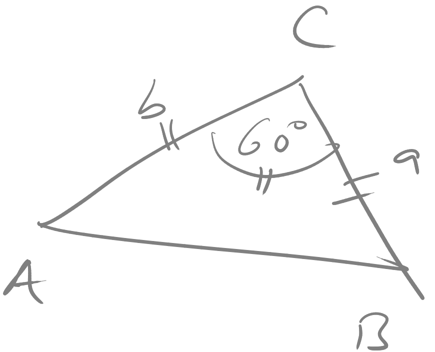
$a = 3$
$b = 5$
$\gamma = 60^\circ$



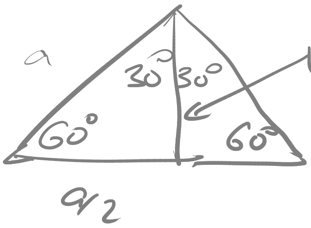
$P = ?$ $P = \frac{3 \cdot 5}{2} \cdot \frac{\sqrt{3}}{2} = \frac{15}{4}\sqrt{3}$

$v_b = ?$ $\frac{v_b \cdot b}{2} = P$ $v_b = \frac{2P}{b}$

https://www2.irb.hr/korisnici/zskoda/mat2t220614.pdf

$ab \sin \gamma / 2$
$\shortparallel$
$\|\vec{a} \times \vec{b}\| / 2$

$\sin 60^\circ = \frac{\sqrt{3}/2 \, \cancel{\alpha}}{\cancel{\alpha}} = \frac{\sqrt{3}}{2}$

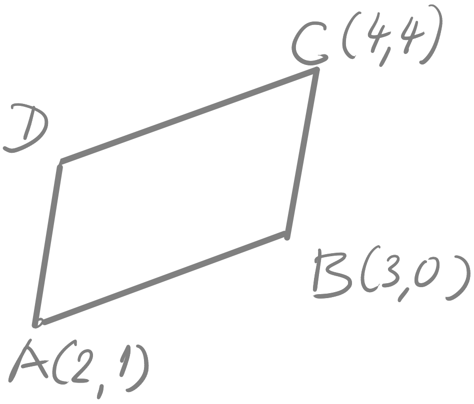


$\sqrt{a^2 - \left(\frac{a}{2}\right)^2}$
$\shortparallel$
$\frac{\sqrt{3}}{2} a$

$v_b = \frac{2 \cdot \frac{15\sqrt{3}}{\cancel{4}\,2}}{5} = \frac{3\sqrt{3}}{2}$

$v_b = a \sin \gamma = 3 \cdot \frac{\sqrt{3}}{2}$

Ako su tri susjedna vrha paralelograma redom A(2, 1), B(3, 0), C(4, 4), nadji koordinate ˇcetvrtog vrha.



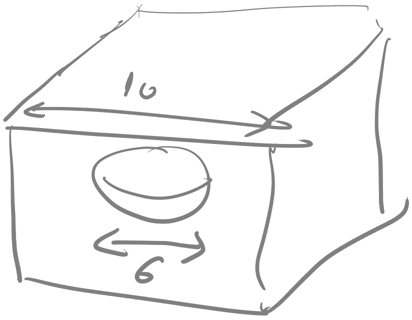
$\vec{AD} = \vec{BC}$

$D = A + \vec{AD} := t_{\vec{AD}}(A)$
$\uparrow \quad \uparrow \quad \uparrow_{\vec{BC}}$

$D = (2,1) + (4-3, 4-0)$

$D = (3,5)$

3. Ako je u kocki duljine stranice a = 10 izdubljena kugla radijusa r = 3 s centrom u centru kocke (sjeciˇste prostornih dijagonala), nadji omjer volumena kocke Va i volumena kugle Vr. Je li obujam ostatka kocke koji preostaje nakon dubljenja veˊci ili manji od volumena kugle koja je izdubljena ?



$V_{\square} = a^3 = 10 \times 10 \times 10 = 1000$

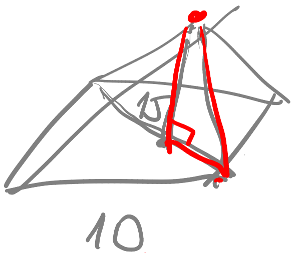
$V_{\circ} = \frac{4}{3} r^3 \pi = \frac{4}{3} \cdot 27\pi = 36\pi = 36 \times 3.14$

≈ 113

$\begin{array}{r} 36 \times 3.14 \\ \hline 108 \\ 36 \\ 144 \\ \hline 113.04 \end{array}$

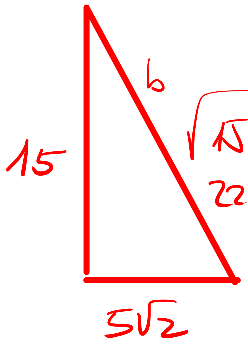
$V_{ostatka} = V_{\square} - V_{\circ} = 1000 - 113$
$= 887 > 113$
$V_{\circ}$

4. Uspravna ˇcetverostrana piramida ima za osnovicu kvadrat stranice a = 10, a visina piramide je h = 15. a) Nadji duljinu svakog od 4 brida b koji spajaju vrh piramide s vrhovima kvadrata u osnovici. b) Nadji povrˇsinu svakog od 4 trokuta na poboˇcju piramide. Svakako skiciraj piramidu s oznakama, a, b, h.



$d = 10\sqrt{2} = \sqrt{10^2+10^2}$

$\frac{10\sqrt{2}}{2} = 5\sqrt{2}$



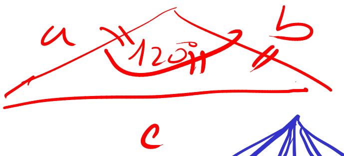
$\sqrt{\underset{225}{15^2} + \underset{25\cdot 2}{(5\sqrt{2})^2}} = \sqrt{275}$

$= \sqrt{25\cdot 11}$

$= 5\sqrt{11}$

5. Nadji stranicu c u trokutu u kojem su stranice a = 18, b = 12 i kut izmedju njih je tup kut od 120 stupnjeva

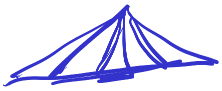
$c^2 = a^2 + b^2 - 2ab\cos\overset{\gamma}{\sphericalangle}(a,b)$

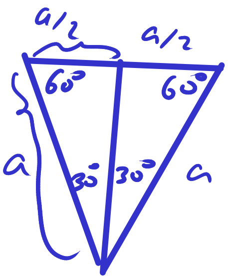


$c^2 = 18^2 + 12^2 - 2\cdot 18\cdot 12 \cdot \underset{(-1/2)}{\cos 120^\circ}$

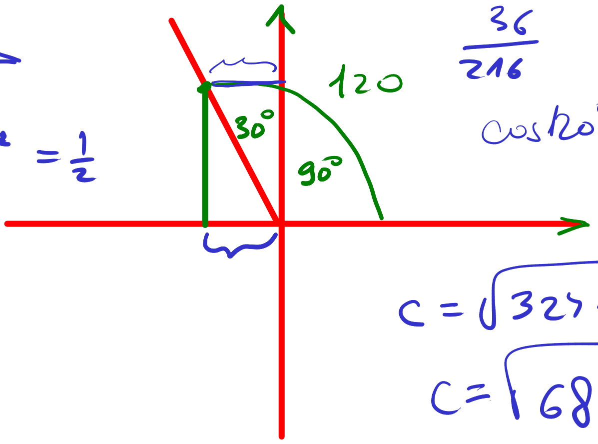
$\begin{array}{r} 180 \\ 36 \\ \hline 216 \end{array}$

$\cos\alpha = \frac{x}{r}$





$\sin 30^\circ = \frac{a/2}{a} = \frac{1}{2}$



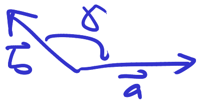
$\cos 120^\circ = -\sin 30^\circ = -\frac{1}{2}$

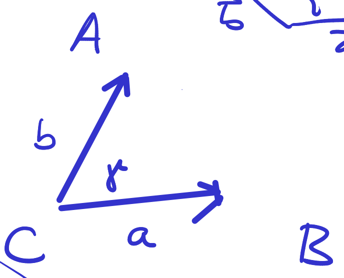
$c = \sqrt{324 + 144 + 216}$

$c = \sqrt{684}$

$\vec{i} + 2\vec{k}$ $\vec{i} + 3\vec{j} - \vec{k}$

6. Ako vektori ~CB = ~i + 2~k i ~CA = ~i + 3~j − ~k odgovaraju dvjema stranicama trokuta i idu iz istog vrha C, nadji kut γ izmedju te dvije stranice i vektor ~AB koji odgovara tre´coj stranici. Nadji i povrˇsinu trokuta 4ABC koriste´ci definiciju vektorskog umnoˇska





$P = \|\vec{a} \times \vec{b}\|$

$\vec{a}\cdot\vec{b} = \|\vec{a}\| \, \|\vec{b}\| \cos\gamma$

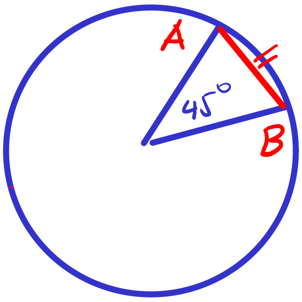
$\cos\gamma = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\,\|\vec{b}\|} = \frac{(\vec{i}+2\vec{k})(\vec{i}+3\vec{j}-\vec{k})}{} = \frac{-1}{\underbrace{\sqrt{1^2+2^2}}_{\sqrt{5}}\ \underbrace{\sqrt{1^2+3^2+\underbrace{(-1)^2}_{1^2}}}_{\sqrt{11}}}$

$= \frac{-1}{\sqrt{55}}$

$\gamma = \cos^{-1}\left(-\frac{1}{\sqrt{55}}\right)$

$P = \|\vec{a} \times \vec{b}\|$

Nadji duljinu kružnog luka l, površinu kružnog isječka P , i duljinu pripadne tetive d(A, B) ako je pripadni središnji kut α = 45◦ i promjer kruga je D = 10.



$D=10$

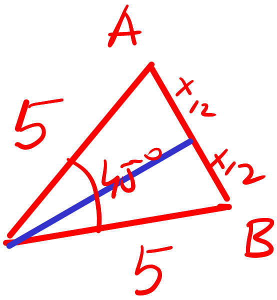
$P_O = r^2\pi = \left(\frac{D}{2}\right)^2\pi = \frac{D^2\pi}{4} = \frac{100}{4}\pi = 25\pi$

$P_{\sphericalangle} : P_O = 45° : 360°$

$P_{\sphericalangle} = \frac{45°}{360°} 25\pi = \frac{25}{8}\pi$

$l : 2r\pi = 45° : 360°$

$l = \underset{D}{\boxed{2r}}\pi \cdot \frac{1}{8} = \frac{10\pi}{8} = 1.25\pi = \frac{5}{4}\pi$



$x = ?$

$\frac{x}{2} = 5\sin\frac{45°}{2}$

$x = 10\sin\frac{45°}{2}$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

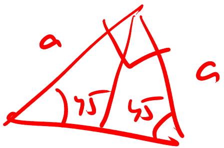
$\left.\begin{array}{l} 1 = \cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2} \\ \cos\alpha = \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2} \end{array}\right\} -$

$\cos\left(\frac{\alpha}{2}+\frac{\alpha}{2}\right) = \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}$

$\frac{1-\cos\alpha}{2} = \sin^2\frac{\alpha}{2}$

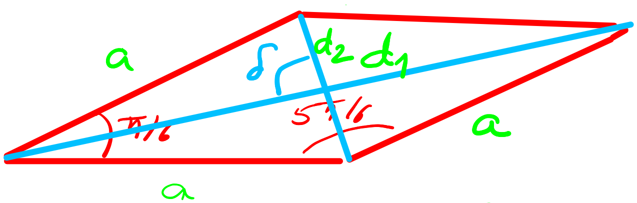
$\cos 45° = \sin 45° = \frac{\sqrt{2}}{2}$

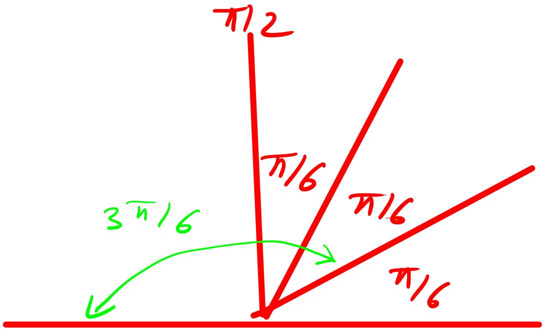
$\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1-\sqrt{2}/2}{2}}$



$\sqrt{a^2+a^2} = a\sqrt{2}$

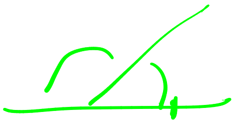
8. Romb (paralelogram kojem su sve 4 stranice jednake) ima jedan od kuteva α = π/6 radijana. a) Ako je površina romba P = 52, nadji stranicu a tog romba. b) Nadji kut δ medju dijagonalama romba.





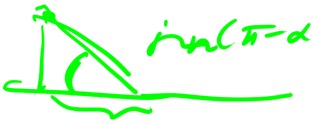
$$P = a^2 \underbrace{\sin\frac{\pi}{6}} = \frac{a^2}{2} \Rightarrow a^2 = 2P = 104$$

$$a = \sqrt{104}$$



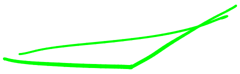
$\cos(\pi-\alpha) = -\cos\alpha$

$\sin(\pi-\alpha) = \sin\alpha$



$$d_1^2 = a^2 + a^2 - 2a\cdot a\cos\frac{5\pi}{6}$$

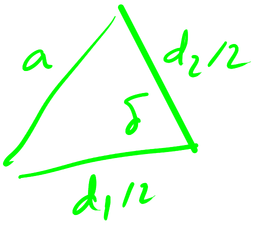
$-\sqrt{3}/2$



$$d_2^2 = a^2 + a^2 - 2a^2\underbrace{\cos\frac{\pi}{6}}_{\sqrt{3}/2}$$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$$\cos\gamma = \frac{a^2+b^2-c^2}{2ab}$$



$$\cos\delta = \frac{d_1^2/4 + d_2^2/4 - a^2}{2\frac{d_1}{2}\frac{d_2}{2}} \rightarrow \text{u računalo}$$

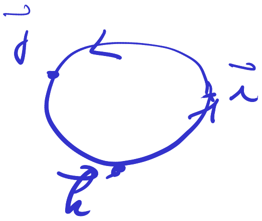
6. $P = \|\vec{a} \times \vec{b}\|$

$$(\vec{i} + 2\vec{k}) \times (\vec{i} + 3\vec{j} - \vec{k})$$

$$= 3\vec{k} - (-\vec{j}) + 2\vec{j} - 6\vec{i}$$

$$= -6\vec{i} + 3\vec{j} + 3\vec{k}$$

$$P = \sqrt{6^2 + 3^2 + 3^2} = \sqrt{54} = 3\sqrt{6}$$



1

$\cos^2\gamma$ $\sin^2\gamma$

$$a^2 b^2 = (\vec{a}\cdot\vec{b})^2 + \|\vec{a}\times\vec{b}\|^2$$

$$55 = (-1)^2 + 54$$

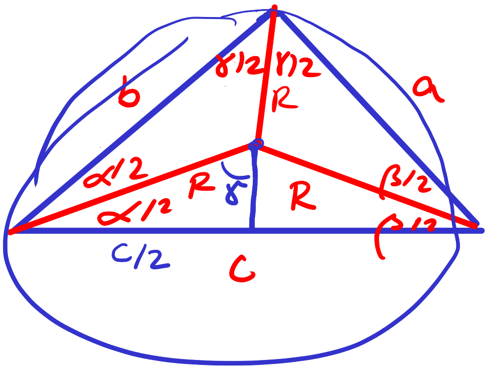
provjera

$1+0-2$

$$\vec{a}\cdot\vec{b} = -1$$

$$\|\vec{a}\| = \sqrt{5} = a$$

$$\|\vec{b}\| = \sqrt{11} = b$$



$$\frac{c/2}{R} = \sin\gamma$$

$$\sin\alpha = \frac{y}{r}$$

$$R = \frac{c}{2\sin\gamma} = \frac{a}{2\sin\alpha}$$

$$\operatorname{tg}\alpha = \frac{y}{x}$$

$$\frac{c}{a} = \frac{\sin\gamma}{\sin\alpha}$$