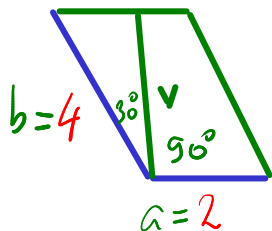


zadarmat2 8.lipnja 2022. IME i PREZIME:

Heronova formula $P = \sqrt{s(s-a)(s-b)(s-c)}$, $s = \frac{a+b+c}{2}$, implicitna jednadžba ravnine u prostoru $Ax + By + Cz + D = 0$ okomita na vektor $(A, B, C) = A\vec{i} + B\vec{j} + C\vec{k}$. $\cos 30^\circ = \frac{\sqrt{3}}{2} = \sin 60^\circ$, $\sin 30^\circ = \frac{1}{2} = \cos 60^\circ$. Vektorski umnožak $\vec{a} \times \vec{b} = (a_y b_z - a_z b_y)\vec{i} + (a_z b_x - a_x b_z)\vec{j} + (a_x b_y - a_y b_x)\vec{k}$, $\|\vec{a} \times \vec{b}\| = \|\vec{a}\| \|\vec{b}\| \sin(\vec{a}, \vec{b})$.

3 boda

1. Nadj površinu paralelograma koja ima stranice $a = 2$, $b = 4$ i kut između te dvije stranice je 120 stupnjeva.



$$v = 4 \sin 120^\circ = 4 \sin 60^\circ = 4 \cos 30^\circ = 4 \frac{\sqrt{3}}{2}$$

$$P = a \cdot v = 2 \cdot 4 \frac{\sqrt{3}}{2} = 4 \sqrt{3}$$

($v = \text{visina}$)

3 boda

2. Nadj parametarsku jednadžbu pravca koji prolazi točkom $T(2, 1, 5)$ i okomit je na ravninu $2x + 3y - 2z + 1 = 0$, tj. koji je uzduž smjera normale na tu ravninu.

$$\vec{r}(t) = \vec{r}_T + t \vec{n} \quad \vec{n} = (2, 3, -2)$$

$$x(t) = 2 + 2t$$

$$y(t) = 1 + 3t$$

$$z(t) = 5 - 2t$$

↑
T



4 boda

3. Koliko dijagonala ima 17-terokut? Koliko radijana ima svaki njen unutarnji kut (kod svakog vrha 17-terokuta)?

od svakog vrha do svakog drugog osim samog sebe i dva susjedna po jedna dijagonala svaka se broji dva put

$$d = \frac{n \cdot (n-3)}{2} = \frac{17 \cdot 14}{2} = 17 \cdot 7 = 70 + 49 = \underline{119}$$

$$\sum \text{kutova} = (n-2) \cdot \pi \text{ rad} = 15\pi \text{ rad}$$

$$17 \text{ kutova, dakle svaki je } d = \frac{15\pi}{17} \text{ rad.}$$

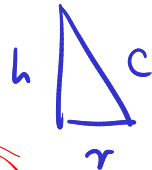
$$(15 * \pi) / 17 = 2.77199351787$$

5 bodova

4. Uspravni konus (stožac) ima kao osnovicu kružnicu polumjera $r = 2$, a svaka njena izvodnica jednaka je $c = 5$. Nadji volumen V i oplošje P stošca.



$$V = B \frac{h}{3}$$



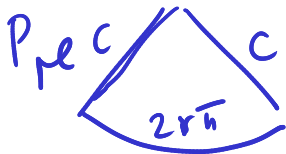
$$h = \sqrt{c^2 - r^2} \quad \text{PITAGORA}$$

$$h = \sqrt{25 - 16} = \sqrt{9} = 3$$

$$B = r^2 \pi = 4\pi$$

$$V = 4\pi \cdot \frac{\sqrt{5^2 - 4^2}}{3} = 4\pi$$

$$P = B + P_{pe} = 4\pi + 2 \cdot 5 \cdot \pi = 14\pi$$



$$P_{pe} : c^2 \pi = 2r\pi : 2c\pi$$

$$P_{pe} = rc\pi$$

$$\vec{a} \cdot \vec{b} = (2\vec{i} - 4\vec{k}) \cdot (3\vec{i} + \vec{j} - \vec{k}) = 2 \cdot 3 + (-4) \cdot (-1) = 10$$

4 boda

5. Nadji skalarni umnožak, vektorski umnožak i kosinus kuta između vektora $\vec{a} = 2\vec{i} - 4\vec{k}$, $\vec{b} = 3\vec{i} + \vec{j} - \vec{k}$.

$$\vec{a} \times \vec{b} = (2\vec{i} - 4\vec{k}) \times (3\vec{i} + \vec{j} - \vec{k}) = 2\vec{k} + 2\vec{j} - 12\vec{j} + 4\vec{i} + \vec{0} = 4\vec{i} - 10\vec{j} + 2\vec{k}$$

vidi formulu za vektorski umnožak na prvoj stranici

$$\cos 30^\circ = \frac{\sqrt{3}}{2} = \cos^2 15^\circ - \sin^2 15^\circ = 1 - 2\sin^2 15^\circ \Rightarrow \sin 15^\circ = \sqrt{\frac{1 - \cos 30^\circ}{2}}$$

5 bodova

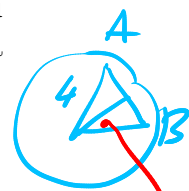
6. Nadji duljinu kružnog luka l , površinu kružnog isječka P , i duljinu pripadne tetive $d(A, B)$ ako je pripadni središnji kut $\alpha = 30^\circ$ i promjer kruga je $D = 8$.

$$2r\pi = D\pi = 8\pi \quad D = 2r \quad r = 4$$

$$\frac{\alpha}{360^\circ} = \frac{30^\circ}{360^\circ} = \frac{1}{12}$$

$$\frac{1}{12} D\pi = l$$

$$\frac{1}{12} \cdot 2r\pi = P$$



$$15^\circ = \frac{30^\circ}{2}$$

$$d(A, B) = 2 \cdot r \sin 15^\circ = 8 \sin 15^\circ = 2.07055... \quad l = \frac{1}{12} 8\pi = \frac{2}{3}\pi \quad P = \frac{16\pi}{12} = \frac{4\pi}{3}$$

$$= 8 \sqrt{\frac{1 - \sqrt{3}/2}{2}} = 8 \sqrt{\frac{2 - \sqrt{3}}{2}}$$

2+2 boda

7. Nadji površinu P i visinu na stranicu a u trokutu kojem su stranice $a = 5$, $b = 4$ i $c = 7$.

vidi Heronovu formulu na str.1`

$$S = \frac{a+b+c}{2} = 8$$

$$P = \sqrt{s(s-a)(s-b)(s-c)}$$

$$P^2 = 8 \cdot (8-5) \cdot (8-4) \cdot (8-7)$$

9.79795897113

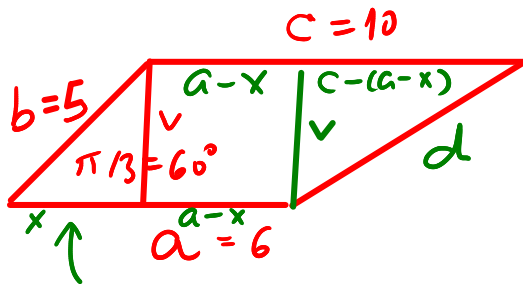
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$$P = \sqrt{8 \cdot 3 \cdot 4 \cdot 1} = \sqrt{96} = \sqrt{6 \cdot 16} = 4\sqrt{6}$$

$$P = \frac{a \cdot v_a}{2} \Rightarrow v_a = \frac{2P}{a} = \frac{2 \cdot 4\sqrt{6}}{5} = \frac{8\sqrt{6}}{5}$$

2+2+3 boda

8. Trapez s osnovicama $a = 6$, $c = 10$ i krakovima b i d , gdje znamo da je $b = 5$, a d ne znamo, ima kut pri vrhu u kojem se spajaju osnovica a i krak b jednak $\pi/3$ radijana. Nadjite visinu v trapeza, krak d i površinu P trapeza.



$$v = b \sin 60^\circ = b \frac{\sqrt{3}}{2} = 5 \frac{\sqrt{3}}{2}$$

$$P = \frac{a+c}{2} \cdot v = \frac{16}{2} \cdot 5 \frac{\sqrt{3}}{2} = 20\sqrt{3}$$

$$x = 5 \sin \frac{\pi}{3} = 5 \sin 30^\circ = 5 \frac{1}{2}$$

$$a-x = 6 - \frac{5}{2} = \frac{12-5}{2} = \frac{7}{2}$$

$$c-(a-x) = 10 - \frac{7}{2} = \frac{20-7}{2} = \frac{13}{2}$$

$$d^2 = v^2 + (c-a+x)^2 = \left(5 \frac{\sqrt{3}}{2}\right)^2 + \left(\frac{13}{2}\right)^2 = \frac{75}{4} + \frac{169}{4} = \frac{244}{4} = 61$$

$$d = \sqrt{61} \approx 7.81$$