moguće 56 bodova

prolaz od 28

2 od 28
3 od 35
4 od 42
5 od 49

Matematika 3, 12.2.2021. rok 2

4 boda

1. Nađi za koje je vrijednosti od x slijedeća funkcija nula, tj. koje su nultočke funkcije

$$f(x) = \sin\left(\frac{1}{x+1}\right)$$

Rješenje. Sinus kuta računamo tako da stavimo prvo krak na os x, a na drugom kraku gledamo točku koja je udaljena za 1 od ishodišta i očitamo njenu apscisu. Dakle nula je za kuteve $0, \pm\pi, \pm 2\pi, \ldots$, dakle $m\pi$ gdje je $m \in \mathbf{Z}$. Argument kad se to dešava je $\frac{1}{x+1}$ pa ga izjednačimo s $m\pi$, dakle $m(x+1)\pi = 1$ što je nemoguće za $m = 0$, a ako je $m \neq 0$, tada je $x + 1 = 1/(m\pi)$ pa su nultočke $x = 1/(m\pi) - 1$ gdje je $m = \pm 1, \pm 2, \pm 3, \ldots$.

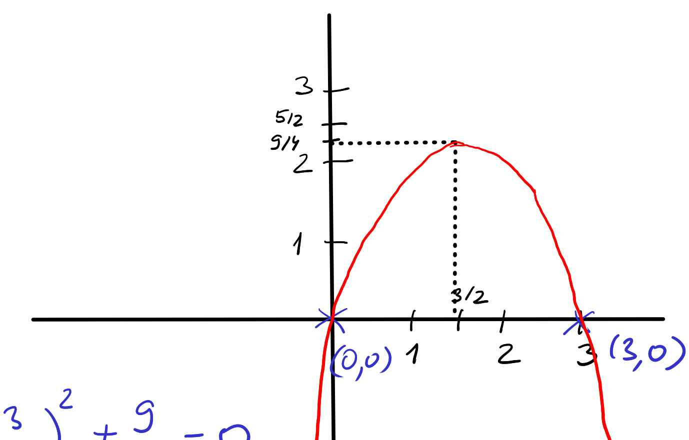
6 bodova

2. Skiciraj parabolu $y = -x^2 + 3x$, nađi njeno tjeme metodom dopunjavanja do na kvadrat (tj. ne preko gotovih formula) i, ako postoje, nađi koordinate sjecišta s osima x i y.

$$y = -x^2 + 3x = -(x^2 - 3x)$$
$$= -\left(\left(x - \tfrac{3}{2}\right)^2 - \left(\tfrac{3}{2}\right)^2\right)$$
$$y = -\underbrace{\left(x - \tfrac{3}{2}\right)^2}_{\text{tjeme je kad je to } 0} + \tfrac{9}{4}$$

$x_T = \frac{3}{2}$, $y_T = -0 + \frac{9}{4} = \frac{9}{4}$

$T\left(\frac{3}{2}, \frac{9}{4}\right)$



Os x... $y = 0$, $-\left(x - \frac{3}{2}\right)^2 + \frac{9}{4} = 0$

$x - \frac{3}{2} = \pm\sqrt{\frac{9}{4}}$

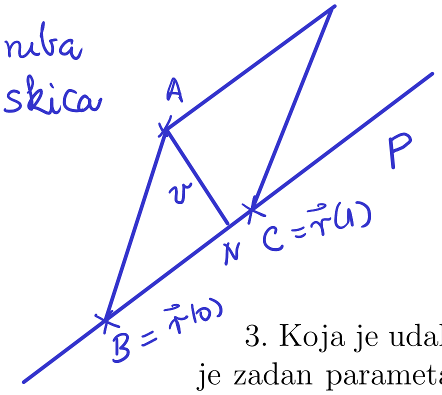
$x = \frac{3}{2} \pm \frac{3}{2}$ $\quad x_1 = 0 \quad (0,0)$, $\quad x_2 = 3 \quad (3,0)$

Os y... $x = 0$

$y = -x^2 + 3x$

$y = 0$, $(0,0)$

gruba skica



$$\overrightarrow{BC} = (1,2,1)$$

$$B = \vec{r}(t=0) = (1,0,-1)$$

$$C = \vec{r}(t=1) = (2,2,0)$$

$$d(A,N) = ?$$

$$d(A,N) = v = \frac{P_{\square}}{d(B,C)}$$

$$d(B,C) = \|(1,2,1)\| = \sqrt{1^2+2^2+1^2} = \sqrt{6}$$

6 bodova

3. Koja je udaljenost u prostoru među točkom $A(1,0,0)$ od pravca p koji je zadan parametarski $t \mapsto (t+1, 2t, t-1)$. Nađi i koordinate točke B na pravcu takve da je $\overline{AB}$ najkraća spojnica točke A s pravcem p.

$$P_{\square} = \|\overrightarrow{BC} \times \overrightarrow{BA}\|$$

$$\overrightarrow{BC} = (1,2,1)$$

$$\overrightarrow{BA} = (0,0,1)$$

$B(1,0,-1)$

$A(1,0,0)$

$C(2,2,0)$

$$\overrightarrow{BC} \times \overrightarrow{BA} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} \vec{i} - \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \vec{j} + \begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} \vec{k} = 2\vec{i} - \vec{j}$$

$$\|\overrightarrow{BC} \times \overrightarrow{BA}\| = \sqrt{2^2 + (-1)^2} = \sqrt{5}$$

$$d(A,N) = v = \frac{\|\overrightarrow{BC} \times \overrightarrow{BA}\|}{\|\overrightarrow{BC}\|} = \frac{\sqrt{5}}{\sqrt{6}}$$

4 boda

4. Podijeli polinom $-2x^3 + x^2 + x - 3$ polinomom $2x + 1$ s ostatkom.

$$(-2x^3 + x^2 + x - 3) : (2x+1) = -x^2 + x$$

$$-(-2x^3 - x^2)$$

$$2x^2 + x$$

$$-(2x^2 + x)$$

$$0x - 3$$

PROVJERA: $(2x+1)\cdot(-x^2+x) + (-3) =$

$$= -2x^3 - x^2 + 2x^2 + x - 3$$

$$= -2x^3 + x^2 + x - 3$$

2+2 boda

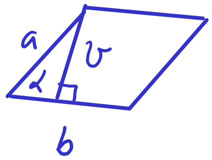
5. Neka su $\vec{a} = \vec{i} + \vec{j}$ i $\vec{b} = 2\vec{i} - \vec{j} + \vec{k}$ dva vektora u prostoru.
i) Nađi njihov skalarni umnožak $\vec{a} \cdot \vec{b}$
ii) Nađi vektorski umnožak $\vec{a} \times \vec{b}$. Pazi na predznake.

i) $(\vec{i} + \vec{j}) \cdot (2\vec{i} - \vec{j} + \vec{k}) = 1 \cdot 2 + 1 \cdot (-1) + 0 \cdot 1 = 1$

ii)
$$\vec{a} \times \vec{b} = \begin{vmatrix} 1 & 1 & 0 \\ 2 & -1 & 1 \\ \vec{i} & \vec{j} & \vec{k} \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix} \vec{i} - \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} \vec{j} + \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} \vec{k} =$$

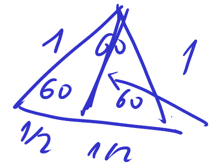
$$= \vec{i} - \vec{j} - 3\vec{k}$$

6.



$v_b = a\sin\angle\alpha = 5\sin 60^\circ = 5\frac{\sqrt{3}}{2}$

$P = b\,v_b = 7\cdot 5\frac{\sqrt{3}}{2} = \frac{35}{2}\sqrt{3}$



$\sqrt{1-(1/2)^2} = \sqrt{\frac{3}{4}}$

$\Rightarrow \sin 60 = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

ili: $P = a\,v_a = 5\,v_a = 5\cdot 7\sqrt{3}/2 = \frac{35}{2}\sqrt{3}$

$v_a = b\sin\alpha = 7\frac{\sqrt{3}}{2}$

6. Koristeći trigonometrijske funkcije, nađi visinu v i površinu P paralelograma kojem su stranice duljina $a = 5$ i $b = 7$, a kut između njih je $\gamma = 60^\circ$. 4 boda

Rješenje u terminima cijelih brojeva i korijena je u redu, npr. izraz tipa $7\frac{\sqrt{5}}{12}$ je prihvatljiv (kad bi bio točan), s obzirom da nemate kalkulatore.

7. Riješi sustav jednadžbi

5 boda

$$\begin{array}{l|rcrcrcr} \text{I} & 2x & - & y & + & 4z & = & 4 \\ \text{II} & 3x & + & y & - & z & = & 3 \\ \text{III} & x & - & y & + & z & = & -1 \end{array}$$

Gaussovom metodom eliminacije.

I − 2III, II − 3III

$$\begin{cases} y + 2z = 6 \\ 4y - 4z = 6 \quad /:2 \\ x - y + z = -1 \end{cases}$$

I − ½II

$$\begin{cases} y + 2z = 6 \\ 2y - 2z = 3 \\ x - y + z = -1 \end{cases}$$

$$\begin{cases} 3z = \frac{9}{2} \\ 2y - 2z = 3 \\ x - y + z = -1 \end{cases}$$

$z = 3/2$

$2y = 3 + 2\cdot\frac{3}{2} = 6 \Leftarrow 2y - 2z = 3$

$y = 3$

$x - 3 + \frac{3}{2} = -1 \Rightarrow x = -1 + 3 - 3/2 = 1/2$

Provjera

$2\cdot\frac{1}{2} - 3 + 4\,\frac{3}{2} = 4$

$3\cdot\frac{1}{2} + 3 - \frac{3}{2} = 3$

$\frac{1}{2} - 3 + \frac{3}{2} = -1$

DRUGI NAČIN

$$\left[\begin{array}{rrr|r} 2 & -1 & 4 & 4 \\ 3 & 1 & -1 & 3 \\ 1 & -1 & 1 & -1 \end{array}\right] \begin{array}{l} \\ -\frac{3}{2}\text{I} \\ -\frac{1}{2}\text{I} \end{array} \sim \left[\begin{array}{rrr|r} 2 & -1 & 4 & 4 \\ 0 & 5/2 & -7 & -3 \\ 0 & -1/2 & -1 & -3 \end{array}\right] \begin{array}{l} \\ \\ +\frac{1}{5}\text{II} \end{array}$$

$$\sim \left[\begin{array}{rrr|r} 2 & -1 & 4 & 4 \\ 0 & 5/2 & -7 & -3 \\ 0 & 0 & -\frac{12}{5} & -\frac{18}{5} \end{array}\right] \begin{array}{l} \cdot 1/2 \\ \cdot 2/5 \\ \cdot\left(-\frac{12}{5}\right) \end{array} \sim \left[\begin{array}{rrr|r} 1 & -1/2 & 2 & 2 \\ 0 & 1 & -14/5 & -6/5 \\ 0 & 0 & 1 & 3/2 \end{array}\right]$$

$\to x = 2 - 2\,\frac{3}{2} + \frac{1}{2}\,3 = \frac{4-6+3}{2} = \frac{1}{2}$

$\to y = -6/5 + 14/5\,\frac{3}{2} = \frac{-12+42}{10} = 3$

$\to z = 3/2$

4 boda

8. Nađi skup rješenja nejednadžbe

$$(x-3)(x+4)(x-1) < 0$$

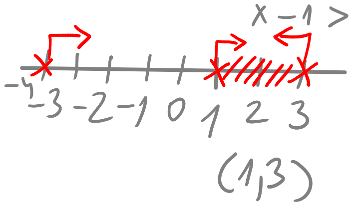
Slučajevi

I: $x-3<0$, $x<3$; $x+4>0$, $x>-4$; $x-1>0$, $x>1$

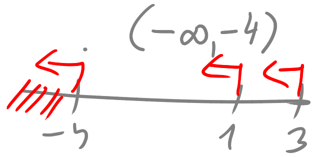
II: $x-3<0$; $x+4<0$; $x-1<0$

III: $x-3>0$; $x+4<0$; $x-1>0$

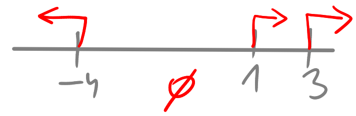
IV: $x-3>0$; $x+4>0$; $x-1<0$

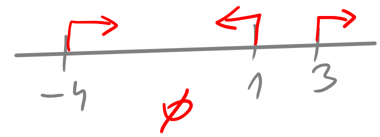


$(1,3)$



$(-\infty,-4)$





$S = (1,3) \cup (-\infty,-4)$

9. Pomnoži matrice

3 boda

$$\begin{pmatrix} 1 & 5 \\ 4 & 0 \end{pmatrix} \cdot \begin{pmatrix} -2 & 1 & 5 \\ 1 & 0 & 6 \end{pmatrix} = \begin{pmatrix} 1\cdot(-2)+5\cdot 1 & 1\cdot 1+5\cdot 0 & 1\cdot 5+5\cdot 6 \\ 4\cdot(-2)+0\cdot 1 & 4\cdot 1+0\cdot 0 & 4\cdot 5+0\cdot 6 \end{pmatrix}$$

Kad pišete rješenje treba biti jasno kojeg je matrica tipa.

$$= \begin{pmatrix} 3 & 1 & 35 \\ -8 & 4 & 20 \end{pmatrix}$$

2×3

10. Za slijedeće veličine napiši njihove egzaktne vrijednosti ako su definirane, a ako nisu definirane napiši da nisu definirane. Sve funkcije gledamo kao funkcije realne varijable osim e) i g) kad gledamo i kompleksne vrijednosti.

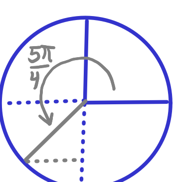
2 a) $\log_3(9^{3/4})$ $= \log_3((3^2)^{3/4}) = \log_3(3^{6/4}) = 6/4 = 3/2$

2 b) $(\cos(x))^2$ ako je $\sin(x) = 0.21$ $\cos^2 x + \sin^2 x = 1 \Rightarrow \cos^2 x = 1-\sin^2 x = 1-0.21^2 = 1-0.0441 = 0.9559$

2 c) $\log_3(9x^4)$ ako je $\log_3(x) = 3$ $\log_3(9x^4) = \log_3 9 + \log_3 x^4 = \log_3 3^2 + 4\log_3 x = 2+4\cdot 3 = 14$

d) $\frac{\log_3(z^6)}{\log_9(z^9)} = \frac{4}{3}$ 3 boda

e) $\exp(5i\pi/4)$ gdje je $i=\sqrt{-1}$ 2 boda



$\cos\frac{5\pi}{4} = \sin\frac{5\pi}{4} = -1/\sqrt{2} = -\sqrt{2}/2$

2 b f) Izračunaj $(i^2+i^3+i^4)\cdot(i+1)$ gdje je $i=\sqrt{-1}$ kao kompleksni broj u obliku $a+bi$, $a,b\in\mathbf{Z}$ $(\cancel{-1}-i+\cancel{1})(i+1) = 1-i = 1-1\cdot i$ $a=1, b=-1$

3 b g) (dijeljenje dva kompleksna broja, rješenje u obliku $x+yi$ gdje $x,y\in\mathbf{Q}$)

e) $\exp\frac{5\pi}{4}i = \cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4} = \frac{-\sqrt{2}}{2} - \frac{i\sqrt{2}}{2}$

$$\frac{4-3i}{1+4i} = \frac{4-3i}{1+4i}\cdot\frac{1-4i}{1-4i} = \frac{4-3i-16i-12}{1+16} = \frac{-8}{17} + \frac{(-19)}{17}i$$

d) $3^{\log_3 x} = x$

$9^{\log_9 x} = x$

$\| $

$(3^2)^{\log_9 x} = 3^{2\log_9 x}$

$2\log_9 x = \log_3 x \Rightarrow \frac{\log_3 z^6}{\log_9 z^9} = \frac{6\log_3 z}{9\log_9 z} = \frac{6\cdot 2\cancel{\log_9 z}}{9\cdot\cancel{\log_9 z}} = \frac{12}{9} = \frac{4}{3}$

$\uparrow \log_3 9$