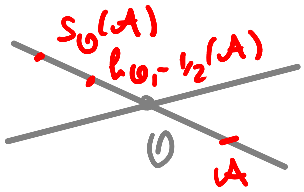
Matematika 2, 7. travnja 2021.



Prošli put: homotetija $h_{O,\lambda}$, $O \in M^2$, $\lambda \neq 0$, $\lambda \in \mathbb{R}$

$\lambda = 1$ $h_{O,\lambda} = id$

$\lambda \neq 1$ O je jedina fiksna točka

$\lambda = -1$ centralna simetrija $h_{O,\lambda} = S_O$

$d(O, h_{O,\lambda}(A)) = |\lambda|\, d(O,A)$

Svaka homotetija je $\underbrace{\text{preslikavanje sličnosti}}$ s koeficijentom sličnosti apsolutna vrijednost od lambda

$f : M^2 \to M^2$, $\lambda d(A,B) = d(f(A), f(B))$

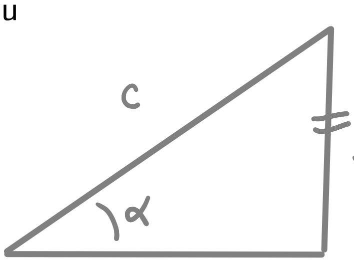
$\lambda > 0$ je koeficijent sličnosti

Za $\lambda = 1$ to je IZOMETRIJA

4 teorema o sličnosti trokuta

i svaka sličnost je kompozicija izometrije i homotetije

Kod pravokutnih trokuta to daje osnovu trigonometrije, naime iz teorema o sličnosti trokuta slijedi da omjer dviju kateta ili omjer katete i hipotenuze ovisi samo o kutevima u pravokutnom trokutu



$= a$ kuta α nasuprotna kateta

$\frac{a}{c} = \sin\alpha$ $\qquad$ $\frac{a}{b} = tg\,\alpha = \frac{\sin\alpha}{\cos\alpha}$

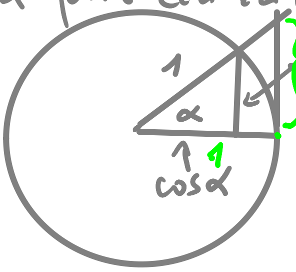
$\frac{b}{c} = \cos\alpha$ $\qquad$ $\frac{b}{a} = ctg\,\alpha = \frac{\cos\alpha}{\sin\alpha}$

b kuta α priležeća kateta

Pitagorin teorem

$a^2 + b^2 = c^2 \quad /:c^2$

$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$



TRIGONOMETRIJSKA KRUŽNICA

jedinična kružnica

$\frac{?}{1} = \frac{\sin\alpha}{\cos\alpha}$

$? = tg\,\alpha$

$\boxed{\cos^2\alpha + \sin^2\alpha = 1}$ OSNOVNI TRIGONOMETRIJSKI IDENTITET

$/:\cos^2\alpha$

$1 + \frac{\sin^2\alpha}{\cos^2\alpha} = \frac{1}{\cos^2\alpha}$

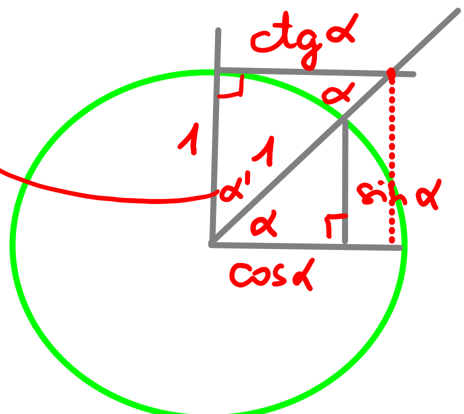
$1 + tg^2\alpha = \frac{1}{\cos^2\alpha}$ $\Big\}$ varijante

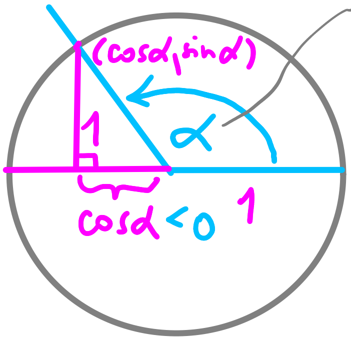
$1 + ctg^2\alpha = \frac{1}{\sin^2\alpha}$

$\cos\alpha \in [-1, 1]$

$\cos\alpha \in (0, 1)$ ako $\alpha < 90°$

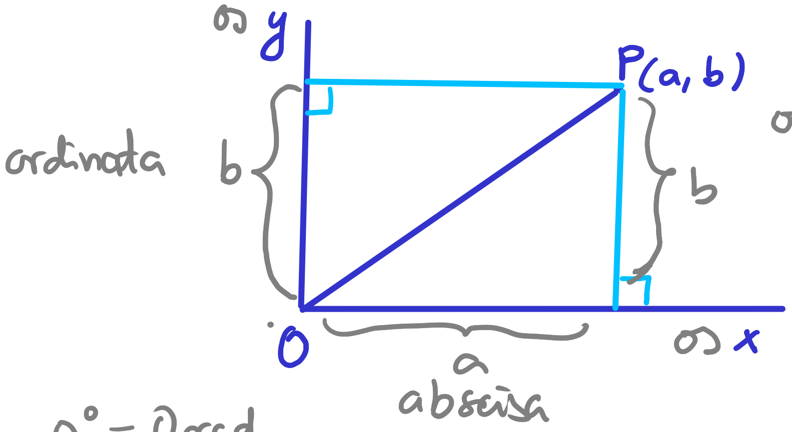
$\sin\alpha \in [-1, 1]$ $\qquad$ $ctg\,\alpha \in \mathbb{R}$, $tg\,\alpha \in \mathbb{R}$

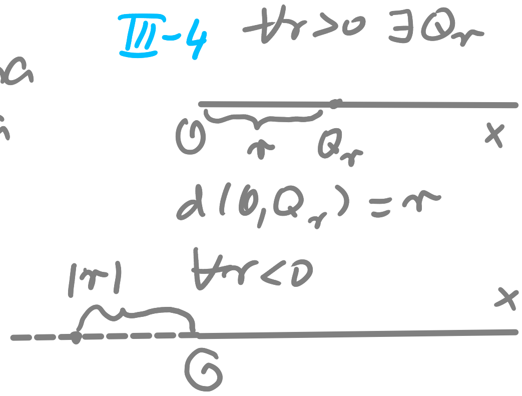




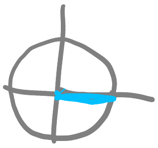
Teorem o pravokutnom koordinatnom sustavu u ravnini

Neka je O točka u ravnini i Ox polupravac i Oy polupravca takav da je Ox okomit na Oy. Tada postoji jedinstvena bijekcija iz skupa parova realnih brojeva u ravninu (ili obratno, njen inverz iz ravnine u skup parova realnih brojeva) koja pridružuje paru (a,b) realnih brojeva jedinstvenu točku P u ravnini M takvu da



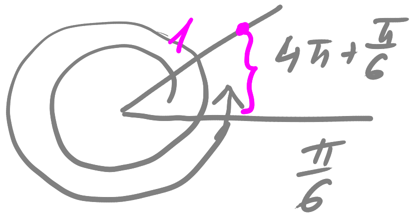


$0^\circ = 0\,\text{rad}$



$\cos 0^\circ = 1$ $\quad \mathrm{tg}\,0 = \frac{0}{1} = 0$

$\sin 0^\circ = 0$ $\quad \mathrm{ctg}\,0 \cancel{= \frac{1}{0}}$ nije definirano



$\sin(2\pi + \alpha) = \sin\alpha$

$\cos(2\pi + \alpha) = \cos\alpha$

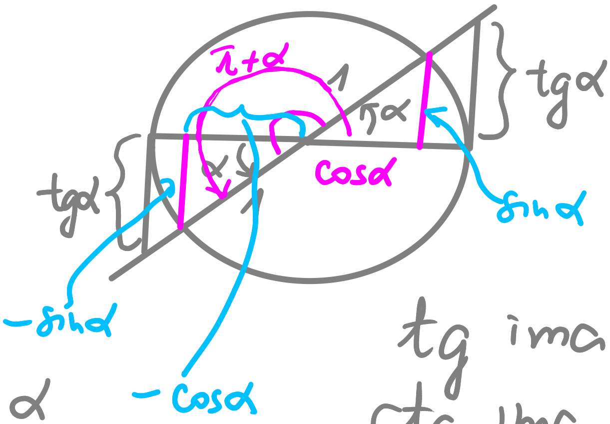
$T = 2\pi$ je perioda funkcija sin i cos

$x \in D \Rightarrow x + T \in D$ i $f(x) = f(x+T)$

$$\overset{\text{domena}}{f : D} \to \overset{\text{kodomena}}{K} \text{ je realna f-ja realne varijable}$$

$D \subseteq \mathbb{R}$, $K \subseteq \mathbb{R}$

realne varijable



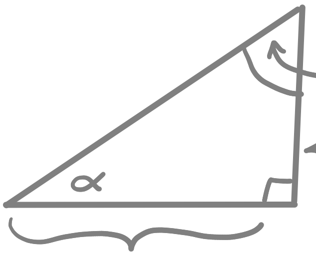
$$\mathrm{tg}(\alpha+\pi) = \frac{\sin(\alpha+\pi)}{\cos(\alpha+\pi)} = \frac{-\sin\alpha}{-\cos\alpha} = \mathrm{tg}\,\alpha$$

tg ima periodu π

ctg ima periodu π

$\cos(\pi + \alpha) = -\cos\alpha$ $\qquad \cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$

$\sin(\pi + \alpha) = -\sin\alpha$ $\qquad \sin\underbrace{\left(\frac{\pi}{2} - \alpha\right)}_{\text{komplementarni kut od } \alpha} = \cos\alpha$



nasuprotna od α = priležeća od $(\frac{\pi}{2}-\alpha)$ } $\sin\alpha = \cos(\frac{\pi}{2}-\alpha)$

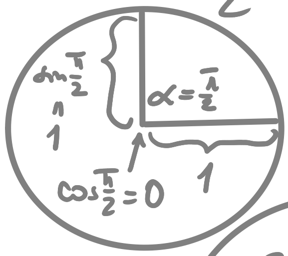
priležeća kateta od α } $\cos\alpha$

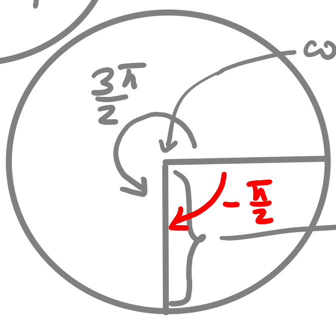
= nasuprotna od $\frac{\pi}{2}-\alpha$ } $\sin(\frac{\pi}{2}-\alpha)$

$\cos\alpha = \sin(\frac{\pi}{2}-\alpha)$

$\operatorname{tg}\alpha = \operatorname{ctg}(\frac{\pi}{2}-\alpha)$

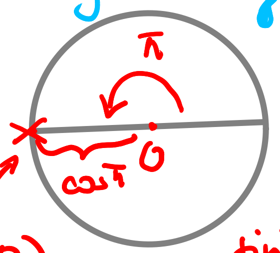
$\operatorname{ctg}\alpha = \operatorname{tg}(\frac{\pi}{2}-\alpha)$





$\cos\frac{3\pi}{2}=0$

$\sin\frac{3\pi}{2}=-1$



$(-1,0)$

$(\cos\pi, \sin\pi)$

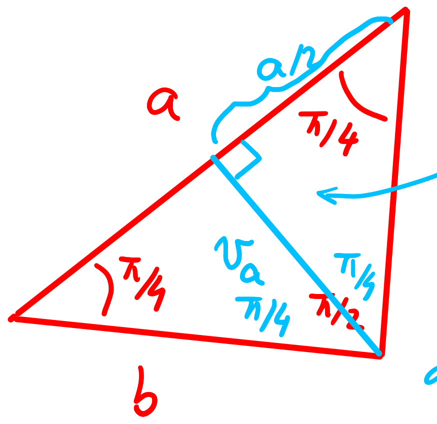
$\sin\pi = 0$

$\cos\pi = -1$

$\cos(-\frac{\pi}{2}) = \cos\frac{3\pi}{2} = 0$

$\sin(-\frac{\pi}{2}) = \sin\frac{3\pi}{2} = -1$

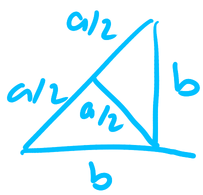
trigonometrijske funkcije za pi/2 radijana (45 stupnjeva)



$b^2 = \frac{a^2}{4} + v_a^2$

PITAGORA $v_a^2 = b^2 - (a/2)^2$ } jednakokračni

Ovdje $v_a = \frac{a}{2}$



$b^2 = \frac{a^2}{4} + \frac{a^2}{4} = \frac{a^2}{2}$

$b = \sqrt{\frac{a^2}{2}} = \frac{a}{\sqrt{2}} = \frac{\sqrt{2}}{2}\cdot a$

$\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2^2}} = \frac{\sqrt{2}}{2}$

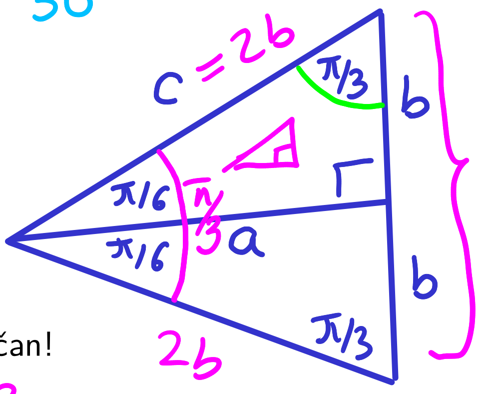
$\cos\frac{\pi}{4} = \frac{\text{PRILEŽEĆA}}{\text{HIPOTENUZA}} = \frac{b}{a} = \frac{\frac{\sqrt{2}}{2}\cancel{a}}{\cancel{a}} = \frac{\sqrt{2}}{2} \approx 0.707$

$\sin\frac{\pi}{4} = \frac{\text{NASUPROTNA}}{\text{HIP}} = \frac{b}{a} = \frac{\sqrt{2}}{2}$, $\sqrt{2} = 1.41421356\ldots$

$\frac{\pi}{6}$ ili 30°

$$\frac{\pi}{6}+\frac{\pi}{3}=\frac{\pi+2\pi}{6}=\frac{3\pi}{6}=\frac{\pi}{2}$$

$$30^\circ+60^\circ=90^\circ$$



jednakostraničan!

$$tg\,\frac{\pi}{6}=\frac{\text{NASUPROTNA}}{\text{PRILEŽEĆA}}=\frac{b}{a}=\frac{b}{\sqrt{(2b)^2-b^2}}=\frac{b}{\sqrt{4b^2-b^2}}=\frac{b}{\sqrt{3b^2}}$$

$ctg\,\pi/3$

$\parallel$

$$tg\,\frac{\pi}{6}=\frac{\cancel{b}}{\sqrt{3}\cdot\cancel{b}}=\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}}{3}=0.577\ldots$$

$\cos\frac{\pi}{3}$

$\parallel$

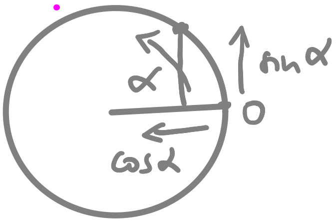
$$\sin\frac{\pi}{6}=\frac{\text{NASUPROTNA}}{\text{HIPOTENUZA}}=\frac{b}{c}=\frac{b}{2b}=\frac{1}{2}=0.5$$

$$\cos\frac{\pi}{6}=\frac{\text{PRILEŽEĆA}}{\text{HIPOTENUZA}}=\frac{a}{c}=\frac{\sqrt{3b^2}}{2b}=\frac{\sqrt{3}\cdot\cancel{b}}{2\cancel{b}}=\frac{\sqrt{3}}{2}$$

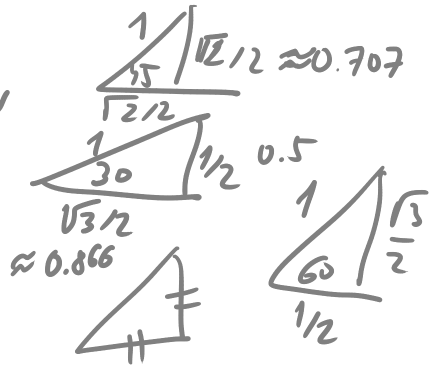
≈ 0.866

$\parallel$

$\sin\frac{\pi}{3}$



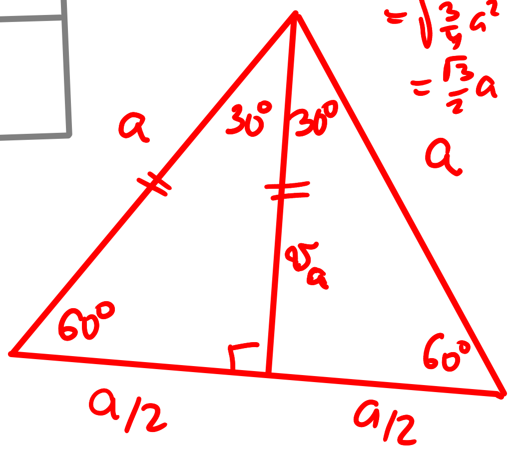
$\alpha \uparrow_{0}^{\pi}$ $\sin\alpha \uparrow_{0}^{1}$ $\cos\alpha \downarrow$



	30°	45°	60°	90°	
α	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\cos\alpha$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\sin\alpha$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$ctg\,\alpha$	—	$\sqrt{3}$	1	$\sqrt{3}/3$	0
$tg\,\alpha$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	—

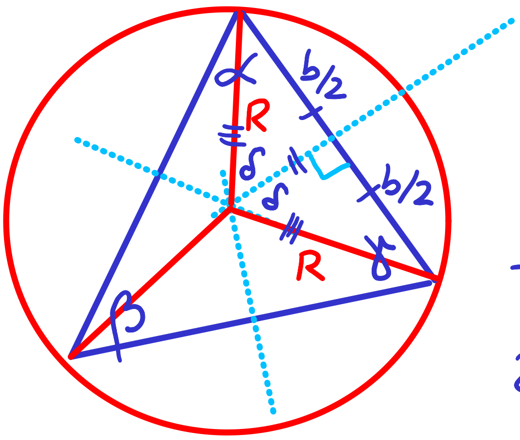
ili $1/\sqrt{3}$

$$v_a=\sqrt{a^2-a^2/4}=\sqrt{\tfrac{3}{4}a^2}=\frac{\sqrt{3}}{2}a$$

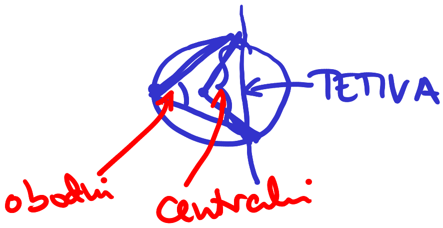


$$\cos 30^\circ=\frac{v_a}{a}=\frac{\sqrt{3}}{2}$$

$$\sin 30^\circ=\frac{a/2}{a}=1/2$$



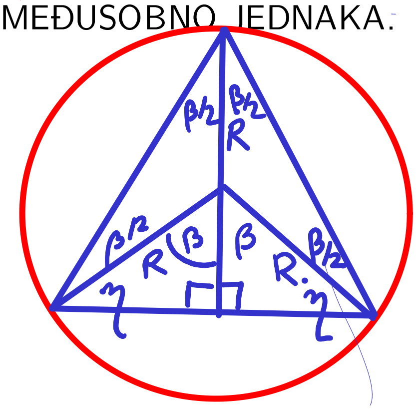
$$\sin\delta = \frac{b/2}{R}$$



TVRDNJA: $\delta = \beta$ (TM O CENTRALNOM I OBODNOM KUTU)

$2\delta = 2\beta$ centralni kut je jednak dvostrukom obodnom kutu nad istom tetivom na kružnici i s iste strane „nad istim lukom"

SVAKA DVA OBODNA KUTA NAD ISTOM TETIVOM I S ISTE STRANE (LUKA) SU MEĐUSOBNO JEDNAKA.



$$\alpha + \beta + \gamma = \pi$$

$$2\eta + 2\beta = \pi$$

$$\eta = \frac{\pi}{2} - \beta$$