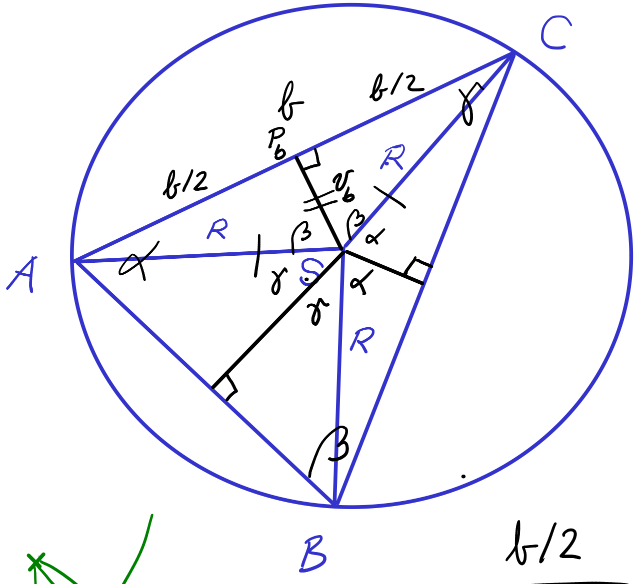


$R, v_b, 90^\circ$

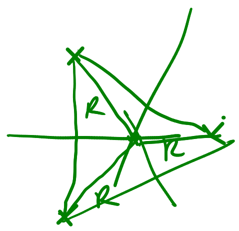
nasuprot većoj stranici

$\Rightarrow$

$\triangle ASP_b \cong \triangle CSP_b$

$\Rightarrow |\overline{AP_b}| = |\overline{P_bC}| = \frac{b}{2}$

Po teoremu o središnjem i obodnom kutu $\measuredangle ASP_b = \measuredangle CBP_b = \beta = \measuredangle ABC$



$\frac{b/2}{R} = \sin\beta$

$b = 2R\sin\beta$

$a = 2R\sin\alpha$

$c = 2R\sin\gamma$

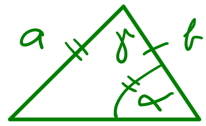
$b : a = \sin\beta : \sin\alpha$

SINUSOV TEOREM

u bilo kojem trokutu!

Zadatak Neka su zadani a, b, α

$\Rightarrow$ naći γ računski



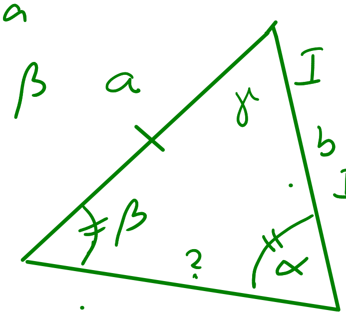
$a > b$

Rješenje: β iz $\sin\beta = \frac{b}{a}\sin\alpha$

i onda $\gamma = \pi - \alpha - \beta$

Zadatak α, β, a

Naći c



I $b = a\frac{\sin\beta}{\sin\alpha}$

II $\gamma = \pi - \alpha - \beta$

III $c^2 = a^2 + b^2 - 2ab\cos\gamma$

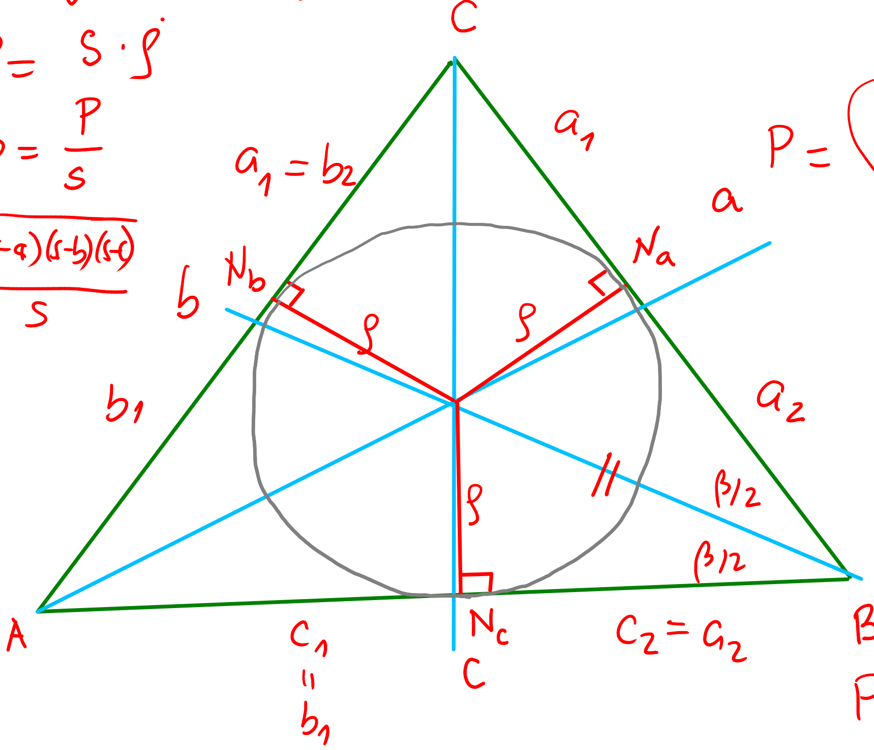
KOSINUSOV TEOREM

$P = \sqrt{s(s-a)(s-b)(s-c)}$ HERONOVA FORMULA

$P = s \cdot \rho$

$\rho = \frac{P}{s}$

$\rho = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$



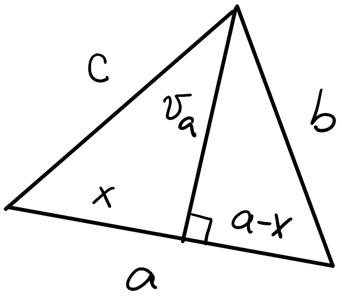
$P = \frac{b_1 \cdot \rho}{2} + \frac{b_1\rho}{2} + \frac{b_2\rho}{2} + \frac{a_1\rho}{2} + \frac{a_2\rho}{2} + \frac{a_2\rho}{2}$

$= \frac{b\rho}{2} + \frac{c\rho}{2} + \frac{a\rho}{2}$

$P = \frac{a+b+c}{2}\rho$

Heronova formula za površinu trokuta kojem su zadane sve tri stranice a,b,c

$$P = \frac{a \cdot v_a}{2}$$



$$v_a, x = ?$$

$$v_a^2 = c^2 - x^2 \quad \mathrm{I}$$

$$v_a^2 = b^2 - (a-x)^2$$

$$v_a^2 = b^2 - a^2 + 2ax - x^2 \quad \mathrm{II}$$

$$\mathrm{I} - \mathrm{II} \qquad 0 = c^2 - b^2 + a^2 - 2ax$$

$$x = \frac{c^2 + a^2 - b^2}{2a} \rightsquigarrow \mathrm{I}$$

$$v_a^2 = c^2 - \left(\frac{c^2 + a^2 - b^2}{2a}\right)^2$$

RAZLIKA KVADRATA $\quad š^2 - ž^2 = (š - ž)(š + ž)$

$$v_a^2 = \left(c - \frac{c^2 + a^2 - b^2}{2a}\right)\left(c + \frac{c^2 + a^2 - b^2}{2a}\right)$$

$$v_a^2 = \frac{2ac - c^2 - a^2 + b^2}{2a} \quad \frac{2ac + c^2 + a^2 - b^2}{2a}$$

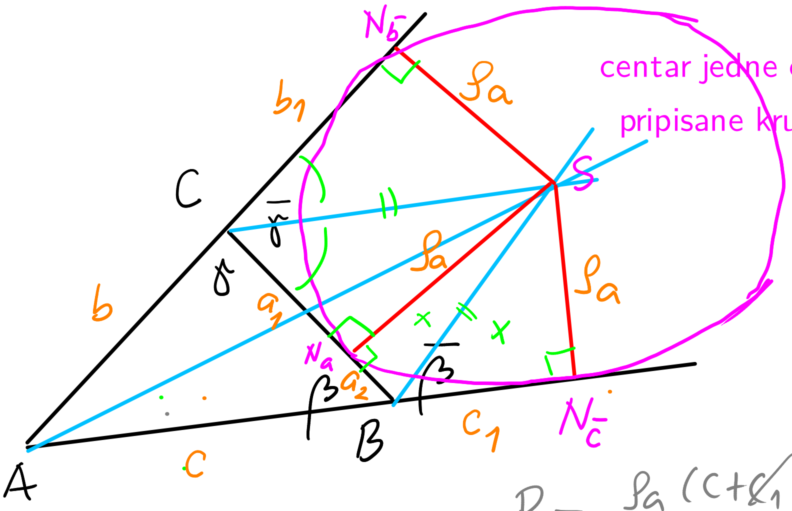
$$v_a^2 = \frac{-(a-c)^2 + b^2}{2a} \quad \frac{(a+c)^2 - b^2}{2a} \cdot = \frac{b^2 - (a-c)^2}{2a} \quad \frac{(a+c)^2 - b^2}{2a}$$

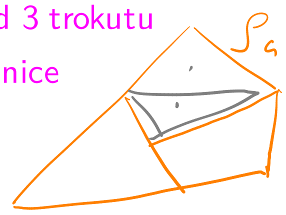
$$a^2 v_a^2 = \frac{(b - a - c)}{2} \quad \frac{b + a - c}{2} \quad \frac{a + c - b}{2} \quad \frac{a + c + b}{2} \quad 4$$

$$\frac{a^2 v_a^2}{4} = (s-a)\ (s-c)\ (s-a) \cdot s$$

$$P = \frac{a \cdot v_a}{2} \qquad P = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\rho = \frac{P}{s} = \frac{P}{\sqrt{s^2}} = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$





$$P = \frac{\rho_a(c+\cancel{c_1})}{2} + \frac{\rho_a(b+\cancel{b_1})}{2} \overbrace{- \frac{\rho_a \cancel{c_1}}{\cancel{2}} - \frac{\rho_a a_2}{2} - \frac{\rho_a \cancel{b_1}}{\cancel{2}} - \frac{\rho_a a_1}{2}}^{a}$$

$$\rho = \frac{P}{s} = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

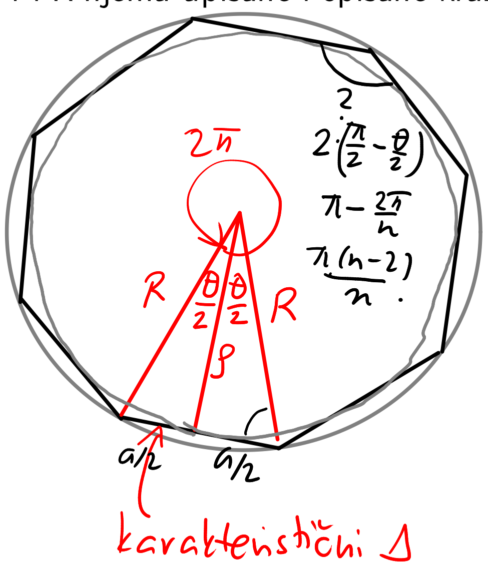
$$\rho_a = \frac{P}{s-a} = \sqrt{\frac{s(\cancel{s-a})(s-b)(s-c)}{(s-a)(\cancel{s-a})}} = \sqrt{\frac{s(s-b)(s-c)}{s-a}}$$

$$P = \frac{\rho_a}{2}(c+b-a)$$

$$\rho_a = \frac{2P}{c+b-a} = \frac{P}{s-a} \qquad \underbrace{\frac{c+b+a}{2} - \frac{2a}{2}}_{s-a}$$

Primjer. Pravilni poligoni. Poligon kojemu su sve stranice jednake i susjedne stranice zatvaraju jednake unutarnje kuteve. Ako je stranica pravilnog n-gona jednaka a, kolika je površina tog poligona i koliki su radijusi r i R njemu upisane i opisane kružnice ?

$\underline{\underline{n, a}}$



$$2 \cdot \left(\frac{\pi}{2} - \frac{\theta}{2}\right) = \pi - \frac{2\pi}{n} = \frac{\pi(n-2)}{n}$$

n-gon

$$P = n P_\Delta = n \cdot \frac{\rho a}{2}$$

$$\theta = \frac{2\pi}{n}$$

$$\rho = R\cos\frac{\theta}{2}$$

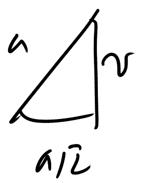
$$\frac{a}{2} = R\sin\frac{\theta}{2}$$

$$\frac{\rho}{a/2} = \operatorname{ctg}\frac{\theta}{2}$$

$$\rho = \frac{a}{2}\operatorname{ctg}\frac{\theta}{2} = \frac{a}{2\operatorname{tg}\theta/2}$$

$$P = n\frac{a^2}{4\operatorname{tg}\theta/2}$$

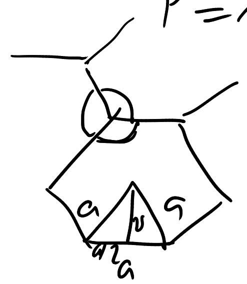
$$P_{O\,\text{upis}} = \rho^2\pi = \frac{a^2\pi}{4\operatorname{tg}^2\theta/2} = \frac{3a^2\pi}{4}$$



$$n = 6, \quad \frac{2\pi}{6} = \frac{\pi}{3} = 60^\circ$$

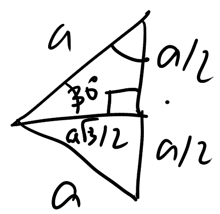
$$\sqrt{a^2 - (a/2)^2} = \sqrt{\frac{3}{4}a^2} = \frac{\sqrt{3}}{2}a$$

$$P = 6\frac{a^2}{4\operatorname{tg}\pi/6} = \frac{3}{2}a^2\sqrt{3}$$



$$\frac{a^2\sqrt{3}}{4} = \frac{a \cdot a\sqrt{3}/2}{2}$$

$$P_{\text{hex}} = \frac{6a^2\sqrt{3}}{4} = \frac{3}{2}a^2\sqrt{3}$$

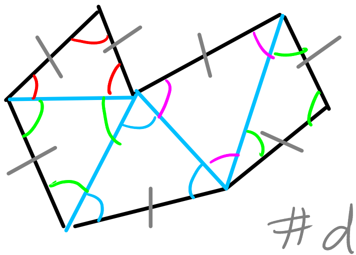


$$\operatorname{tg}\frac{\pi}{6} = \frac{\cancel{a}/\cancel{2}}{\cancel{a}\sqrt{3}/\cancel{2}} = \frac{\sqrt{3}}{3}$$

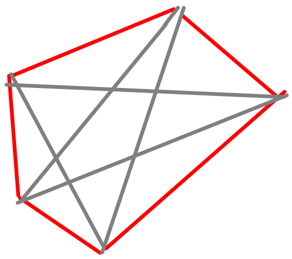
$n = 7$

$\# \Delta = n - 2$

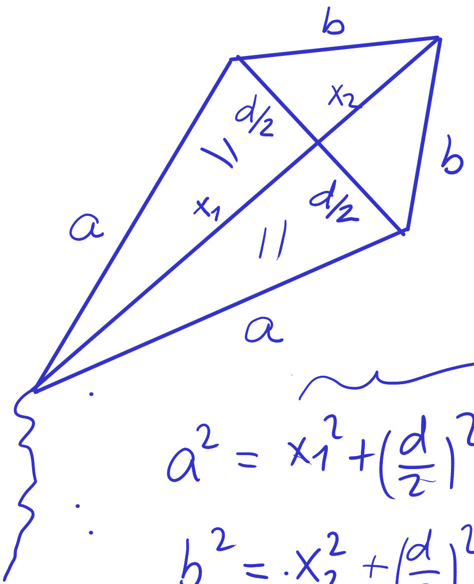
$(n-2)\cdot \pi =$ zbroj kuteva u n-terokutu



$$\# d = \frac{(n-3)\cdot n}{2} = \frac{(\cancel{5}-3)5}{\cancel{2}} = 5$$



$n = 5$



deltoid $P_{\text{deltoid}} = ?$

$$P = 2\frac{x_1 \frac{d}{2}}{2} + 2\frac{x_2 \frac{d}{2}}{2}$$

$$= \underbrace{(x_1+x_2)}\frac{d}{2} = \frac{f\,d}{2}$$

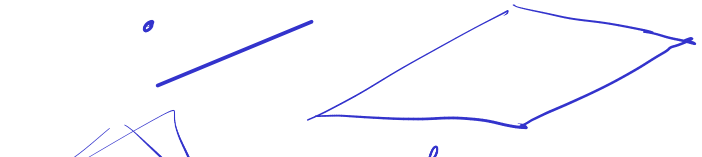
f druga dijagonala

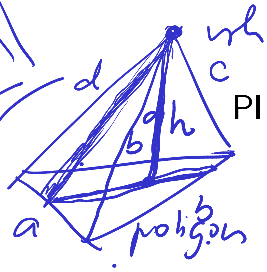
$$a^2 = x_1^2 + \left(\frac{d}{2}\right)^2$$

$$b^2 = x_2^2 + \left(\frac{d}{2}\right)^2$$

Primjene u prostoru

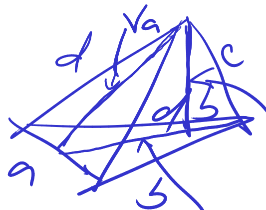
U prostoru koristimo ravninske prereze i ravninsku geometriju u njima

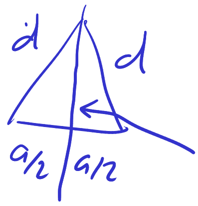




PIRAMIDA

a, b, c, d

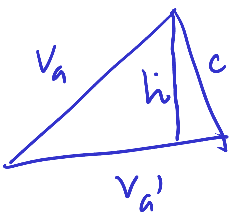




$$\sqrt{d^2 - \left(\frac{a}{2}\right)^2}$$

PREREZ (PRESJEK)

$$v_a' = \sqrt{b^2 - \left(\frac{a}{2}\right)^2}$$

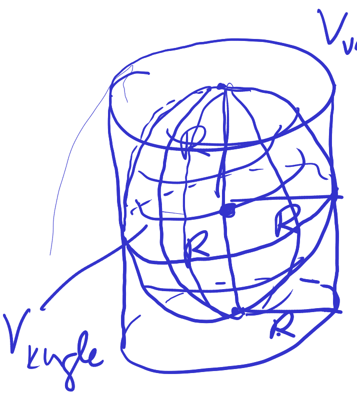


$$\frac{h \cdot v_a'}{2} = \sqrt{\tilde{s}(\tilde{s}-c)(\tilde{s}-v_a)(\tilde{s}-v_a')}$$

$$\tilde{s} = \frac{v_a + v_a' + c}{2} = \frac{\sqrt{b^2-(a/2)^2} + \sqrt{d^2-(a/2)^2} + c}{2}$$

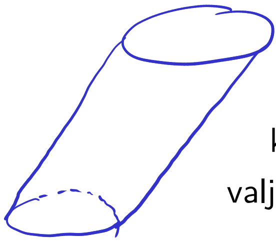
$$h = \frac{2}{v_a'}\sqrt{}$$

$$h = \frac{2}{\sqrt{b^2-(a/2)^2}}\sqrt{\tilde{s}(\tilde{s}-c)\left(\tilde{s}-\sqrt{d^2-(a/2)^2}\right)\left(\tilde{s}-\sqrt{b^2-(a/2)^2}\right)}$$

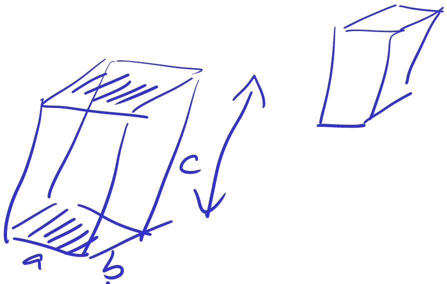


$\boxed{h = 2R}$

uspravni valjak s upisanom kuglom je poseban

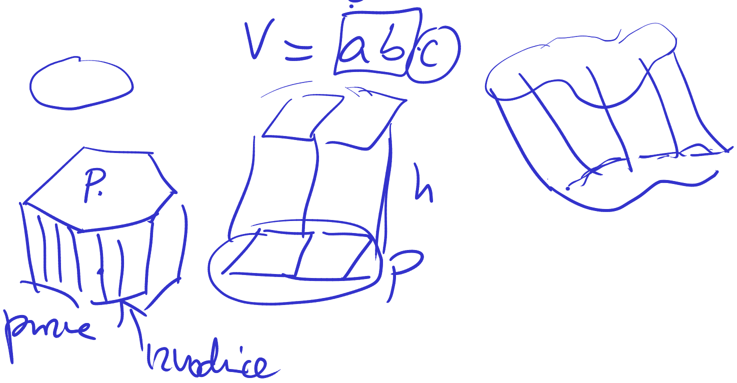


kosi valjak

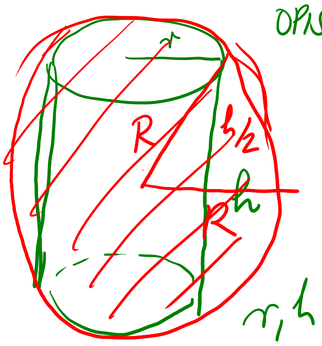


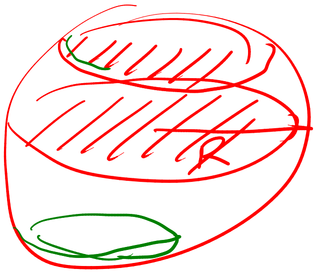
$V = \boxed{abc}$

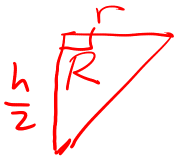
$$\frac{V_{valjka}}{V_{kugle}} = \frac{\cancel{R^2}\cancel{\pi} \cdot 2\cancel{R}}{\frac{4}{3}\cancel{R^3}\cancel{\pi}} = \frac{2}{\frac{4}{3}} = \frac{\frac{2}{1}}{\frac{4}{3}} = \frac{6}{4} = \frac{3}{2}$$



OPIŠEMO KUGLU



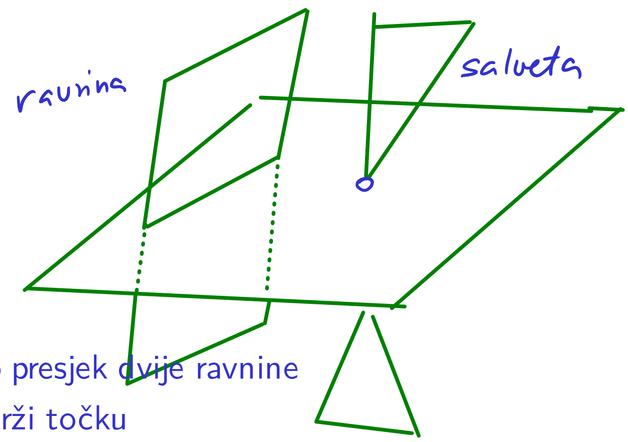




$$R = \sqrt{r^2 + \left(\frac{h}{2}\right)^2}$$

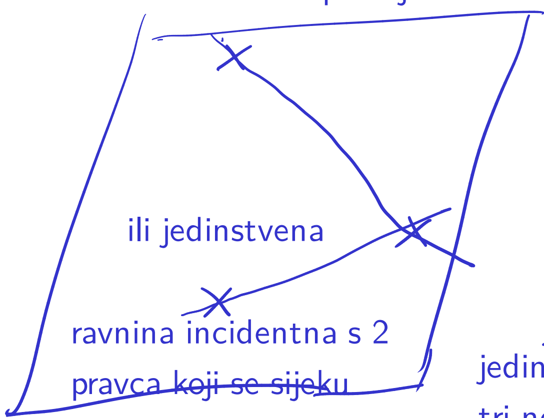
$$\sqrt{x} = x^{1/2}$$

$$\frac{V_{kugle}}{V_{valjka}} = \frac{\frac{4}{3}\left(r^2 + \frac{h^2}{4}\right)^{3/2}\cancel{\pi}}{r^2\cancel{\pi} \cdot h} = \frac{4}{3}\,\frac{\left(r^2 + \frac{h^2}{4}\right)^{3/2}}{r^2 h}$$



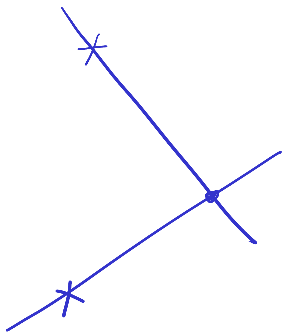
ako presjek dvije ravnine
sadrži točku
sadrži i neki pravac kroz tu točku

Van svake točke postoji ravnina!



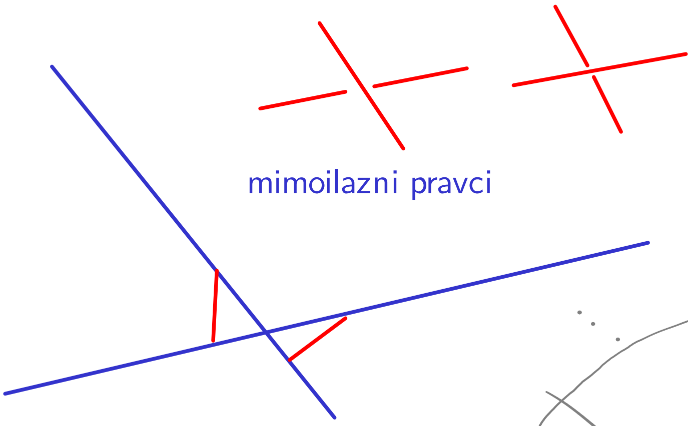
ili jedinstvena

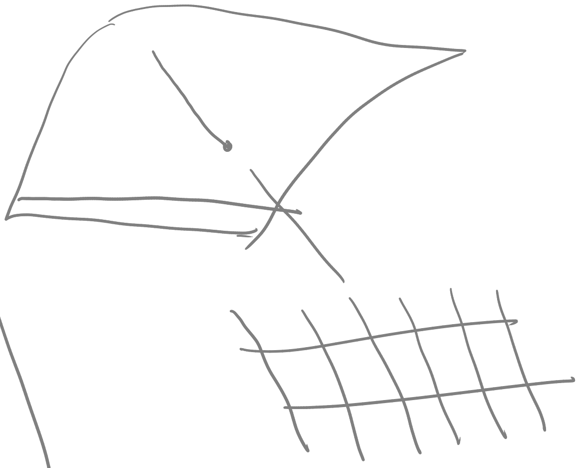
ravnina incidentna s 2
pravca koji se sijeku
u točno 1 točki

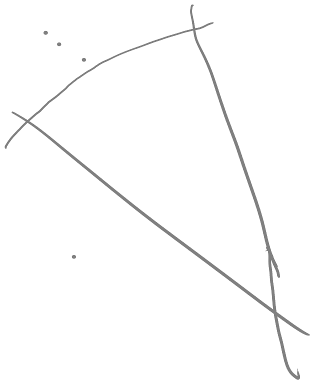


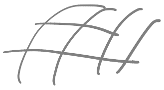
jedinstvena ravnina kroz
tri nekolinearne točke

Aksiomi stereometrije na stranici https://ncatlab.org/zoranskoda/show/aksiomi+stereometrije









dva su pravca u prostoru paralelna ako postoji ravnina koja sadrži oba
i u toj ravnini su paralelni u ravninskom smislu (ili se ne sijeku ili su identični)