

inverzija u odnosu na kružnicu

$$d(S, f(P)) \cdot d(S, P) = r^2$$

$$f(P_\alpha) = P_\alpha(d_0, d_0 \cdot \tan \alpha)$$

$\alpha = 0$  najbljeg točke

$$d(S, P_\alpha) = \frac{d_0}{\cos \alpha}$$

$$d(S, f(P_\alpha)) = \frac{r^2}{\frac{d_0}{\cos \alpha}} = \frac{r^2 \cos \alpha}{d_0}$$

$$f(P_\alpha) = \left( \frac{r^2 \cos^2 \alpha}{d_0}, \frac{r^2 \cos \alpha \sin \alpha}{d_0} \right)$$

$$d(S', f(P_\alpha))$$

$$\sqrt{(y_{f(P_\alpha)} - y_{S'})^2 + (x_{f(P_\alpha)} - x_{S'})^2}$$

$$d^2(S', f(P_\alpha)) = \left( \frac{r^2}{d_0} \left( \cos^2 \alpha - \frac{1}{2} \right)^2 + \left( \frac{r^2}{d_0} \right)^2 \cos^2 \alpha \sin^2 \alpha \right)$$

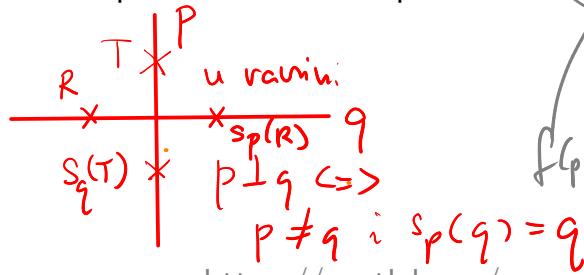
$$= \left( \frac{r^2}{d_0} \right)^2 \left[ (\cos^2 \alpha)^2 - \cos^2 \alpha + \frac{1}{4} + \cos^2 \alpha \sin^2 \alpha \right]$$

$$\cos^2 \alpha \frac{(\cos^2 \alpha - 1)}{-\sin^2 \alpha}$$

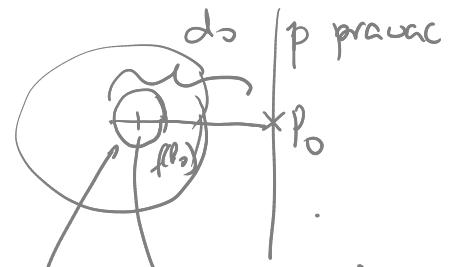
$$d^2(S', f(P_\alpha)) = \left( \frac{r^2}{d_0} \right)^2 \frac{1}{4}$$

$$d(S', f(P_\alpha)) = \frac{1}{2} \frac{r^2}{d_0}$$

POLOŽAJI pravaca i ravnina u prostoru

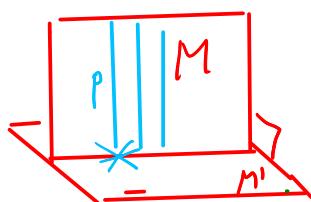
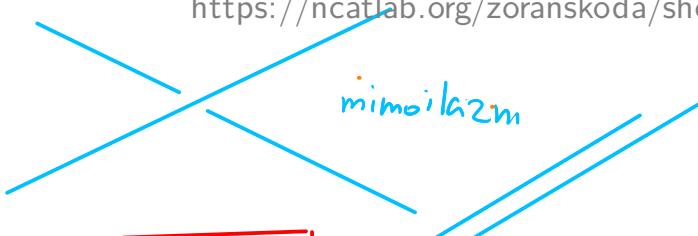


<https://ncatlab.org/zoranskoda/show/aksiomi+stereometrije>



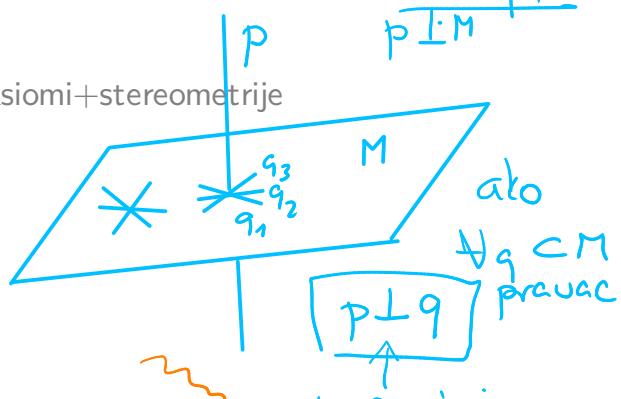
$f(p)$  pravac !

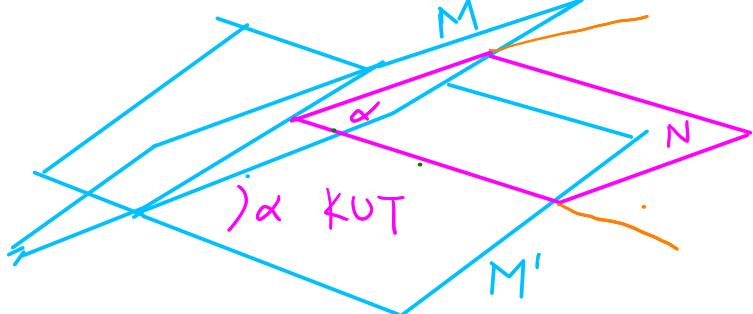
Definicija  
 $p \perp M$



$M \perp M' \Leftrightarrow \exists p \subset M$  pravac,  $p \perp M'$

u ravini određenog s  $p:q$





$$\alpha = \angle O_p p'$$

$$O = p \cap p' \in M \cap M'$$

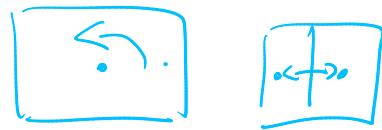
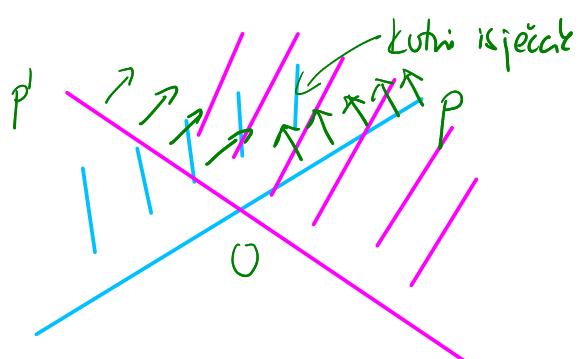
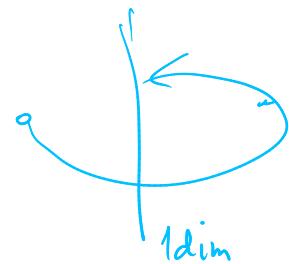
$p \subset h, p' \subset h'$

$$p = h \cap N, p' = h' \cap N'$$

$$N \perp h, h'$$

$\alpha$  je najveći od svih kutova  $\angle (N \cap h, N \cap h')$   
za sve  $N$

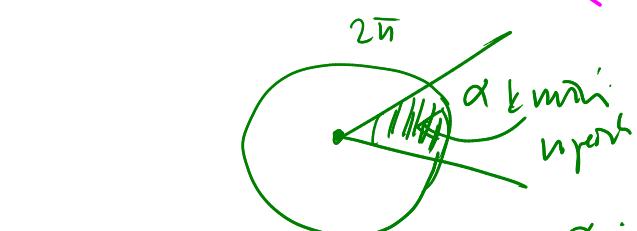
Virtualna slika je iza ogledala



Op, Op' polupravci

$\Rightarrow$  kutni isječak -- presjek dviju poluravnina

i to: s obzirom na pravac  $p$  uzmemmo onu poluravninu u kojoj je polupravac  $Op'$  i obratno, s obzirom na  $p'$  uzmemmo onu poluravninu u kojoj je  $Op$



$$\frac{\alpha}{2\pi} = \frac{l}{2R}$$

$$\frac{\alpha}{2\pi} : \frac{2\pi}{2} = \frac{P_D}{P_A} : \frac{P_D}{R \sin \frac{\alpha}{2}}$$

$$\frac{P_D}{P_A} = \frac{\frac{\alpha}{2\pi}}{\frac{2\pi}{2}} r^2 \rightarrow$$

$$\frac{P_D}{P_A} = \frac{r^2 \alpha}{2\pi}$$

kutni odječak

$$P_D - P_A$$

$$\frac{R^2 \alpha}{2} - \frac{1}{2} 2 R \sin \frac{\alpha}{2} R \cos \frac{\alpha}{2}$$

$$\left\{ 2 R \frac{\alpha}{2} \cos \frac{\alpha}{2} \right\}$$

$$\frac{R^2}{2} (\alpha - 2 \sin \alpha \cos \alpha)$$

$$\sin(\alpha + \beta) = \cos \alpha \sin \beta + \sin \alpha \cos \beta$$

$$\Leftrightarrow (\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\sin(2\alpha) = \cos \alpha \sin \alpha + \sin \alpha \cos \alpha$$

$$= 2 \sin \alpha \cos \alpha$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$



$$R^2 \pi - r^2 \pi$$

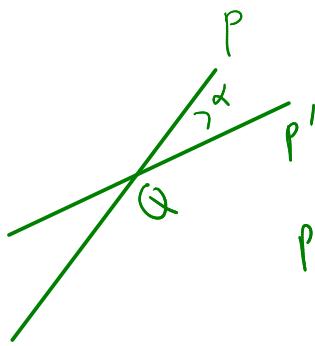
kružni vijenac

$$(R^2 - r^2) \pi \cdot h$$



kut između ravnina  $M$  i  $M'$  koje se sijeku u jednom pravcu je NAJVEĆI od svih kuteva

između pravaca koji su presječnice s trećom ravninom  $N$  gdje su  $N$  sve moguće treće ravnine i to se dešava upravo kad je ta treća ravnina okomita na  $M$  i  $M'$  (tj. okomita na njihovu presječnicu) (ako su  $M$  i  $M'$  paralelne onda je kut između njih 0)

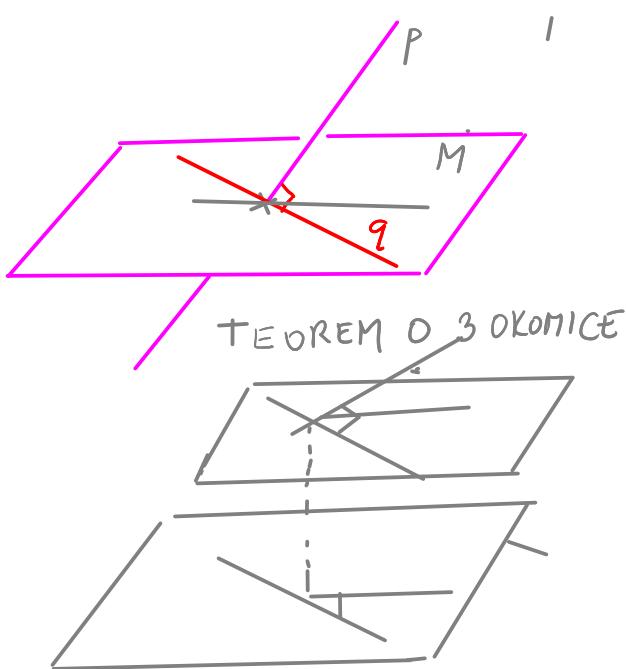
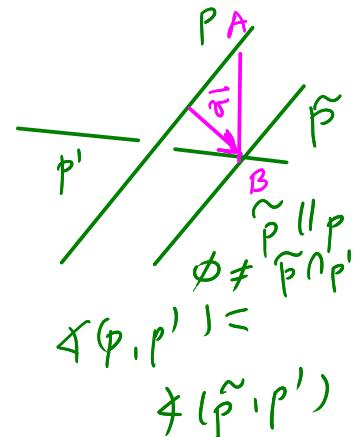


$$p \cap p' \neq \emptyset \Rightarrow \exists M \ni p, p'$$

$$\angle(Q_p, Q_{p'})$$

$$\{Q\} = V \cap P'$$

$$p \parallel p' \Rightarrow \alpha = 0$$



$$\pi: E^3 \rightarrow M$$

$$x^T$$

$$\pi(T)$$

$$\bar{x}(T)$$

$$Ax + By + Cz + D = 0$$

$$\vec{n}(A, B, C)$$

$$\lambda \vec{n}, \lambda \in \mathbb{R}$$

$$(x_0, y_0, z_0) \in M$$

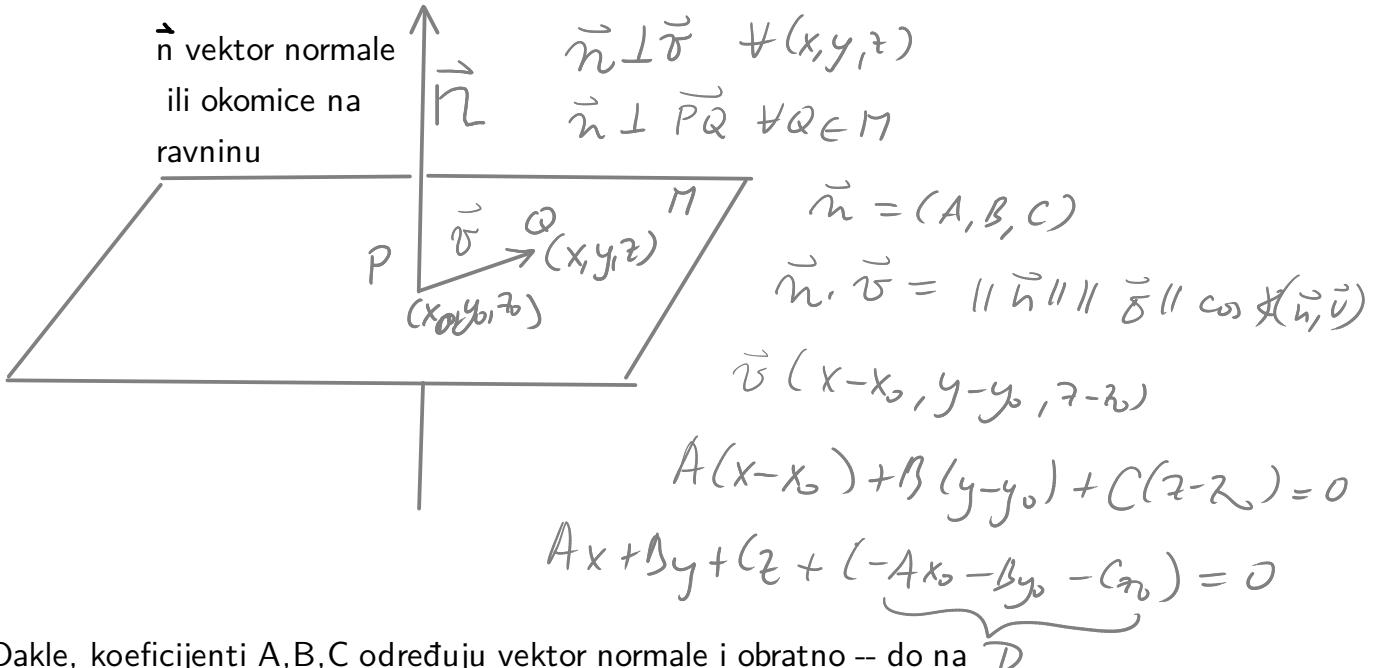
$$\vec{n} \perp M, \vec{n} \perp \vec{v}$$

$$(x, y, z) \in M \quad \vec{v} = (x - x_0, y - y_0, z - z_0)$$

<https://ncatlab.org/zoranskoda/show/aksiomi+stereometrije>

<https://ncatlab.org/zoranskoda/show/analitička+geometrija>

<https://ncatlab.org/zoranskoda/show/analiticki+zadaci>



Dakle, koeficijenti A,B,C određuju vektor normale i obratno -- do na konstantu proporcionalnosti. Dvije ravnine s istim vektorom normale su međusobno paralelne.

Neka razmatranja vezana uz implicitnu jednadžbu ravnine vidi na stranici  
<https://ncatlab.org/zoranskoda/show/analytic%8Dka+geometrija>  
 posebice u poglavljju Ravnina u prostoru

Dobro je konzultirati i knjigu Horvatića, Linearna algebra, poglavljje 6,  
 Elementi analitičke geometrije u  $E^3$