MATEMATIKA 1, 24/01/18

Malo još o kompleksnim brojevima $(2,3) = 2+3i$

$(0,1) = i \qquad (1,0) = 1 \in \mathbb{R}$

$\mathbb{R} \subset \mathbb{C} \qquad 1 \longmapsto (1,0) \qquad 2.73 \longmapsto (2.73, 0)$

$z = (\underset{\mathrm{Re}\, z}{2}, \underset{\mathrm{Im}\, z}{3}) \qquad 2 + 3i \qquad i^2 = -1$

realno

$\sqrt{-1} = \begin{cases} +i \\ -i \end{cases} \qquad (-i)^2 = ((-1)\cdot i)^2 = 1\cdot(-1) = -1$

$\mathrm{Im}\, i^3 = -1, \ \mathrm{Re}\, i^3 = 0$

$i^3 = i \cdot i^2 = i\cdot(-1) = -i$

$i^4 = i^2 \cdot i^2 = (-1)\cdot(-1) = +1$

$i^{57} = i^{14\cdot 4 + 1} = i^{14\cdot 4}\cdot i^1$

$= (i^4)^{14}\cdot i$

$= 1^{14}\cdot i = i$

$(a^m)^n = a^{m\cdot n}$

$a^{m+n} = a^m \cdot a^n$

$57 : 4 = 14$
17
1

$57 = 14 \times 4 + 1$

$m : n = s$ ostatak r

$0 \leq r < n$

$0 \leq s \quad \begin{cases} m = n\cdot s + r \\ 0 \leq r < n \end{cases}$

kongruento modulo

$57 \equiv 1 \mod 4$

$i^1 = i \qquad i^{57} = i^1 = i$

$i^2 = -1 \qquad i^{58} = i^2 = -1$

$i^3 = -i \qquad i^{59} = i^3 = -i$

$i^4 = 1$

korjenovanje -- svaki komplesni broj ima kvadratni korijen -- zapravo svaki osim nule ima dva kvadratna korijena

$\sqrt{i} = u + vi, \quad u, v \in \mathbb{R} \qquad /^2$

$i = (u+vi)^2$

$i = (u+vi)\cdot(u+vi) = u^2 + uvi + uvi + v^2 i^2$ $\quad i^2 = -1$

$= (u^2 - v^2) + 2uvi$

$i = (u^2 - v^2) + 2uv\, i$

$\mathrm{Re}\, z = 0 \qquad \mathrm{Re}\, z' = u^2 - v^2$

$\mathrm{Im}\, z = 1 \qquad \mathrm{Im}\, z' = 2uv$

$\begin{cases} 0 = u^2 - v^2 \\ 1 = 2\cdot u \cdot v \end{cases} \Rightarrow v = \frac{1}{2u}$

$0 = u^2 - \frac{1}{(2u)^2} = u^2 - \frac{1}{4u^2} \qquad / u^2$

$0 = u^4 - 1/4 \qquad \underset{>0}{u^2} = t > 0 \qquad 0 = t^2 - 1/4$

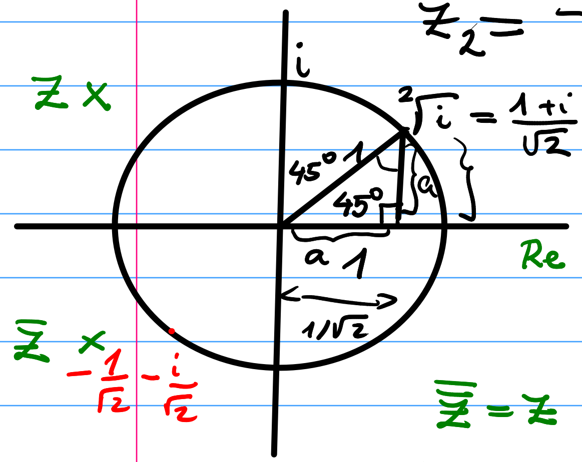
$t^2 = \frac{1}{4}$ $\quad t = \frac{1}{2} > 0$ $\qquad \frac{2}{\sqrt{2}} = \frac{(\sqrt{2})^{\cancel{2}}}{\cancel{\sqrt{2}}} = \sqrt{2}$

(1) $\underline{u^2 = \frac{1}{2}} \Rightarrow u = \frac{1}{\sqrt{2}}$

$v = \frac{1}{2u} = \frac{1}{2\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}}$ $\qquad z_1 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i = \frac{1+i}{\sqrt{2}}$

(2) $u = -\frac{1}{\sqrt{2}}$ $\quad v = \frac{1}{2u} = \frac{1}{2(-\frac{1}{\sqrt{2}})} = \frac{-1}{\sqrt{2}}$

$z_2 = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i = -\frac{1+i}{\sqrt{2}}$



$a^2 + a^2 = 1^2$

$2a^2 = 1$

$a^2 = \frac{1}{2} \Rightarrow a = \frac{1}{\sqrt{2}}$

$\bar{\bar{z}} = z \Leftrightarrow z \mapsto \bar{z}$ je involucija

$|2+3i| = \sqrt{2^2 + 3^2} = \sqrt{13}$

modul ili apsolutna vrijednost kompleksnog broja

$\overline{2+3i} = 2 - 3i$ kompleksno konjugirana vrijednost