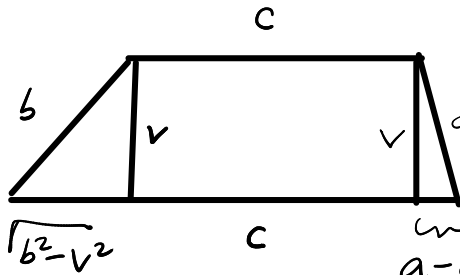
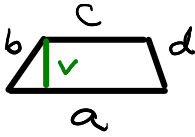


zadarmat2 . 5.7.2023. IME i PREZIME:

4 boda

1. Promatrajte trapez kojem su osnovice duljina $a = 8$ i $c = 4$, oba kuta uz dulju osnovicu šiljasta, visina je $v = 3$ i jedan od krakova je $b = 4$. Nadji duljinu drugog kraka d .



$$d^2 = v^2 + (a - c - \sqrt{b^2 - v^2})^2 = 9 + (4 - \sqrt{7})^2 = 9 + 16 - 2 \cdot 4 \cdot \sqrt{7} + 7 = 32 - 8\sqrt{7}$$

$$a - c - \sqrt{b^2 - v^2} = 8 - 4 - \sqrt{4^2 - 3^2} = 4 - \sqrt{7} \approx 1.35424868894$$

$$d = \sqrt{32 - 8\sqrt{7}} = \sqrt{\text{sqrt}(32 - (8 * (\text{sqrt}(7))))} \approx 3.2915$$

4 boda

2. Ako su dvije stranice u trokutu $a = 7$, $b = 5$ i kut između njih $\gamma = \pi/6$ radijana, nadji treću stranicu c i sinus kuta β nasuprot stranici b .

kosinsov teorem

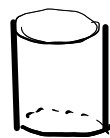
$$c = \sqrt{a^2 + b^2 - 2ab \cos \frac{\pi}{6}} = \sqrt{49 + 25 - 2 \cdot \frac{\sqrt{3}}{2} \cdot 5 \cdot 7} = \sqrt{74 - 35\sqrt{3}} \approx 3.6576$$

sinusov teorem

$$\frac{\sin \beta}{\sin \gamma} = \frac{b}{c} \Rightarrow \sin \beta = \frac{b}{c} \sin \gamma = \frac{5 \cdot \sin \frac{\pi}{6}}{\sqrt{74 - 35\sqrt{3}}} = \frac{5}{2\sqrt{74 - 35\sqrt{3}}} \approx 0.6835$$

4 boda

3. Promatraj uspravni valjak s kružnim promjerom 10 cm i visinom od 8 cm. Nadj njegovu oplošje P i njegov volumen V .



$$P = 2B + P_{\mu} = 2\left(\frac{d}{2}\right)^2 \pi + d\pi \cdot h = 2 \cdot 25\pi + 10\pi \cdot 8 = \underline{130\pi \text{ cm}^2} \approx 408.4 \text{ cm}^2$$

$$V = B \cdot h = \left(\frac{d}{2}\right)^2 \pi \cdot h = 25\pi \cdot 8 \text{ cm}^3 = \underline{200\pi \text{ cm}^3} \approx 628.3 \text{ cm}^3$$

5 bodova

4. Nadj skalarni umnožak, vektorski umnožak i kosinus kuta izmedju vektora $\vec{a} = 5\vec{i} + 3\vec{k}$, $\vec{b} = 2\vec{i} - 2\vec{j} - \vec{k}$.

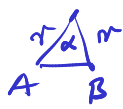
$$3b. \left\{ \begin{array}{l} \vec{a} \cdot \vec{b} = 5 \cdot 2 + 0 \cdot (-2) + 3 \cdot (-1) = 7 \\ \cos \angle(\vec{a}, \vec{b}) = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|} = \frac{7}{\sqrt{5+9} \sqrt{4+4+1}} = \frac{7}{3\sqrt{14}} \approx 0.40016 \end{array} \right.$$

$$2b. \left\{ \begin{array}{l} \vec{a} \times \vec{b} = (5\vec{i} + 3\vec{k}) \times (2\vec{i} - 2\vec{j} - \vec{k}) = \underbrace{-10\vec{i} \times \vec{j}}_{\vec{k}} - \underbrace{5\vec{i} \times \vec{k}}_{(-\vec{j})} \\ \quad + \underbrace{6\vec{k} \times \vec{i}}_{\vec{j}} - \underbrace{6\vec{k} \times \vec{j}}_{-\vec{i}} = 6\vec{i} + 11\vec{j} - 10\vec{k} \end{array} \right.$$

Projice

$$\left. \begin{array}{l} \|\vec{a} \times \vec{b}\|^2 + (\vec{a} \cdot \vec{b})^2 = (36 + 121 + 100) + 49 = 306 \\ \|\vec{a}\|^2 \|\vec{b}\|^2 = 37 \cdot 9 = 306 \end{array} \right\} 306 = 306$$

5 bodova



$$d(A,B) = 2 \frac{d(A,B)}{2} = 2 r \sin \frac{\alpha}{2} = 2 \cdot \frac{D}{2} \sin \frac{40^\circ}{2} = D \sin 20^\circ = 2.05212$$

5. Nadji duljinu kružnog luka l , površinu kružnog isječka P , i duljinu pripadne tetive $d(A, B)$ ako je pripadni središnji kut $\alpha = 40^\circ$ i promjer kruga je $D = 6$.

$$\frac{l}{D\pi} = \frac{\alpha^\circ}{360^\circ} \Rightarrow l = \frac{D\alpha^\circ\pi}{360^\circ} = \frac{6 \cdot 40^\circ \cdot 3.14159}{360^\circ} = \frac{6}{9} \cdot 3.14159 =$$

$$= \frac{2}{3} \cdot 3.14159 = \frac{6.28318}{3} = 2.09439$$

$$\frac{P}{(\frac{D}{2})^2\pi} = \frac{\alpha^\circ}{360^\circ} \Rightarrow P = \frac{1}{9} 9\pi = 3.14159$$

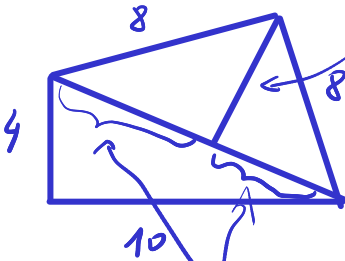
kosinusov teorem:

$$d(A,B)^2 = 2r^2 - 2r^2 \cos \alpha \Rightarrow d(A,B) = 2r^2(1 - \cos \alpha) = \frac{D^2}{2}(1 - \cos \alpha)$$

$$d(A,B) = \sqrt{5.2112} \approx 2.05212 \quad = 18(1 - \cos 40^\circ) = 4.2112$$

5 bodova

6. Četverokut ima stranice redom $a = 4$, $b = 10$, $c = d = 8$ i pravi kut između stranica a i b . Nadji površinu četverokuta (vidi sliku).



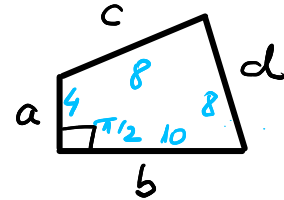
$$\sqrt{64 - 25} = \sqrt{39}$$

$$8^2 - (\sqrt{25})^2$$



$$P = P_{\Delta_1} + P_{\Delta_2} = \frac{40}{2} + \sqrt{25} \cdot \sqrt{39}$$

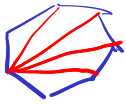
$$= 20 + \sqrt{25 \cdot 39} = 51.859$$



$$\text{ovak: } \frac{1}{2} \sqrt{4^2 + 10^2} = \frac{1}{2} \sqrt{116} = \sqrt{29}$$

3 boda

7. Napiši zbroj S svih unutarnjih kuteva u radijanima u ma kojem 7-kutu i broj dijagonala d u sedmerokutu.



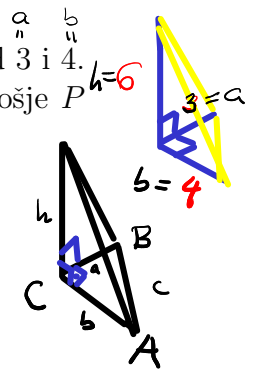
$$S = \underbrace{(7-2)}_{\# \Delta} \cdot \pi = 5\pi \text{ radijana} \approx 15.7 \text{ rad}$$

$$7-3=4$$

$$d = \frac{1}{2} 7 \cdot (7-3) = 14$$

5 bodova

8. Zadana je trostrana piramida nad pravokutnim trokutom s katetama od 3 i 4. Vrh piramide je iznad vrha pravog kuta, na visini 6. Nadj i volumen V i oplošje P piramide.



baza pravokutni trokut, hipotenuza je $\sqrt{3^2+4^2} = \sqrt{25} = 5 = c$

$$B = \frac{a \cdot b}{2} = 6$$

*u
pravok. baze*

$$V = \frac{B \cdot h}{3} = \frac{6 \cdot 6}{3} = 12$$

stranice

$$P_{\Delta_1} = \frac{b \cdot h}{2} = \frac{4 \cdot 6}{2} = 12$$

$$P_{\Delta_2} = \frac{a \cdot h}{2} = \frac{3 \cdot 6}{2} = 9$$

$$P_{\Delta_3} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{9.45965(9.45965 - \sqrt{52}) \dots} = \sqrt{9.45965 \cdot (9.45965 - \sqrt{52}) \cdot (9.45965 - \sqrt{45}) \cdot (9.45965 - \sqrt{25})} = 16.15546$$

Δ_3 stranice $\sqrt{b^2+h^2}, c, \sqrt{a^2+h^2}$

$$\begin{matrix} \downarrow & \downarrow & \downarrow \\ \sqrt{52} & 5 & \sqrt{45} \end{matrix}$$
$$s \approx \frac{1}{2}(5 + \sqrt{45} + \sqrt{52}) = 9.45965$$

$$P = B + P_{\Delta_1} + P_{\Delta_2} + P_{\Delta_3} = 6 + 12 + 9 + 16.15546 = 43.15546$$

