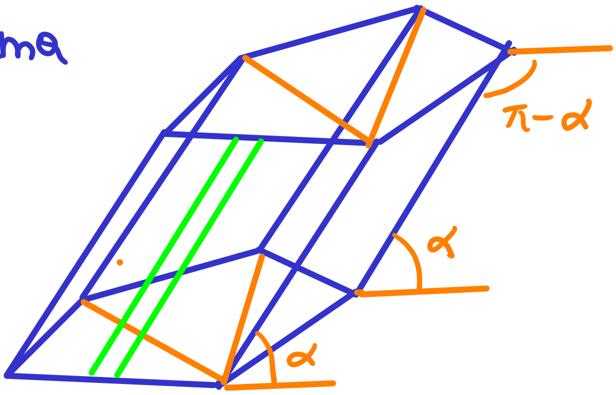
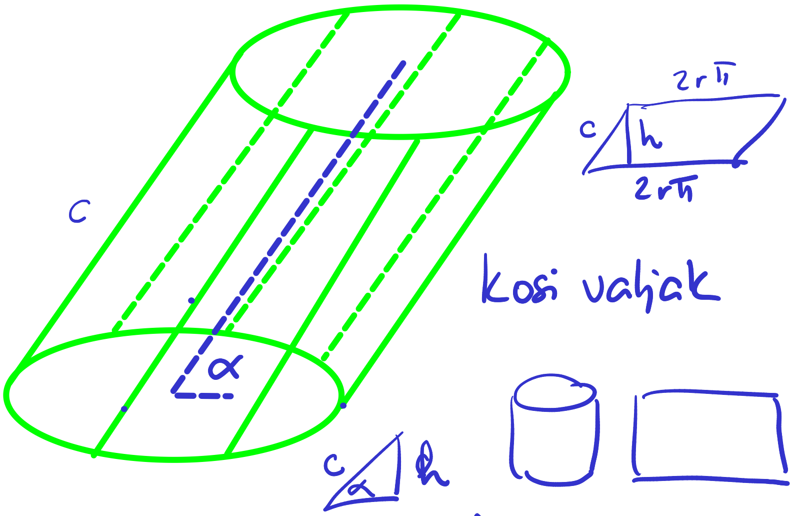
prizma

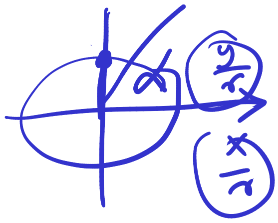




kosi valjak

$V = B \cdot h$

$P = 2B + P_{plohe}$ (plašt)

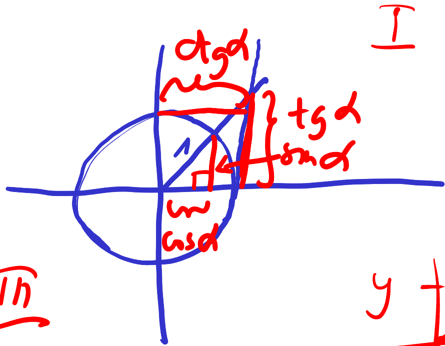


$\frac{y}{x} = \operatorname{tg}\alpha \quad \frac{x}{y} = \operatorname{ctg}\alpha$

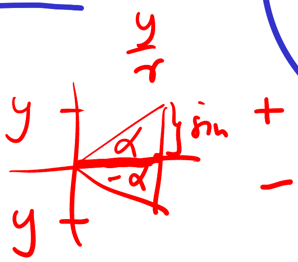
$h, r, \alpha \qquad c = \frac{h}{\sin\alpha}$

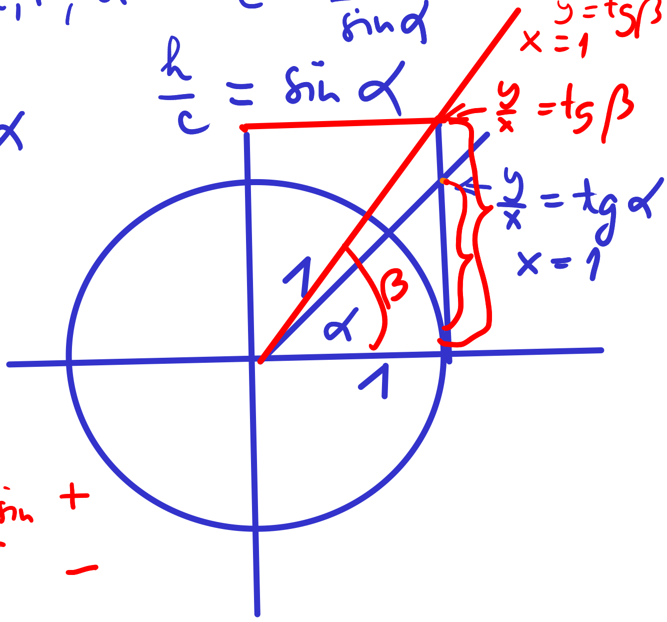
$\frac{h}{c} = \sin\alpha$

I.



III





$\cos(-\alpha) = \cos\alpha$

$\sin(-\alpha) = -\sin\alpha$

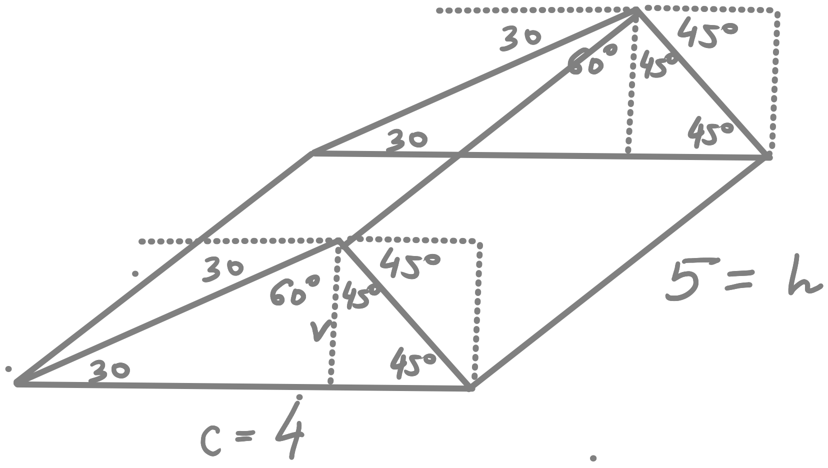
$\operatorname{tg}(-\alpha) = -\operatorname{tg}\alpha$

$\operatorname{ctg}\alpha = \frac{\cos\alpha}{\sin\alpha}$

$\operatorname{ctg}(-\alpha) = \frac{\cos(-\alpha)}{\sin(-\alpha)}$

$= \frac{\cos\alpha}{-\sin\alpha} = -\operatorname{ctg}\alpha$

Zadatak. Zadano je tijelo koje izgleda kao krović, odnosno polegnuta trostrana prizma s raznim naklonima na lijevu i desnu stranu. Ako je duljina gornjeg brida krovića jednaka 5 metara, kutevi naklona su 30 i 45 stupnjeva, i bazni brid s trokutastog boka krovića je 4 metra, koliki je volumen potkrovlja ?

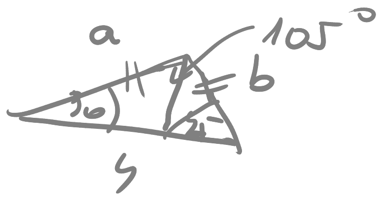


$V = B \cdot 5m$

P_{pob} = opseg baze × duljina izvodnice

uspravna prizma

$B = P_{\triangle}$



$$\frac{\sin(45^\circ)}{\sin(105^\circ)} = \frac{a}{4m}$$

$$a = 4m \frac{\sin 45^\circ}{\sin 105^\circ} = 4\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{4}(1+\sqrt{3})} m$$

$$\sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$$

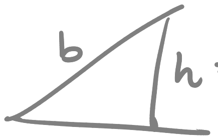
$$= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + \frac{1}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}(1+\sqrt{3})$$

$$= \frac{8}{1+\sqrt{3}}\frac{\sqrt{3}-1}{\sqrt{3}-1} m$$

$$= \underline{4(\sqrt{3}-1)\, m}$$

$$V = v_c = a \cdot \underbrace{\overset{1/2}{\sin 30^\circ}}_{v_c} = \frac{a}{2} \cdot$$

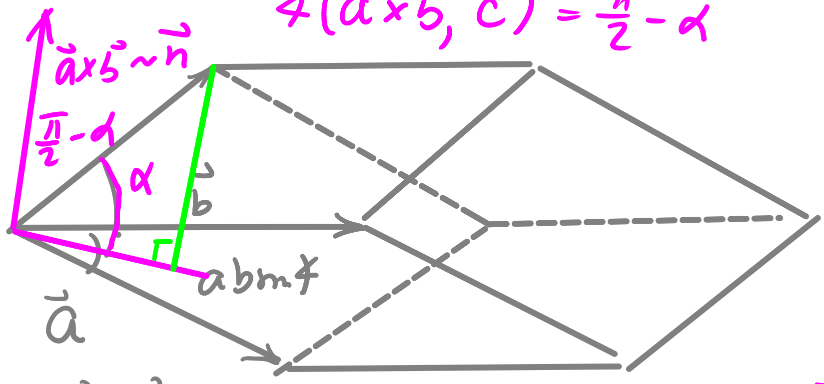
$$B = \overset{v_c}{\frac{4(\sqrt{3}-1)}{2} m} \cdot \overset{c}{4m} \cdot \overset{h}{5m} = (40(\sqrt{3}-1))\, m^3$$

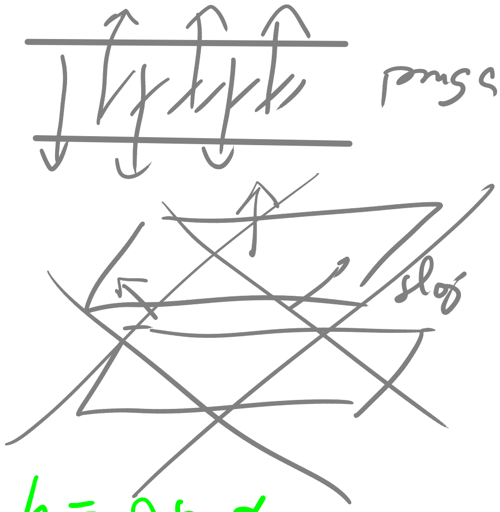


$h = b\sin\alpha$

$a\cdot b\sin\alpha = |a\, v_a|$

$\sphericalangle(\vec a\times\vec b,\vec c) = \frac{\pi}{2}-\alpha$





$\|\vec a\times\vec b\| = \|\vec a\|\|\vec b\||\sin\sphericalangle(\vec a,\vec b)|$ = B · h

$h = c\sin\alpha$

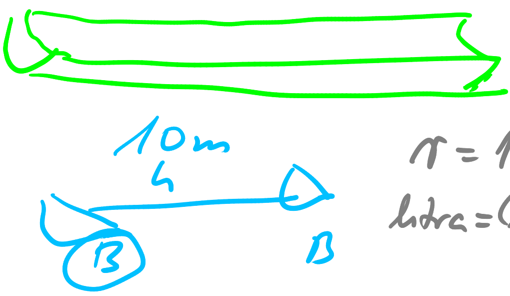
$\underbrace{\cos\left(\frac{\pi}{2}-\alpha\right)}_{\sphericalangle(\vec a\times\vec b,\vec c)} = \sin\alpha$

$\underbrace{\underbrace{\|\vec a\times\vec b\|}_{B}\ \|\vec c\|\ \cos\left(\frac{\pi}{2}-\alpha\right)}$

$\pm V = (\vec a\times\vec b)\cdot\vec c$ = mješoviti produkt

U valjčanom žlijebu koji je položen na zemlju do jedne trećine visine valjka stoji voda. Ako je žlijeb dugačak 10 metara i ima polumjer od 10 centimetara, koliko litara vode stoji u žlijebu ?

pauza do 17:00



$\frac{1}{3}$ visine $= \frac{1}{3}2r$

$r = 10cm = 1dm$

litra $= (1dm)^3$

$V = B\cdot h$

$V = (P_{isj} - P_{\Delta})\cdot h$ (dm³) 100dm 10m = 100dm ⇝ litre

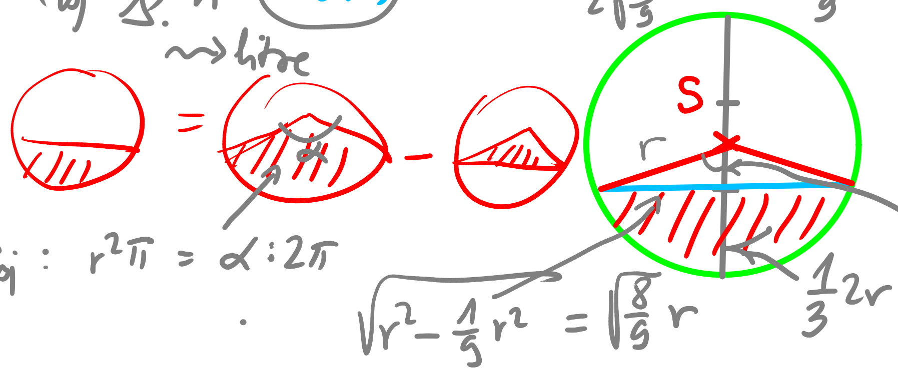
$P_{\Delta} = \frac{1}{2}\cdot\underbrace{\frac{1}{3}r}_{\text{visine}}\cdot\cancel{2}\,\frac{2\sqrt2}{3}r = \frac{2\sqrt2}{9}r^2$

$2\sqrt{\frac{8}{9}}r$

$\sin\frac{\alpha}{2} = \frac{\frac{2\sqrt2}{3}\cancel{r}}{\cancel{r}}$

⇝ $\alpha/2$ ⇝ α

$P_{isj} = \frac{r^2}{2}\alpha$ ← radijani

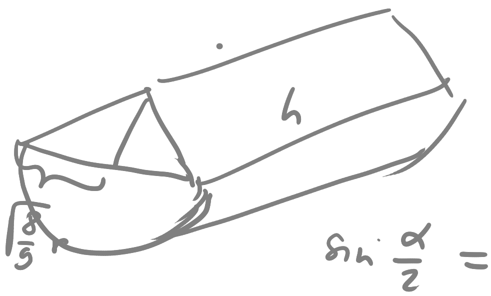


$P_{isj} : r^2\pi = \alpha : 2\pi$

$\sqrt{r^2-\frac{1}{9}r^2} = \sqrt{\frac{8}{9}}r$

$\frac{1}{3}2r$

$r - \frac{2}{3}r = \frac{1}{3}r$



$r = 1\,dm$

$h = 10\,m = 100\,dm$

$$\sin\frac{\alpha}{2} = \frac{\sqrt{\frac{8}{9}}\,r}{r} = \frac{2\sqrt{2}}{3}$$

$l = \alpha r$

$\alpha : 2\pi = 2r\pi : l$

$$\alpha = 2\arcsin\frac{2\sqrt{2}}{3} = 2.4619\ rad$$

$l = \alpha r$

$$P_{isj} = \frac{r^2}{2}\alpha = \frac{dm^2}{2}\,2.4619 = \underline{1.2309}\ dm^2$$

$$P_\Delta = \frac{1}{\cancel{2}}\,\frac{1}{3}r\,\cancel{2}\,\frac{2\sqrt{2}}{3}r$$

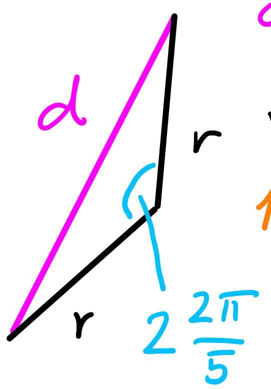
$$= \frac{2\sqrt{2}}{9}r^2 = \frac{2\sqrt{2}}{9}\cdot(1\,dm)^2 = 0.9428\,dm^2$$

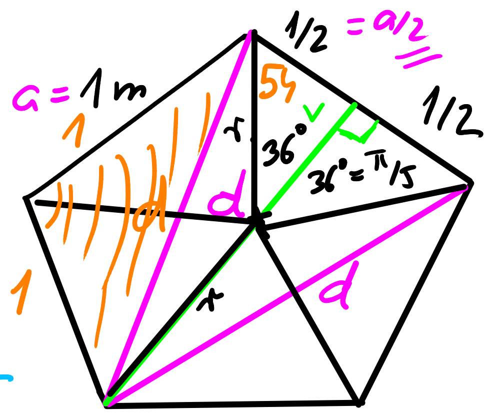
$$B = (1.2309 - 0.9428)\,dm^2 = 0.2881\ dm^2$$

$h = 100\,dm$

$V = 28.81\ dm^3 = 28.81$ <u>litra</u>

Nađi duljinu svake dijagonale pravilnog peterokuta kojem je stranica (a) duljine 1 metar.





$$\frac{1/2}{r} = \sin 36^\circ$$

$$r = \frac{1/2}{\sin \pi/5}$$

$$d = \sqrt{(r+v)^2 + \left(\frac{a}{2}\right)^2}$$

$$\frac{v}{1/2} = ctg\,\pi/5$$

$$v = 2\,ctg\,\pi/5$$

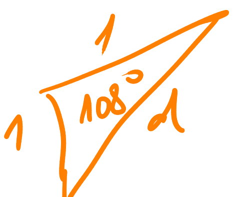
$$d^2 = 1^2 + 1^2 - 2\cdot1\cdot1\cos 108^\circ$$

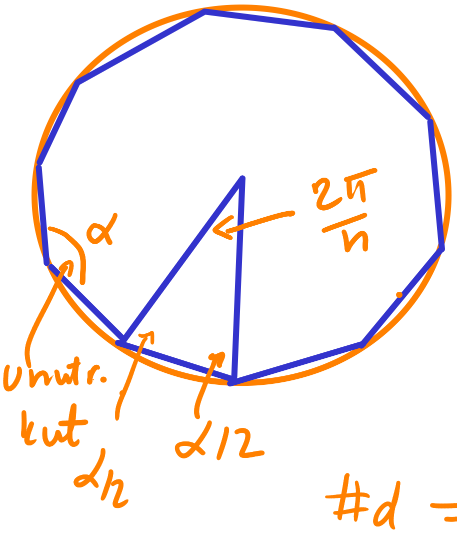
drugi način: uočimo jednakokračan trokut r-r-d

$$d^2 = r^2 + r^2 - 2r\cdot r\cos\frac{2\pi}{5} = 2r^2\left(1 - \cos\frac{2\pi}{5}\right)$$

$$= 2\,\frac{a^2/4}{\sin^2\pi/5}\left(1 - \cos\frac{2\pi}{5}\right)$$

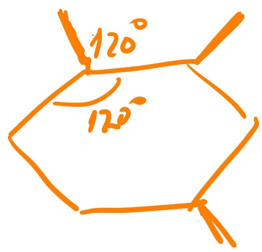
treći način, trokut a,a,d





$$\alpha = \pi - \frac{2\pi}{n} = \pi\left(1 - \frac{2}{n}\right)$$

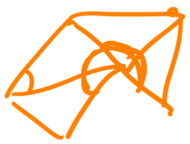
Σ kut u Δ



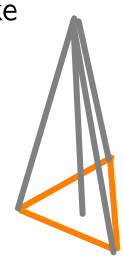
$$\alpha_6 = \pi - \frac{2\pi}{6} = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\#d = \frac{(n-3)\cdot n}{2}$$

$$\Sigma \text{ kuteva} = (n-2)\pi \qquad \frac{(n-2)\pi}{n} = \pi\left(1 - \frac{2}{n}\right)$$

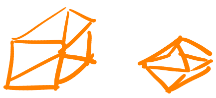


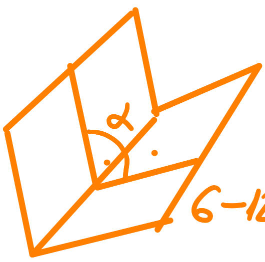
Pravilni poligon: svi unutarnji kutevi jednaki, sve stranice jednake



Pravilni poliedri (Platonova tijela)

1. sve strapenice su međusobno sukladni pravilni poligoni
2. diedarski kut kod svakog brida je jednak





pet: pravilni:

tetraedar (pravilna trostrana piramida kojoj je osnovica sukladna stranicama pobočja) $4-6+4=2$

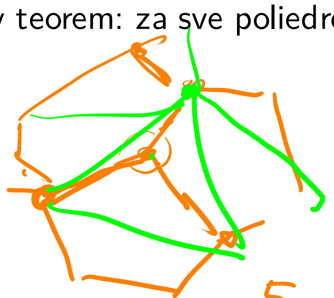
heksaedar (pravilan heksaedar je kocka) $8-12+6=2$

$6-12+8$ oktaedar (osam trokuta, kao dvije četverostrane piramide jedna na drugoj)

dodekaedar (dvaneast pravilnih peterokuta) $20-30+12$

ikosaedar (20 jednakostraničnih trokuta) $12-30+20$

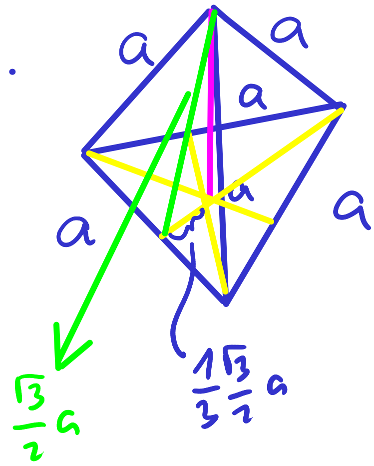
Eulerov teorem: za sve poliedre bez rupa ograničene vrijedi V-B+S = 2

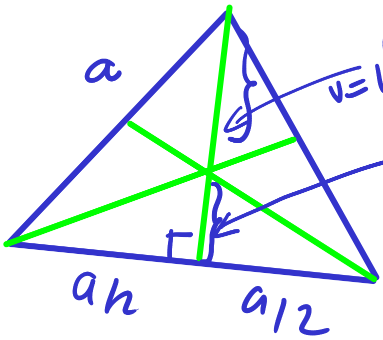


$$\frac{5}{3}\cdot 12 = 20$$

$$12\cdot\frac{5}{2}$$

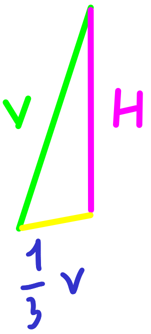
Izračunaj volumen pravilnog tetraedra kojem je stranica a.





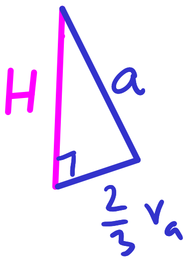
$v=\sqrt{a^2-(a/2)^2}=\frac{\sqrt3}{2}a$

$\frac{1}{3}\frac{\sqrt3}{2}a$



$$H=\sqrt{v^2-(\tfrac{1}{3}v)^2}$$
$$=\sqrt{\tfrac{3}{4}a^2-\tfrac{1}{9}\tfrac{3}{4}a^2}$$
$$=\sqrt{\tfrac{3}{4}a^2\tfrac{8}{9}}=\sqrt{\tfrac{2}{3}}\,a$$

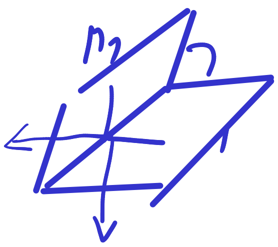
$$V=\frac{1}{3}B\cdot H=\frac{1}{3}\frac{a\,v_a}{2}\cdot H$$
$$=\frac{1}{3}a\frac{\sqrt3}{2}a\,\frac{1}{2}\sqrt{\frac{2}{3}}\,a=\frac{\sqrt2}{12}a^3$$



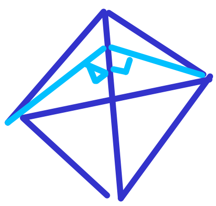
$H^2=\sqrt{v^2-(\tfrac{1}{3}v)^2}$ $\quad V=V_a$

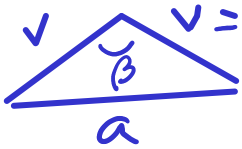
$$H^2=\sqrt{a^2-(\tfrac{2}{3}v)^2}=\sqrt{a^2-(\tfrac{2}{3}\tfrac{\sqrt3}{2}a)^2}$$
$$=\sqrt{a^2-\tfrac{1}{3}a^2}$$
$$=\sqrt{\tfrac{2}{3}a^2}=a\sqrt{\tfrac{2}{3}}$$

Izračunaj kut između dva brida pravilnog tetraedra



$$\measuredangle(\vec{n}_1,\vec{n}_2)$$
$$=\measuredangle(n_1,n_2)$$





$v=\frac{\sqrt3}{2}a$

$$\cos\beta=\frac{v^2+v^2-a^2}{2v\cdot v}=\frac{\frac{3}{4}a^2+\frac{3}{4}a^2-a^2}{\frac{3}{2}a^2}=\frac{-\frac{2}{8}\cancel{a^2}}{\frac{3}{2}\cancel{a^2}}$$

$\beta=1.738$ rad

$$\cos\beta=-\frac{1}{6}$$