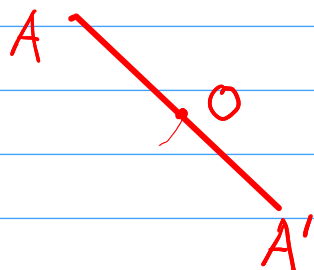


centralna simetrija = osna simetrija za ispruženi kut  
 centralna simetrija oko točke O je izometrija ravnine (ili prostora)  
 koja šalje ma koju točku A u jedinstvenu točku A' koja je  
 na pravcu OA, razlikuje se od A  
 i takva da je  $d(O,A) = d(O,A')$

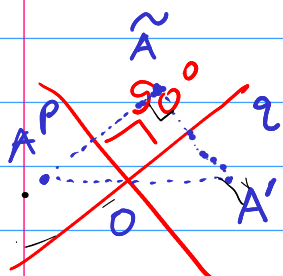


$$\forall A \in M^2$$

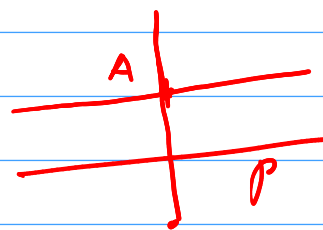
$$A \neq A'$$

$$A \in AO$$

$$d(A,O) = d(A',O)$$

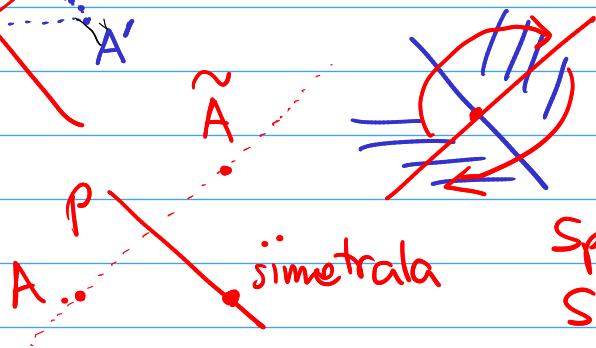


$$p \perp q \Leftrightarrow p \neq q$$



$$s_p(q) = q$$

$$s_p(A)$$

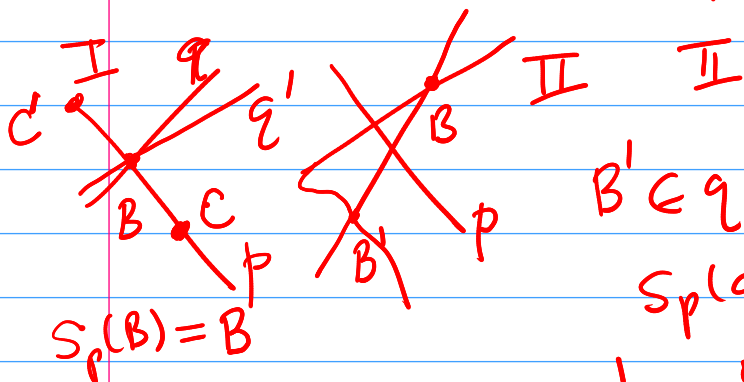


$$s_p(A) = \tilde{A} \quad A \neq \tilde{A}$$

$$s_p(\tilde{A}) = A \quad \left. \begin{array}{l} s_p(A) = \tilde{A} \\ s_p(\tilde{A}) = A \end{array} \right\} p \text{ je simetrala } \tilde{AA}$$

$$\underline{p \perp q \text{ i } p \perp q' \Rightarrow q \parallel q'}$$

Pretpostavimo suprotno,  $q \cap q' = \{B\}$



$$s_p(B) = B$$

$$s_q(C) = C' \in p$$

$$s_{q'}(C) = C'' \in p$$

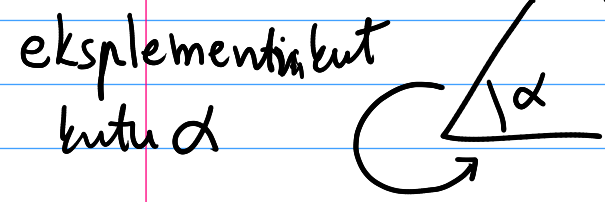
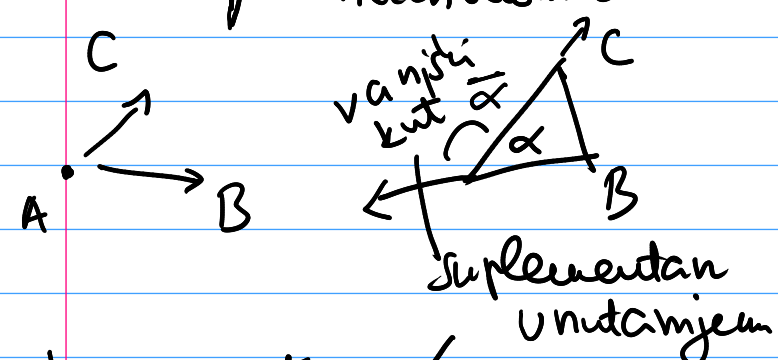
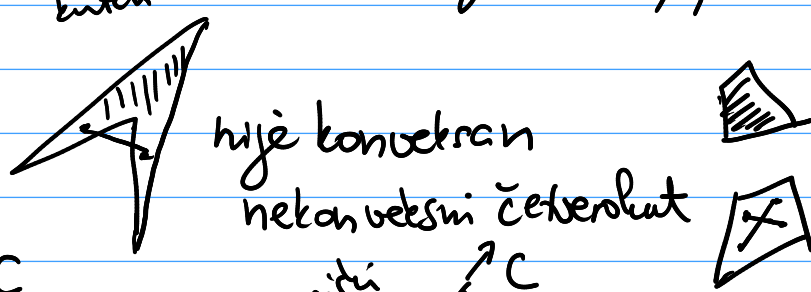
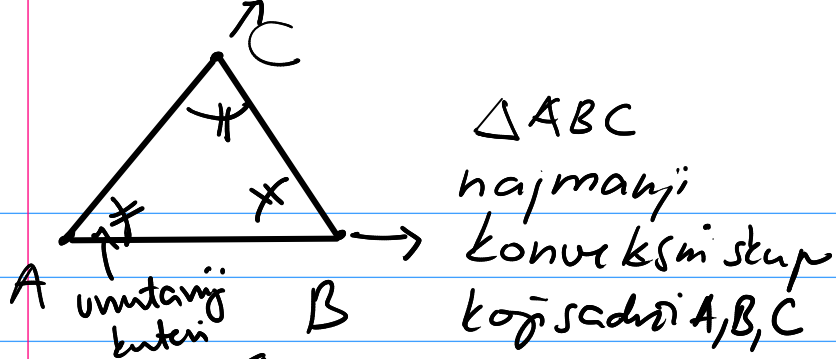
$$B' \in q, B' \in q'$$

$$s_p(q) = q \quad s_p(q') = q'$$

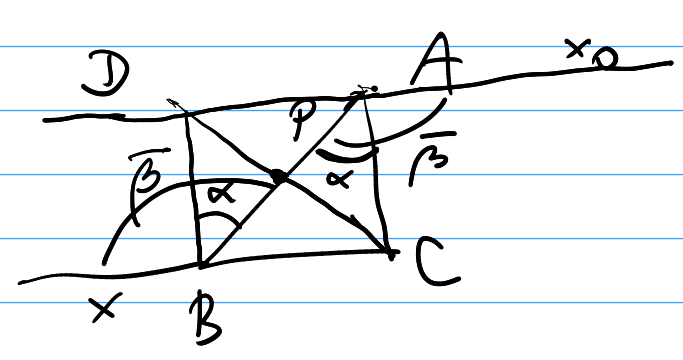
$$q' = BB' = q$$

$$d(C,B) = d(C',B) = d(C'',B)$$

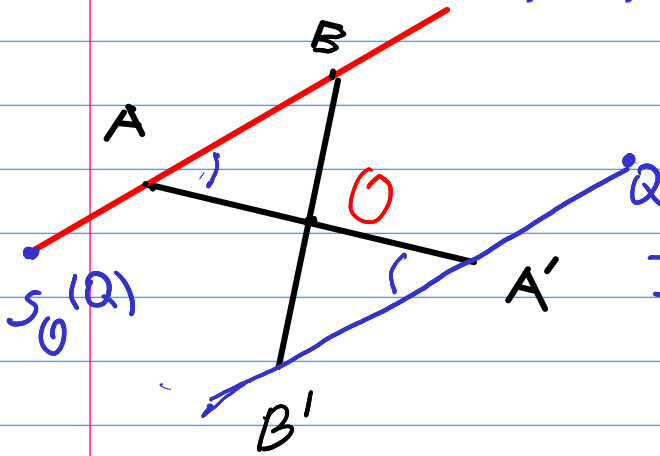
$\Rightarrow C' = C''$  je uzom. s obzirom  
 na simetralu CC' pa je  
 $q = q'$



$\alpha + \bar{\alpha} = 180^\circ$  suplem. kut  
 $\alpha + \alpha' = 90^\circ$  komplem.



centralna simetrija šalje svaki pravac u njemu paralelni pravac!



Dokaz

Pretp.  $Q \in AB \cap A'B'$

$$\text{I } S_O(Q) = Q$$

$$\Rightarrow Q = O$$

$\Rightarrow S_O(AB) = AB$  je paralelan (samom sebi)

$$\text{II } S_O(Q) \neq Q$$

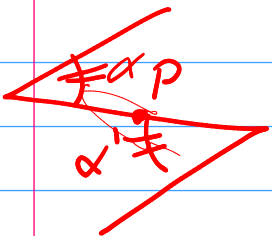
$S_O(Q) \in AB$  jer je  $S_O$  involucija na prave  $A'B'$  u  $AB$

$$S_O(Q) \in A'B'$$

jer je  $Q \in AB \cap A'B'$

pa je  $Q' \in A'B' \cap AB$

$$\text{Znači } AB = QQ' = A'B' \Rightarrow \Leftarrow$$



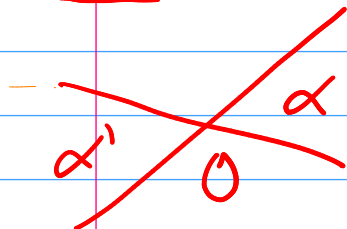
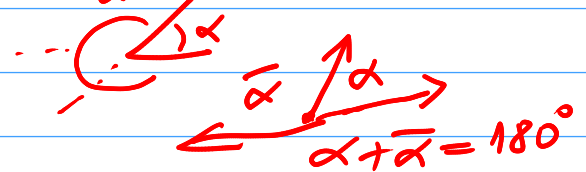
Z-teorem

$S_P$  centralna sim.

$$\alpha \mapsto \alpha'$$

$$\alpha = \alpha'$$

kutevi s paralelnim krajevima

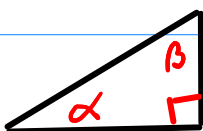


$$\alpha = \alpha' \text{ jer } S_O(\alpha) = \alpha'$$

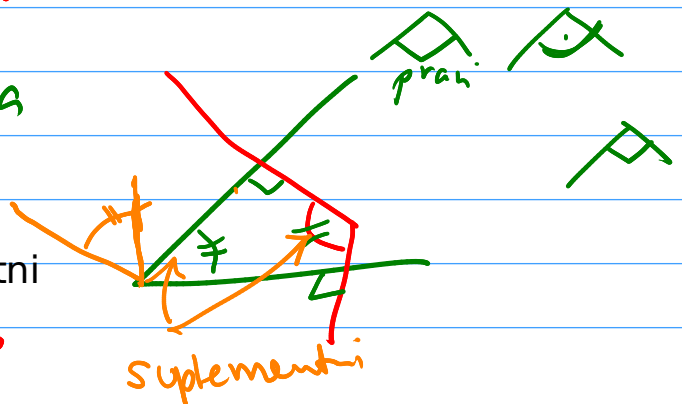
UNAKRSNI KUTEVI

Kutevi s okomitim krajevima

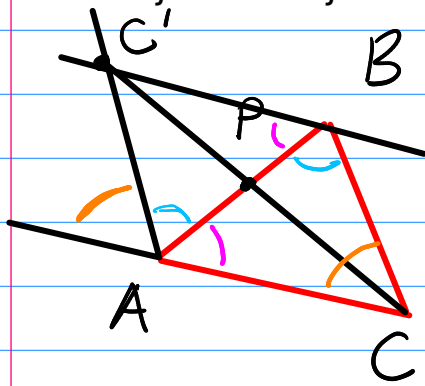
Ako je jedan krak paralelan, a jedan okomit onda su komplementni



$$\alpha + \beta = 90^\circ$$



zbroj unutarnjih kuteva u trokutu je uvijek ispruženi kut



$A \longleftrightarrow B$   
 $S_P$  centr. simetrija

Konvencija  $\overline{AB} = c$ ,  $\sphericalangle BAC = \alpha$   
 $\overline{AC} = b$ ,  $\overline{BC} = a$ ,  $\sphericalangle BCA = \gamma$

↓ srednji

Srednjica trokuta je spojnica koja spaja polovišta dviju različitih stranica trokuta.

Težišnica trokuta je spojnica vrha trokuta i polovišta njemu nasuprotne stranice.

Visina trokuta je pravac koji prolazi jednim vrhom trokuta, a okomit je na pravac na kojem leži tom vrhu nasuprotna stranica. Duljina visine je udaljenost od vrha trokuta do presjeka visine i pravca na kojem leži nasuprotna stranica.

$$s_c = \frac{1}{2} c$$

