{9}{11} = \frac{50 + 27}{11} = 7$$

$$3 \cdot \frac{25}{11} + (-\frac{9}{11}) = \frac{75 - 9}{11} = \frac{66}{11} = 6$$

$$\begin{cases} x + y + z = 1 \\ 2x - 3y + 4z = 2 \\ x - 4y + z = 3 \end{cases} \begin{matrix} (-2I) \\ (-I) \end{matrix} \begin{cases} x + y + z = 1 \\ -5y + 2z = 0 \\ -5y = 2 \end{cases}$$

$$\begin{cases} x + z + y = 1 \\ 2z + 5y = 0 \\ -5y = 2 \end{cases} \begin{matrix} + \frac{1}{5} III \\ III \end{matrix}$$

$$\begin{cases} x = \frac{12}{5} \\ z = -1 \\ y = -2/5 \end{cases}$$

$$\begin{cases} x + z = \frac{7}{5} \\ 2z = -2 \\ -5y = 2 \end{cases} \begin{matrix} (-1/2) \\ /:2 \\ /:(-5) \end{matrix}$$

$$I \quad \frac{12}{5} - \frac{2}{5} + (-1) = \frac{5}{5} = 1 \quad \checkmark$$

$$II \quad 2 \cdot \frac{12}{5} - 3 \cdot (-\frac{2}{5}) + 4 \cdot (-1) = \frac{10}{5} = 2 \quad \checkmark$$

$$III \quad \frac{12}{5} - 4 \cdot (-\frac{2}{5}) + (-\frac{5}{5}) = \frac{12 + 8 - 5}{5} = 3 \quad \checkmark$$

Gaussova metoda eliminacije

$$2 + \frac{1}{5} \cdot 2 = \frac{12}{5}$$

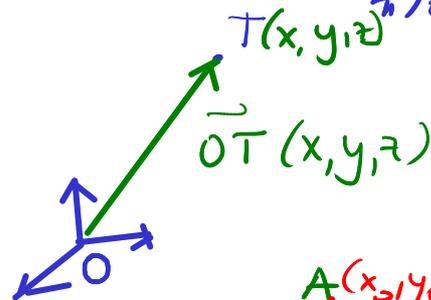
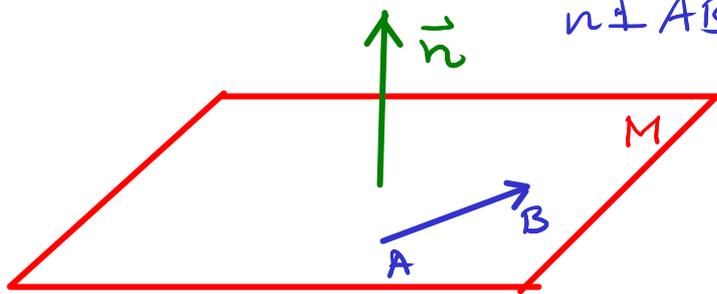
$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & -3 & 4 & 2 \\ 1 & -4 & 1 & 3 \end{array} \right) \begin{matrix} -2I \\ -I \end{matrix} \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -5 & 2 & 0 \\ 0 & -5 & 0 & 2 \end{array} \right) \begin{matrix} \\ \\ -II \end{matrix} \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -5 & 2 & 0 \\ 0 & 0 & -2 & 2 \end{array} \right) \begin{matrix} + \frac{1}{2} III \\ + III \end{matrix} \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & -5 & 0 & 2 \\ 0 & 0 & -2 & 2 \end{array} \right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 12/5 \\ 0 & -5 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array} \right) \begin{matrix} \cdot 2/5 \\ /:(-5) \end{matrix} \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 12/5 \\ 0 & 1 & 0 & -2/5 \\ 0 & 0 & 1 & -1 \end{array} \right) \begin{matrix} = x \\ = y \\ = z \end{matrix}$$

$$\vec{n} = (n_x, n_y, n_z)$$

$$\vec{n} \perp \vec{AB} \Leftrightarrow \vec{n} \cdot \vec{AB} = 0$$

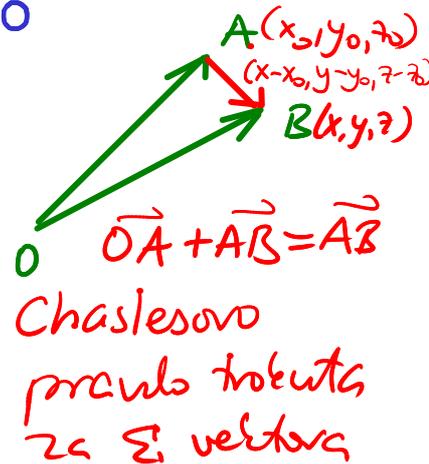
$$\|\vec{n}\| \|\vec{AB}\| \cos \varphi = 0$$



A točka, B pravokutnik
 $A(x_0, y_0, z_0)$ $B(x, y, z)$

$$\vec{AB} (\underline{x-x_0}, \underline{y-y_0}, \underline{z-z_0})$$

$$\vec{AB} = \vec{OB} - \vec{OA}$$



$$\vec{n} \cdot \vec{AB} = 0 \quad \vec{n} \perp \vec{AB}$$

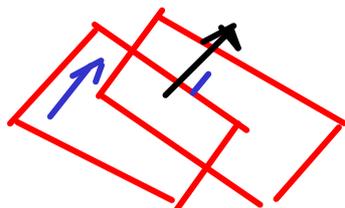
$$(n_x, n_y, n_z) \cdot (x-x_0, y-y_0, z-z_0) = 0$$

$$n_x(x-x_0) + n_y(y-y_0) + n_z(z-z_0) = 0$$

$$n_x x + n_y y + n_z z + (-n_x x_0 - n_y y_0 - n_z z_0) = 0$$

$$Ax + By + Cz + D = 0$$

$(A, B, C) \perp M$
 normala



$$2x + y + z + 3 = 0 \quad /:2$$

$$4x + 2y + 2z + 6 = 0$$

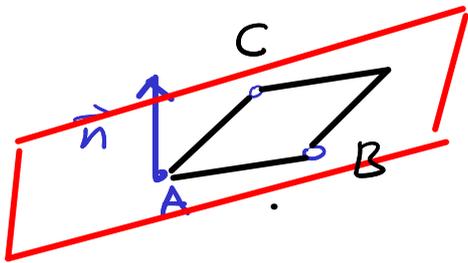
$$4x + 2y + 2z + 7 = 0$$

$$\vec{n} \parallel \vec{n}'$$

Tri točke na ravnini su A(2,1,1), B(2,5,6), C(3,3,3). Nađi neku implicitnu jednadžbu te ravnine.



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$$\vec{n} = \vec{AB} \times \vec{AC} \text{ je } \perp \vec{AB} \text{ i } \perp \vec{AC}$$

$$\vec{AB} (2-2, 5-1, 6-1)$$

$$\vec{AB} (0, 4, 5) \checkmark$$

$$\vec{AC} (3-2, 3-1, 3-1) = (1, 2, 2) \checkmark$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 4 & 5 \\ 1 & 2 & 2 \end{vmatrix} = \begin{vmatrix} 4 & 5 \\ 2 & 2 \end{vmatrix} \vec{i} - \begin{vmatrix} 0 & 5 \\ 1 & 2 \end{vmatrix} \vec{j} + \begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} \vec{k}$$

$$= (4 \cdot 2 - 5 \cdot 2) \vec{i} - (0 \cdot 2 - 5 \cdot 1) \vec{j} + (0 \cdot 2 - 4 \cdot 1) \vec{k}$$

$$\vec{n} = -2\vec{i} + 5\vec{j} - 4\vec{k}$$

$$A(2, 1, 1) \in M$$

$$-2x + 5y - 4z + D = 0$$

$$B(2, 5, 6) \checkmark$$

$$C(3, 3, 3)$$

$$-4 + 5 - 4 + D = 0 \quad D = 3$$

$$-2x + 5y - 4z + 3 = 0$$

$$+ \frac{2}{3}x - \frac{5}{3}y + \frac{4}{3}z$$

$$-4 + 25 - 24 + 3 = 0$$

$$-6 + 15 - 12 + 3 = 0$$

$$a \quad b \quad c$$

$$(\quad)x + (\quad)y + (\quad)z + D = 0$$

$$A(2, 1, 1) \quad 2a + b + c = -D$$

$$B(2, 5, 6) \quad 2a + 5b + 6c = -D$$

$$C(3, 3, 3) \quad 3a + 3b + 3c = -D$$

$$\begin{cases} 2a + b + c = 1 \\ 2a + 5b + 6c = 1 \\ 3a + 3b + 3c = 1 \end{cases}$$

$$\begin{cases} 2 \cdot \frac{2}{3} - \frac{5}{3} + \frac{4}{3} = 1 \\ 2 \cdot \frac{2}{3} + 5 \left(\frac{-5}{3} \right) + 6 \cdot \frac{4}{3} = \frac{3}{3} = 1 \\ 3 \cdot \frac{2}{3} + 3 \left(\frac{-5}{3} \right) + 3 \cdot \frac{4}{3} = \frac{3}{3} = 1 \end{cases}$$