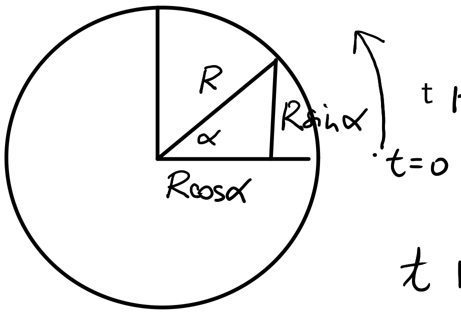
Matematika 2, 5. svibnja 2021.

razni rezultati o kružnim likovima i tijelima (kružnice, sfere, valjci, stošci...)



$t \mapsto (R\cos t,\ R\sin t)$ parametarska jednadžba kružnice

$t \in [0, 2\pi)$

po kružnici se gibamo brže

$t \mapsto (R\cos 2t,\ R\sin 2t)$ opet kružnica

$t \in [0, \pi)$

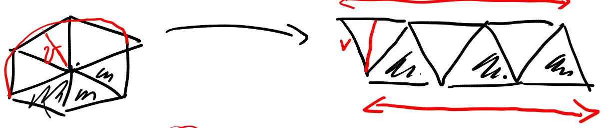
$R\cos \omega t,\ R\sin \omega t$

kutna brzina (prevaljeni kut u jedinici „vremena")

$P = R^2\pi$ $\quad \pi = \dfrac{P_O}{R^2}$

opseg $= 2R\pi$ — jedno je def. π-a drugo teorem

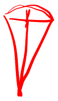
malo manje od pola opsega



$P = v \cdot (\text{pola opsega})$, $v \approx R$

n trokuta

$\frac{n}{2}$ osnovica



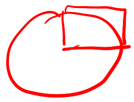
$R \approx v$ malo veći

opseg $\approx$

$$\frac{P}{\frac{1}{2}\text{opseg}} = v$$

$$\boxed{\frac{P_O}{\text{opseg}_O} = \frac{1}{2}v \approx \frac{1}{2}R}$$

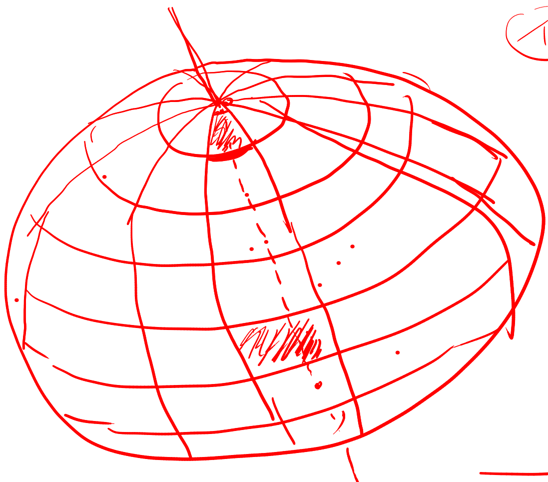
TEOREM



λ sličnost

$P \sim \lambda^2$

$$\frac{P}{O} = \frac{R}{2}$$

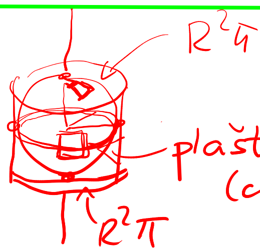


$2R\pi \cdot 2R$

$\parallel$

$4R^2\pi$

$2R = h$

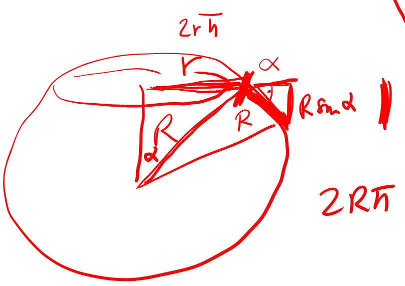


plašt valjka (cilindra)

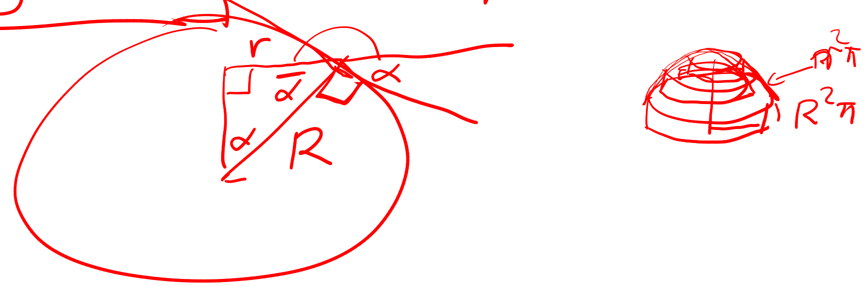
(razmotani) plašt ima površinu $2R\pi \cdot h$

$2R\pi$

$$\boxed{P_{\bigcirc} = 4R^2\pi}$$



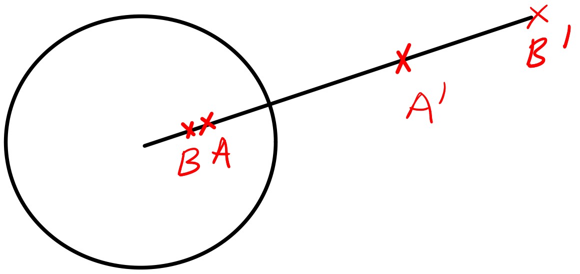
$r = R\sin\alpha$



$$V_{\bigcirc} = \frac{4}{3}R^3\pi$$

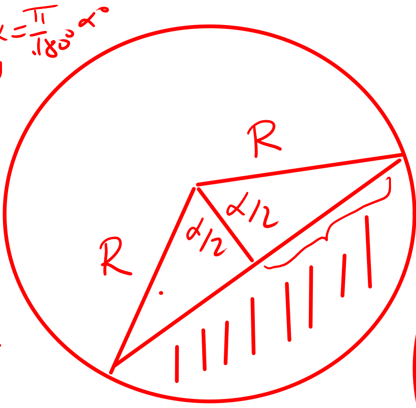
inverzija s obzirom na kružnicu - preslikavanje ravnine osim ishodišta u samu sebe osim ishodišta koje šalje svaku točku na zraci iz ishodišta na udaljenosti x u točku na istoj zraci na udaljenosti x' tako da je x * x' = r * r

čuva kuteve

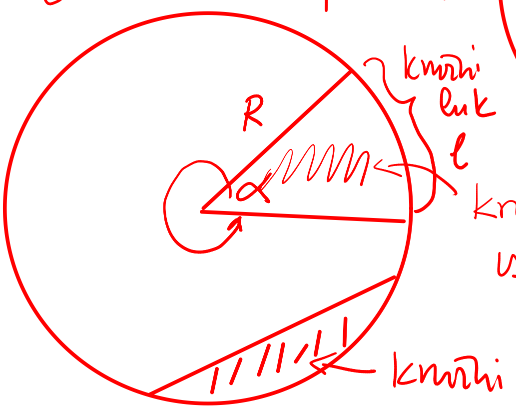


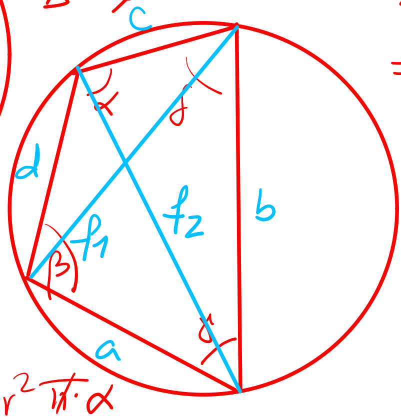
$$\frac{\alpha}{2\pi} = \frac{l}{2\pi r} \qquad \frac{\alpha}{\alpha^\circ} = \frac{\pi}{180^\circ} \qquad \alpha = \frac{\pi}{180^\circ}\alpha^\circ \qquad \alpha^\circ = \frac{180^\circ}{\pi}\alpha$$

$l = \alpha r$ (u radijanima)



$$P_\Delta = \frac{1}{\cancel{2}}\cancel{2}\cdot R\sin\frac{\alpha}{2}\, R\cos\frac{\alpha}{2} = R^2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} = \frac{R^2}{2}\sin\alpha$$





$\alpha + \gamma = 180^\circ$

$\beta + \delta = 180^\circ$

$$\frac{\alpha}{2\pi} = \frac{P}{r^2\pi}$$

$$P_{isj.} = \frac{r^2\cancel{\pi}\cdot\alpha}{2\cancel{\pi}}$$

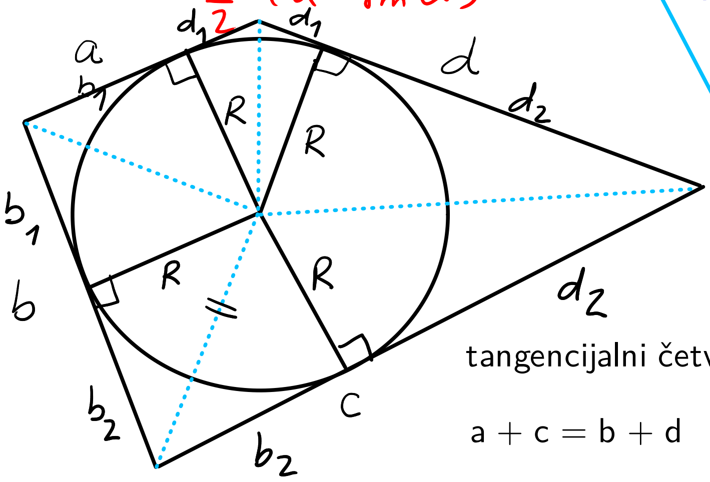
$$P_{isj.} = \frac{r^2\alpha}{2} = \frac{lr}{2}$$

$$P = \frac{R^2\alpha}{2} - \frac{R^2}{2}\sin\alpha = \frac{R^2}{2}(\alpha - \sin\alpha)$$

PTOLOMEJEV TEOREM ZA TETIVNI Č.

a c + b d = f1 f2

zbroj umnožaka duljina nasuprotnih stranica jednak umnošku duljina dijagonala u tetivnom četverokutu

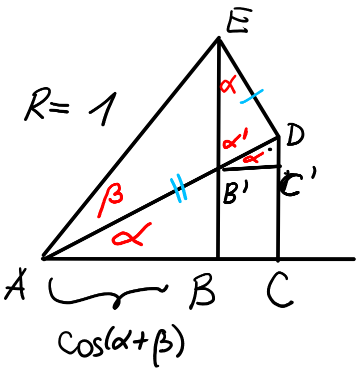


tangencijalni četverokut

a + c = b + d

PAUZA DO 17:06

$$\underbrace{b_1 + b_2 + d_1 + d_2}_{a\ +\ c} = \underline{b + d}$$



$|B'C'| = |BC|$ $\qquad \measuredangle BAE = \alpha + \beta$

$|AB| = \underbrace{|AE|}_{1} \cos(\alpha+\beta)$

$\parallel$

$\underbrace{|AC|}_{|AD| \cdot \cos\alpha} - |BC|$

$\parallel |B'C'| = |B'D| \cdot \cos\alpha$

$\Delta EB'D \rightsquigarrow |B'D| = |ED| \cdot tg\,\alpha$

uvrstimo

$$\overbrace{\cos(\alpha+\beta)}^{|AB|} = \overbrace{|\underbrace{AD}_{\cos\beta}| \cdot \cos\alpha}^{|AC|} - \overbrace{|\underbrace{ED}_{\sin\beta}| \cdot \underbrace{tg\,\alpha}_{|B'D|} \cdot \cos\alpha}^{|BC|}$$

$\sin\alpha$

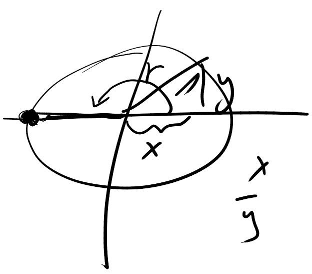
$\frac{NAS}{PRIL} \cdot \frac{PRIL}{HIP}$

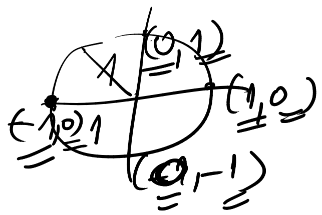
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

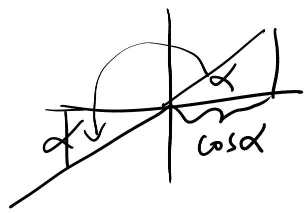
adicioni teorem za funkciju kosinus

$$\cos(\alpha+\pi) = \cos\alpha\underbrace{\cos\pi}_{-1} \underbrace{-\sin\alpha\underbrace{\sin\pi}_{0}}$$

$$\cos(\alpha+\pi) = -\cos\alpha$$





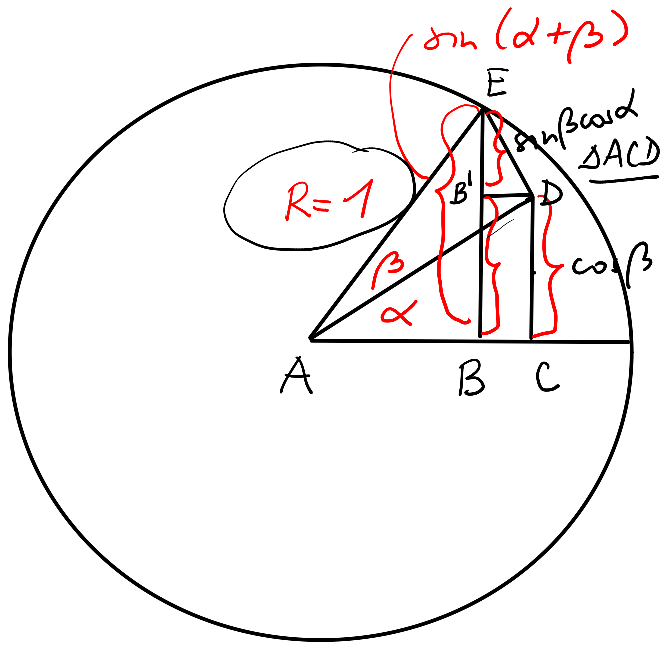


$1 = |AE| = R, \alpha, \beta$

ΔADE

$|AD| = R\cos\beta = \cos\beta$

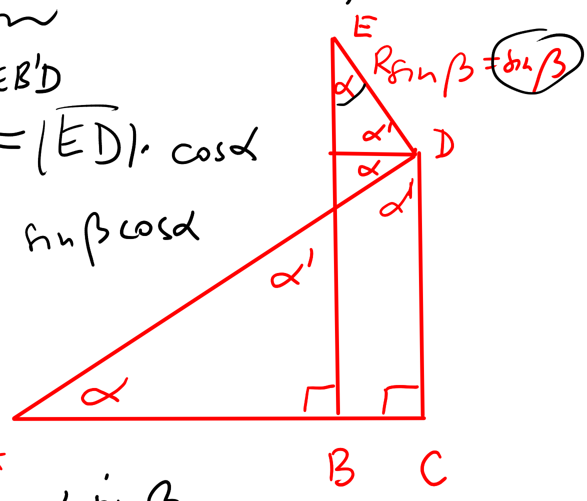
$|CD| = \underbrace{|AD|}\cdot\sin\alpha = \cos\beta\cdot\sin\alpha$



$\Delta EB'D$

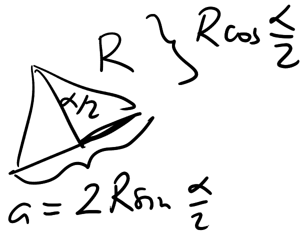
$|B'E| = |ED| \cdot \cos\alpha$

$= \sin\beta\cos\alpha$



$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

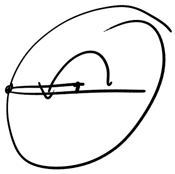
$$\sin\alpha = \sin\left(\frac{\alpha}{2}+\frac{\alpha}{2}\right) = \sin\frac{\alpha}{2}\cos\frac{\alpha}{2}+\cos\frac{\alpha}{2}\sin\frac{\alpha}{2}$$

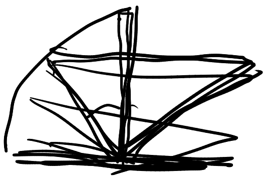


$$\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$$

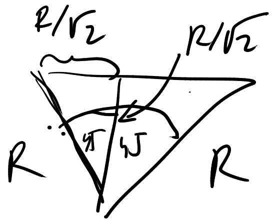
$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$P = \frac{1}{2}a\cdot v_a = \frac{1}{2}\,2R\sin\frac{\alpha}{2}\cdot R\cos\frac{\alpha}{2} = \frac{R^2}{2}\cdot\underbrace{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}$$





$$P = \frac{R^2}{2}\sin\alpha$$



$$\frac{R^2}{2}\underbrace{\sin 90^\circ}_{1}$$