5+5+4+4+6+5+5+6 = 40

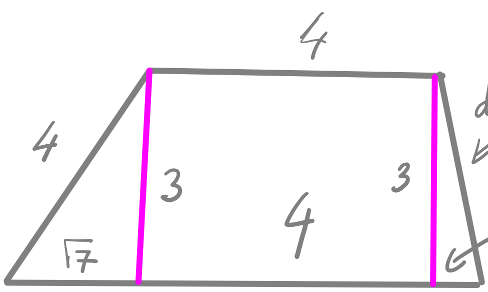
moguće 40 bodova, prolaz 20 bodova, trojka od 25, četvorka od 30, petica od 35

zadarmat2 9.6.2021. IME i PREZIME:

5 bodova

1. Promatrajte trapez kojem su osnovice duljina $a = 8$ i $c = 4$, oba kuta uz dulju osnovicu šiljasta, visina je $v = 3$ i jedan od krakova je $b = 4$. Nadji duljinu drugog kraka d.

d = sqrt((4 - sqrt(7))**2+3**2)



$$d = \sqrt{(8-4-\sqrt{7})^2 + 3^2} = \sqrt{16+9+7-8\sqrt{7}} = \sqrt{32-8\sqrt{7}} = 2\sqrt{8-2\sqrt{7}}$$

$= 3.2915\ldots$

$8-4-\sqrt{7} = 4-\sqrt{7}$

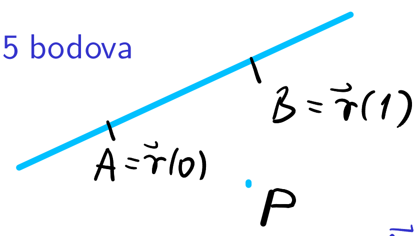
$c - a - \sqrt{b^2-v^2}$

$= 1.3542486889\ldots$

$\sqrt{16-9} = \sqrt{7}$

2. Nadji implicitnu jednadžbu ravnine (dakle oblika $Ax+By+Cz+D=0$ gdje su A, B, C, D realni brojevi) koja **sadrži** pravac na pravac $x = 2t$, $y = 3t+1$, $z = -t-2$, a **prolazi** kroz točku $P(2, 3, 4)$.

5 bodova



$A(0,1,-2)$ $B(2,4,-3)$

$\vec{AB}(2,3,-1)$ $\vec{AP}(2,2,6)$

$\vec{AB} \times \vec{AP} = (2\vec{i}+3\vec{j}-\vec{k}) \times (2\vec{i}+2\vec{j}+6\vec{k})$

$= 4\vec{k} - 12\vec{j} + (-6\vec{k}) + 18\vec{i} - 2\vec{j} + 2\vec{i}$

$= 20\vec{i} - 14\vec{j} - 2\vec{k}$

x	y	z	
0	1	−2	A
2	4	−3	B
2	3	4	P

$20x - 14y - 2z + D = 0 \Rightarrow D = -20x + 14y + 2z = 10$

(x, y, z) uzmi A ili B ili P

$n_x x + n_y y + n_z z + D = 0$

$(n_x, n_y, n_z) \propto \vec{AB} \times \vec{AP}$

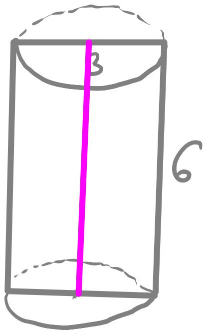
$20x - 14y - 2z + 10 = 0$ ili proporcionalno

npr. $10x - 7y - z + 5 = 0$ (*)

Provjera $A(0,1,-2)$, $B(2,4,-3)$, $P(2,3,4)$ zadovoljavaju (*)

4 boda

3. Neka je središnji presjek uspravnog valjka (presjek koji sadrži po jedan promjer svake osnovice i središnju os simetrije valjka) pravokutnik sa stranicama $a = 3$ i $b = 6$ gdje je b okomit na ravninu osnovice. Nadi površinu plašta (= pobočja) valjka.



$r = \frac{3}{2} \Rightarrow P_M = 2r\pi \cdot h = 2 \cdot \frac{3}{2} \cdot 6\pi = 18\pi$

P = 18 pi

4 boda

4. Nadji skalarni umnožak, vektorski umnožak i kosinus kuta izmedju vektora $\vec{a} = 2\vec{i} + 3\vec{k}$, $\vec{b} = \vec{i} - \vec{j} - \vec{k}$.

$\vec{a} \cdot \vec{b} = 2 \cdot 1 + 0 \cdot (-1) + 3(-1) = -1$

$\|\vec{a}\| = \sqrt{2^2 + 3^2} = \sqrt{13}$ ≈ 3.60555127546

$\|\vec{b}\| = \sqrt{1^2 + (-1)^2 + (-1)^2} = \sqrt{3} \approx 1.73205080757$

$\cos \measuredangle(\vec{a}, \vec{b}) = \frac{-1}{\sqrt{39}}$ = - 0.160128153805...

$\vec{a} \times \vec{b} = 2\vec{i} \times (-\vec{j}) + 2\vec{i} \times (-\vec{k}) + 3\vec{k} \times \vec{i} + 3\vec{k} \times (-\vec{j})$

$= -2\vec{k} + 2\vec{j} + 3\vec{j} + 3\vec{i} = 3\vec{i} + 5\vec{j} - 2\vec{k}$

$\|\vec{a} \times \vec{b}\| = \sqrt{9 + 25 + 4} = \sqrt{38}$ $\quad \sin \measuredangle(\vec{a}, \vec{b}) = \frac{\sqrt{38}}{\sqrt{39}}$

$40^\circ = \frac{2\pi}{9}$ rad $\qquad 20^\circ = \frac{\pi}{9}$ rad $\qquad \sin 2\alpha = 2\sin\alpha\cos\alpha$

6 bodova

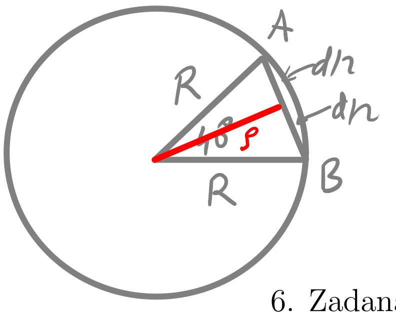
5. Nadji duljinu kružnog luka l, površinu kružnog isječka P, i duljinu pripadne tetive $d(A, B)$ ako je pripadni središnji kut $\alpha = 40^\circ$ i promjer kruga je $D = 6$.

$l = 2R\pi \frac{\alpha}{2\pi} = R\alpha = \frac{D}{2}\alpha = \frac{D}{2}\frac{2\pi}{9} = \frac{2\pi}{3} = 2.0943951\ldots$

$P_{isječka} = R^2\pi = \frac{2\pi}{9} : 2\pi \qquad P = \frac{D^2}{4}\pi\frac{1}{9} \qquad D^2 = 36 \qquad P_{isj} = \pi$

$\frac{d}{2} = R\sin 20^\circ \qquad \varrho = R\cos 20^\circ$

$d = 2R\sin 20^\circ = D\sin\frac{\pi}{9} = 6\sin\frac{\pi}{9} = 2.05212\ldots$



$P_\Delta = \frac{d}{2}\varrho = R^2\sin 20^\circ \cdot \cos 20^\circ = \frac{R^2}{2}\sin 40^\circ$

$P_\Delta \approx 2.89 \qquad = \frac{D^2}{4}\sin 20^\circ\cos 20^\circ$ ili $\frac{D^2}{8}\sin 40^\circ = \frac{9}{2}\sin 40^\circ$

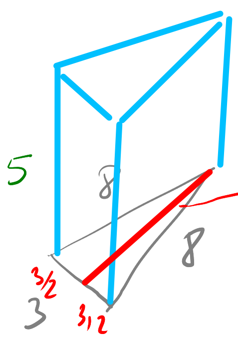
6. Zadana je uspravna trostrana prizma kojoj je visina $h = 5$, a osnovica jednakokračni trokut osnovice $a = 3$ i krakova $b = c = 8$. Nadji volumen i oplošje prizme.

5 bodova

$V = \frac{3}{4}\sqrt{247} \cdot 5 = \frac{15}{4}\sqrt{247} = 58.935876\ldots$

$P = 2P_\Delta + P_{\square} + 2P_{\square} = \frac{3}{2}\sqrt{247} + 15 + 80$

$= 95 + \frac{3}{2}\sqrt{247} = 118.57435\ldots$



$\sqrt{8^2 - (3/2)^2} = \sqrt{64 - \frac{9}{4}} = \frac{\sqrt{256-9}}{2} = \frac{\sqrt{247}}{2}$

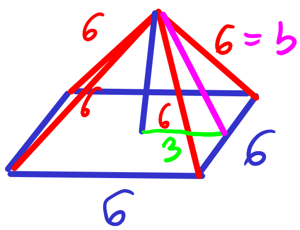
$P_\Delta = \frac{3}{2}\frac{\sqrt{247}}{2} = \frac{3}{4}\sqrt{247}$

5 bodova

7. Zadana je četverostrana pravilna uspravna piramida nad kvadratom sa stranicom $a = 6$. Ako su sva pobočna 4 trokuta jednakostranična (sve 3 stranice iste), kolika je visina h piramide, kolika je visina v svake stranice te koliki su oplošje i volumen piramide ?

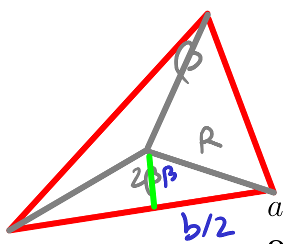
$$V_a = V = \sqrt{b^2-(a/2)^2} = \sqrt{36-9} = \sqrt{27} = 3\sqrt{3} = 5.19615\ldots$$

$$h = \sqrt{v^2-\left(\frac{a}{2}\right)^2} = \sqrt{27-9} = \sqrt{18} = 3\sqrt{2} = 4.24264\ldots$$



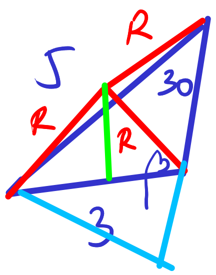
$$P = \frac{a^2\sqrt{3}}{4}\cdot 4 + a^2 = a^2(1+\sqrt{3}) = 36\cdot(1+\sqrt{3}) = 98.353829\ldots$$

$$\frac{a^2\sqrt{3}}{4} = \frac{a\cdot V_a}{2}$$



8. Ako su stranice trokuta $a = 3$, $b = 5$ i kut $\alpha = 30°$ nasuprot stranici a, nadji kut β nasuprot stranici b, površinu trokuta. i polumjer trokutu **opisane** kružnice. Postoje dva rješenja, jedno kad je β tupi kut, a jedno u kojem je šiljasti. Riješi za oba slučaja.

$$\frac{b}{2} = R\sin\beta \Rightarrow R = \frac{b}{2\sin\beta} = \frac{a}{2\sin\alpha} = \frac{c}{2\sin\gamma}$$



$$\frac{\sin\beta}{\sin 30°} = \frac{5}{3}$$

$$R = \frac{a}{2\sin\alpha} = \frac{3}{2\cdot\frac{1}{2}} = 3$$

$$\sin\beta = \frac{5}{3}\sin 30° = \frac{5}{3}\cdot\frac{1}{2} = \frac{5}{6}$$

2 rješenja

šiljasti $\beta < \pi/2$

tupi $\beta > \pi/2$

6 bodova

$\beta_1 = \arcsin(5/6) = 0.98511$ rad $= 56.44269$ deg $= 56°27'$

$\beta_2 = \pi - \arcsin(5/6) = 2.15648$ rad $= 123.5573$ deg $= 123°33'$