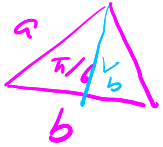


zadarmat2 13.6.2023. IME i PREZIME:

2+4=6b

1. Nadji površinu trokuta i polumjer njemu upisane kružnice ako su dvije stranice $a = 5$, $b = 6$ i kut između te dvije stranice je $\pi/6$ radijana.



$$v_b = a \sin \frac{\pi}{6}$$

$$P = \frac{b \cdot v_b}{2} = \frac{b \cdot a \sin \frac{\pi}{6}}{2} = \frac{5 \cdot 6 \cdot \sin \frac{\pi}{6}}{2} = \frac{30}{2} = 15$$

$$P_{\Delta} = \frac{p \cdot a}{2} + \frac{p \cdot b}{2} + \frac{p \cdot c}{2} \Rightarrow p = \frac{2P}{a+b+c} = \frac{15}{14.0064} =$$

$$c^2 = a^2 + b^2 - 2ab \cos \frac{\pi}{6} = 25 + 36 - 60 \frac{\sqrt{3}}{2} \Rightarrow c = \sqrt{61 - 30\sqrt{3}} \approx 3.0064$$

$$p = \frac{15}{(11 * (\sqrt{61 - (30 * (\sqrt{3}))))}$$

$$p = 0.453576948349$$

2. Promatrajte trapez kojem su osnovice duljina $a = 8$ i $c = 4$, oba unutarnja kuta α, β uz dulju osnovicu šiljasta, visina trapeza je $v = 3$ i jedan od krakova je $b = 4$. Nadji sinus kuta β kojeg zatvara ta stranica b s duljom osnovicom a i sinus kuta α kojeg zatvara drugi krak d s duljom osnovicom a .

2+4 = 6b

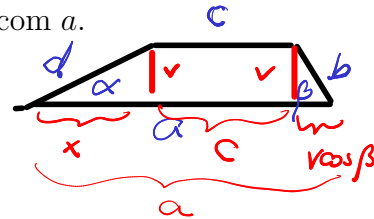
$$d \sin \alpha = v = b \sin \beta$$

$$3 = 4 \sin \beta \Rightarrow \sin \beta = \frac{3}{4}$$

$$\Rightarrow \sin \beta = \frac{3}{4}$$

$$\Rightarrow \cos \beta = \sqrt{1 - \sin^2 \beta} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$$

$$\Rightarrow v \cos \beta = \frac{3\sqrt{7}}{4} = 1.9844$$



$$a = x + c + v \cos \beta$$

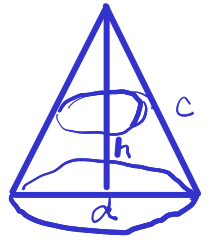
$$x = a - c - v \cos \beta$$

$$= 8 - 4 - \frac{3\sqrt{7}}{4} \approx (8 - 4) - \frac{(3 * (\sqrt{7}))}{4} = 2.0156$$

$$\sin \alpha = \frac{v}{d} = \frac{v}{\sqrt{x^2 + v^2}} = \frac{3}{\sqrt{2.0156^2 + 9}} = 0.2296564$$

2+2=2=6b

3. Uspravni stožac ima kao bazu krug promjera $d = 4$ i izvodnicu duljine $c = 10$. Nadjvi visinu, volumen i površinu (oplošje) stošca.



visina

$$h = \sqrt{c^2 - \left(\frac{d}{2}\right)^2} = \sqrt{100 - 4} = \sqrt{96} = \sqrt{6 \cdot 16} = 4\sqrt{6}$$

volumen $V = \frac{1}{3} B \cdot h = \frac{1}{3} \left(\frac{d}{2}\right)^2 \pi \cdot \sqrt{c^2 - \left(\frac{d}{2}\right)^2} = \frac{1}{3} 4\pi \cdot 4\sqrt{6} = \frac{16\pi\sqrt{6}}{3}$

≈ 9.79795897113

≈ 41.0415945652

oplošje

$$P = B + P_{\text{plast}} = \left(\frac{d}{2}\right)^2 \pi + \underbrace{c \cdot \pi \cdot \frac{d}{2}}_{\text{cm d/2}} = \frac{16}{4} \pi + \frac{4 \cdot 10}{2} \pi = 24\pi$$

≈ 75.3982236862

2+3=2=7b

4. Nadjvi skalarni umnožak, vektorski umnožak i kosinus kuta izmedju vektora $\vec{a} = \vec{i} - 3\vec{k}$, $\vec{b} = \vec{i} + \vec{j} - \vec{k}$.

$$\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1$$

$$\vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{i} = \vec{j} \cdot \vec{k} = \dots = 0$$

$$(\vec{i} - 3\vec{k}) \cdot (\vec{i} + \vec{j} - \vec{k})$$

$$= (1, 0, -3) \cdot (1, 1, -1)$$

$$= 1 \cdot 1 + 0 \cdot 1 - 3(-1)$$

$$= 1 + 0 + 3 = 4$$

$$(\vec{i} - 3\vec{k}) \times (\vec{i} + \vec{j} - \vec{k}) =$$

$$= \vec{k} + \vec{j} - 3\vec{j} + 3\vec{i}$$

$$= 3\vec{i} - 2\vec{j} + \vec{k}$$

vektorski: $\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0$

$$\vec{i} \times \vec{j} = -\vec{j} \times \vec{i} = \vec{k}$$

$$\vec{j} \times \vec{k} = -\vec{k} \times \vec{j} = \vec{i}$$

$$\vec{k} \times \vec{i} = -\vec{i} \times \vec{k} = \vec{j}$$

$$\cos \angle(\vec{a}, \vec{b}) = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|} = \frac{4}{\sqrt{1+3^2} \sqrt{1+1+1}} = \frac{4}{\sqrt{10} \sqrt{3}} = \frac{4\sqrt{30}}{30} = \frac{2\sqrt{30}}{15}$$

≈ 0.73029674334

ili $\sqrt{\frac{16}{30}} = \sqrt{\frac{8}{15}}$

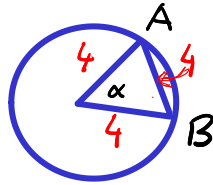
PROVJERA: $(\vec{a} \cdot \vec{b})^2 + (\vec{a} \times \vec{b})^2 = a^2 b^2$

$$16 + (3^2 + 2^2 + 1^2) = 30$$

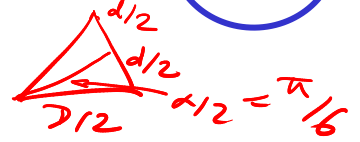
$$16 + 14 = 30 \text{ ŠTIMA!}$$

2+2+2=6b

5. Nadjj duljinu kružnog luka l , površinu kružnog isječka P , i duljinu pripadne tetive $d(A, B)$ ako je pripadni središnji kut $\alpha = 60^\circ$ i promjer kruga je $D = 8$.



$$l: \overset{2r\pi}{\cancel{2\pi}} = \alpha: 2\pi \Rightarrow l = \frac{\alpha D}{2} = \frac{(\pi/3) \cdot 8}{2} = \frac{4\pi}{3}$$

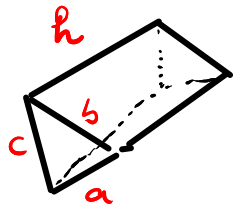


$$P: \underbrace{\left(\frac{D}{2}\right)^2 \pi}_{r^2 \pi} = \alpha: 2\pi \Rightarrow P = \frac{\left(\frac{D}{2}\right)^2 \pi \alpha}{2\pi} = \frac{D^2 \alpha}{8} = \frac{64 \cdot \pi/3}{8} = \frac{8\pi}{3}$$

$$d = 2 \frac{d}{2} = 2 \frac{D}{2} \cdot \sin \frac{\pi}{6} = 8 \sin 30^\circ = 4$$

6 bodova

6. Zadana je uspravna trostrana prizma kojoj je visina $h = 5$, a osnovica jednakokrani trokut osnovice $a = 3$ i krakova $b = c = 8$. Nadjj volumen i oplošje prizme.



$$B = \frac{a v_a}{2} = \frac{3 \cdot \sqrt{61.75}}{2} = \frac{(3/2) \cdot (\text{sqrt}((8 \cdot 8) - (9/4)))}{2} \approx 11.7871752341$$

$$P = 2B + \underbrace{a \cdot h + b \cdot h + c \cdot h}_{3 \text{ pravokutne plohe}}$$

$$= a \cdot v_a + \overset{15}{a \cdot h} + \overset{40}{b \cdot h} + \overset{40}{c \cdot h} = 3 \sqrt{61.75} + 95 = 118.574350468$$



$$V = B \cdot h = \frac{a v_a h}{2} = \frac{a \sqrt{b^2 - (a/2)^2}}{2} h = 12 \sqrt{61.75} = 94.297401873$$

$$8 \times 11.787 \dots$$

$$v_a = \sqrt{b^2 - (a/2)^2} = \sqrt{64 - (3/2)^2} = \sqrt{61.75}$$

$$v_a \approx 7.85811682275$$

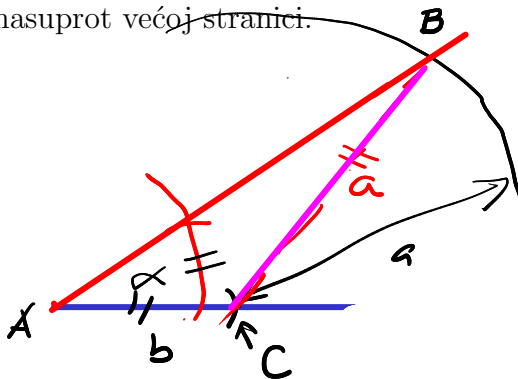
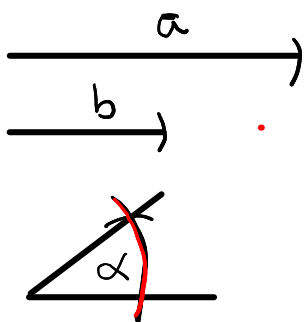
2 boda

7. Definiiraj konveksni skup. (Kad je podskup ravnine konveksan)

Podskup ravnine S je konveksan ako sa svake dvije točke sadrži i njihovu spojnicu, tj. ako za svake dvije točke A i B u S cijela dužina \overline{AB} je podskup od S .

3 boda

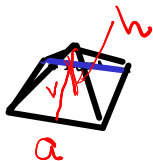
8. Skiciraj geometrijsku konstrukciju trokuta kojem su zadane dvije stranice i kut nasuprot većoj stranici.



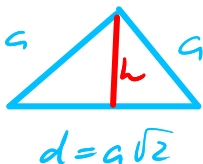
Prvo prenesemo kut α i odrezak duljine b na jednom kraku ~~sa krajnjom~~ točkom C , onda crtamo kružni luk oko C s radijusom a i njegov presjek s drugim krakom kuta α je točka B .

4 boda

9. Zadana je četverostrana pravilna uspravna piramida nad kvadratom sa stranicom $a = 6$. Ako su sva pobočna 4 trokuta jednakostranična (sve 3 stranice iste), kolika je visina h piramide i kolika je visina v svake pobočne stranice piramide?



Prvo rješenje: presjek nad dijagonalom kvadrata ima za osnovicu dijagonalu kvadrata i krakove a .



$$v = \sqrt{a^2 - (a/2)^2}$$

$$= \sqrt{\frac{3}{4} a^2} = \frac{\sqrt{3}}{2} a = 0.866 \cdot a$$

$$= 3 \cdot (\text{sqrt}(3))$$

$$\approx 5.19615242271$$

$$h = \sqrt{a^2 - (d/2)^2} = \sqrt{a^2 - \left(\frac{a\sqrt{2}}{2}\right)^2}$$

$$= \sqrt{a^2 - \frac{a^2}{2}} = \sqrt{\frac{a^2}{2}}$$

$$= \frac{a}{\sqrt{2}} = 0.707 a$$

$$= \frac{\sqrt{2}}{2} 6 = 3\sqrt{2} = \sqrt{18}$$

$$= 4.24264...$$

Drugo rješenje: koristimo presjek nad polovicom kvadrata



$$h^2 = a^2 - (a/2)^2 = \frac{3}{4} a^2 - \frac{a^2}{4} = \frac{2}{4} a^2 = \frac{a^2}{2}$$

$$\Rightarrow h = \frac{a\sqrt{2}}{2} \approx 4.24$$