COUNTER-TERMS IN COULOMB GAUGE QCD

RAB 2007

OUTLINE

INTRODUCTION

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7. COUNTER-TERMS IN X-SPACE

CONCLUSION

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— JUST UV POLE PARTS TO ORDER g^2

FEYNMAN RULES

A_0 INSTANTANEOUS COULOMB PROPAGATOR

A_i TRANSVERSE PROPAGATOR

E_i MOMENTUM CONJUGATE TO A_i

i - - - A_i - - - j $\qquad -\dfrac{1}{k^2+i\epsilon}\left[\delta_{ij}-\dfrac{k_i k_j}{\underset{\sim}{k}^2}\right]$

$0 \cdots A_0 \cdots 0 \qquad -\dfrac{1}{\underset{\sim}{k}^2}$

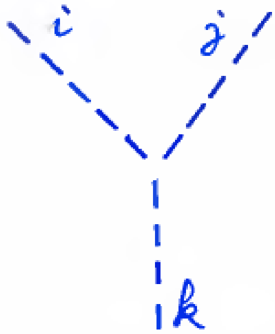
m —— n $\qquad -\dfrac{\underset{\sim}{k}^2}{k^2+i\epsilon}\left[\delta_{mn}-\dfrac{k_m k_n}{\underset{\sim}{k}^2}\right]$

WHERE $\quad k^2 = k_0^2 - \underset{\sim}{k}^2$

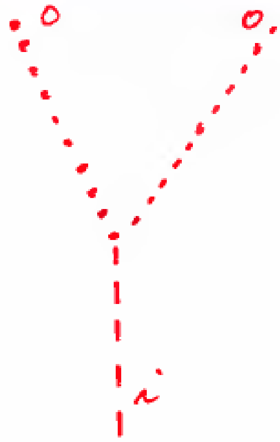
A_i - - - > —— E_j $\qquad \dfrac{ik_0}{k^2+i\epsilon}\left[\delta_{ij}-\dfrac{k_i k_j}{\underset{\sim}{k}^2}\right]$

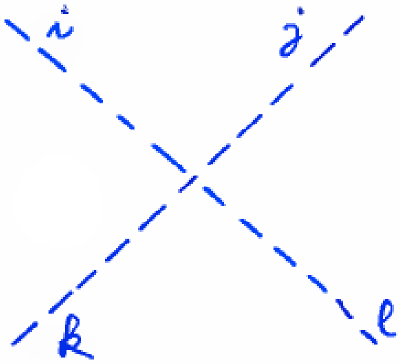
$A_0 \cdots >$ —— E_i $\qquad -\dfrac{ik_i}{\underset{\sim}{k}^2}$

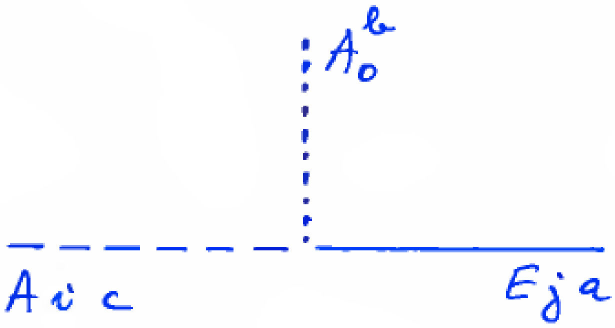
VERTICES



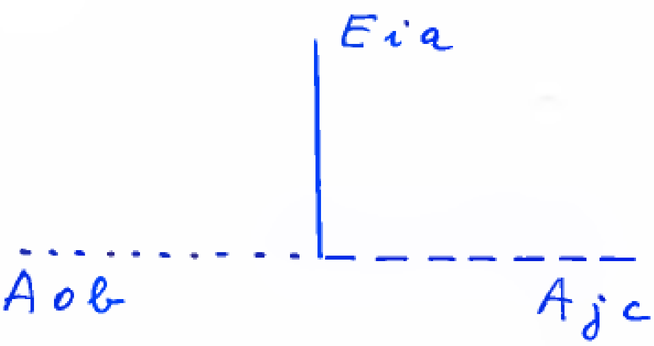
NOTE, NO SUCH VERTEX!







$$g f^{abc} \delta_{ij}$$



$$g f^{abc} \delta_{ij}$$

REMEMBER GHOSTS

$$\frac{1}{2}(\partial_i A_i)^2 \longrightarrow c^* \partial_i D_i(A) c$$

$D(A)$ – COVARIANT DERIVATIVE

CLOSED LOOPS WITH MINUS SIGN

CANCEL EACH OTHER

FORGET GHOSTS AS LONG AS WE OMIT COULOMB CLOSED LOOPS

MOTIVATION

$$\sigma_{Coul} \equiv \lim_{R\to\infty} V(R)/R \qquad \text{STRING TENSION}$$

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POTENTIAL
$$V(K) = \lim_{k_0\to\infty} g^2 D^{A_0 A_0}(k_0, K)$$

$k_0 \to \infty$

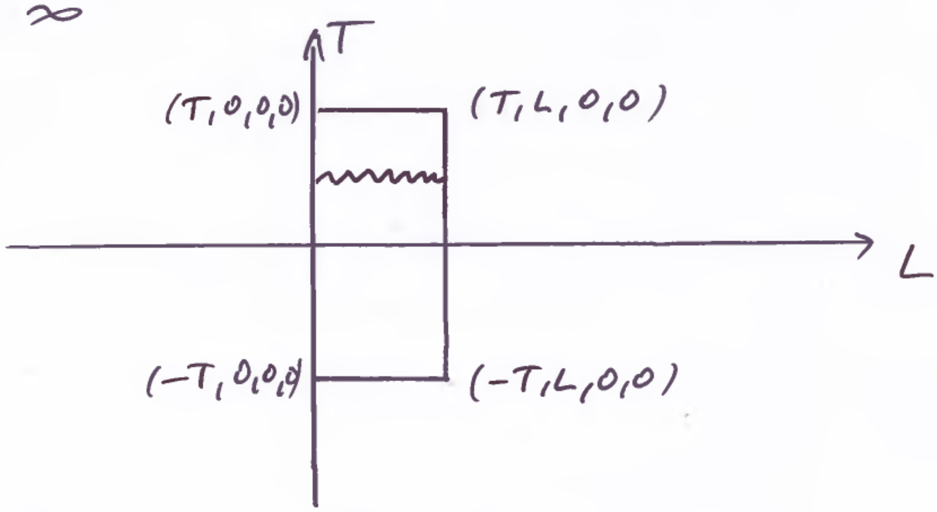
$\delta(x_0)$ – INSTANTANEOUS IN POSITION SPACE

J.C. TAYLOR $k_0 \to 0$ QUARK–ANTIQUARK POTENTIAL

WILSON LOOP

$(T,0,0,0)$, $(-T,0,0,0)$, $(T,L,0,0)$, $(-T,L,0,0)$

$T \to \infty$



$$D_{\mu\nu}(k_0,K) \longrightarrow D_{00}(k_0,K)$$

$$W = \int d^4k \int_{-T}^{T} dt \int_{-T}^{T} dt'\, e^{i(t-t')k_0}\, e^{iL\cdot K}\, D_{00}(k_0,K)$$

$$= \int d^4k \left(\frac{2\sin Tk_0}{k_0}\right)^2 e^{iL\cdot K}\, D_{00}(k_0,K)$$

REPREZENTATION OF δ

$$\delta(k_0) = \frac{1}{\pi} \lim_{T\to\infty} \frac{\sin Tk_0}{k_0}$$

i.e. $\dfrac{2\sin Tk_0}{k_0} \times 2\pi\delta(k_0)$

$$W = 4\pi T \int d^3K\, e^{iK\cdot L}\, D_{00}(k_0\to 0, K)$$

ZERO'th ORDER $\to$ COULOMB POTENTIAL $\dfrac{1}{L}$

D_{00} PROPAGATOR

$$D^{A_0A_0} = \frac{1}{K^4}\left[\Gamma^{A_0A_0} + iK_n\Gamma^{A_0E_n}\right]$$
$$+ \frac{iK_m}{K^4}\left[\Gamma^{E_mA_0} + iK_n\Gamma^{E_mE_n}\right]$$

$$D^{A_0A_0} = c\,(K^2)^{-2} \times \Big\{ \frac{11}{3}\Gamma\left(\frac{\epsilon}{2}\right)K^2 - \frac{5}{3}K^2 \ln\frac{(-k^2-i\eta)}{\mu^2}$$
$$+ \frac{1}{2}k^2(k^2+2k_0^2)\times D \quad - 2K^2 \ln\frac{K^2}{\mu^2}$$
$$+ \frac{k^2}{2k_0K}(k^2+2k_0^2)\ln\frac{k_0+K-i\eta}{k_0-K+i\eta} \times \ln\frac{K^2}{(-k^2-i\eta)}$$
$$-(3k_0^2-K^2)\ln\frac{K^2}{(-k^2-i\eta)} \quad -(6k_0^2+2K^2)\ln 2$$
$$+ 6k_0^2 + \frac{31}{9}K^2 \Big\}$$

LOOK FOR THE LIMTIS

$$k_0 \longrightarrow \infty$$

$$k_0 \longrightarrow 0$$

QUARK – ANTIQUARK POTENTIAL

D. ZWANZIGER

$$P_{coul} \equiv \lim_{R\to\infty} \frac{V(R)}{R}$$

NON-ZERO VALUE OF P_{coul} – SIGNAL FOR COLOR CONFINEMENT

MOMENTUM SPACE

$$V(K) = \lim_{k_0\to\infty} g^2 D^{A_0A_0}(k_0,K)$$

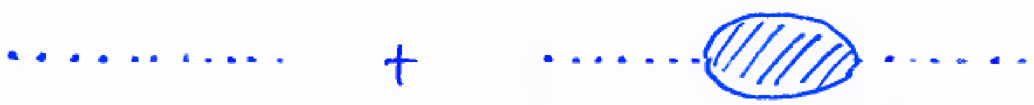
$$\lim_{k_0\to\infty} D^{A_0A_0}(k_0,K) = \frac{c}{K^2}\left\{\frac{11}{3}\Gamma\left(\frac{\epsilon}{2}\right) - \frac{11}{3}\ln\frac{K^2}{\mu^2} - i\pi - \frac{28}{3}\ln 2 + \frac{103}{9} - 2\ln\frac{K}{k_0}\right\}$$

LIMIT $k_0 \to \infty$ DOES NOT EXIST !

J.C.TAYLOR $\qquad k_0 \to 0$

$$\lim_{k_0\to 0} D^{A_0A_0}(k_0,K) = \frac{c}{K^2}\left\{\frac{11}{3}\Gamma\left(\frac{\epsilon}{2}\right) - \frac{11}{3}\ln\frac{K^2}{\mu^2} + \frac{31}{9}\right\}$$

EVALUATE THE POTENTIAL



$$V(L) = -4\pi g_R^2 \int_0^\infty dK \frac{\sin KL}{KL} \Big\{ 1 + \epsilon \ln \frac{\mu}{K} - 2 \times \frac{11}{3} \frac{c}{\epsilon} - \frac{11}{3} \ln \frac{\mu^2}{K^2}$$

$$+ c\left(1 + \epsilon \ln \frac{\mu}{K}\right)\left(\frac{11}{3} \Gamma\left(\frac{\epsilon}{2}\right) - \frac{11}{3} \ln \frac{K^2}{\mu^2} + \frac{31}{9} \right) \Big\}$$

$$V(L) = -4\pi g_R^2 \int_0^\infty dK \frac{\sin KL}{KL} \Big\{ 1 - \frac{11}{3} c\gamma - \frac{11}{3} c \ln \frac{K^2}{\mu^2} + \frac{31}{9} c \Big\}$$

$$\int_0^\infty \frac{\sin ax}{x}\, dx = \frac{\pi}{2} \qquad \text{for } a > 0$$

$$\int_0^\infty \ln x \sin ax \, \frac{dx}{x} = -\frac{\pi}{2} (\gamma + \ln a) \qquad \text{for } a > 0$$

$$V(L) = -2\pi^2 g_R^2(\mu) \frac{1}{L} \Big\{ 1 + \frac{31}{9} c + \frac{11}{3} c\gamma + \frac{11}{3} c \ln (\mu L)^2 \Big\}$$

ASSUME

$$L \times \mu = 1$$

$$g_R(\mu) = g_R\left(\frac{1}{L}\right)$$

SUPPOSE

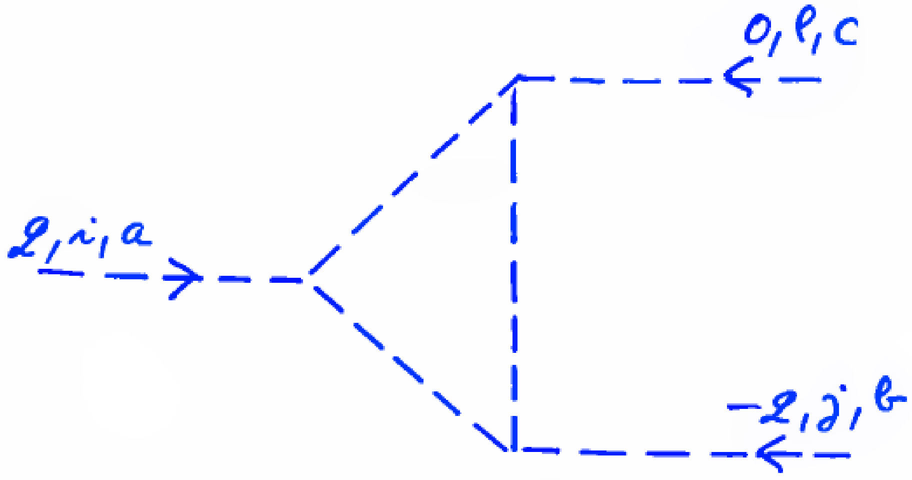
$$g_R\left(\frac{1}{L}\right) \longrightarrow 0 \quad \text{for } L \to 0$$

$$g_R\left(\frac{1}{L}\right) \longrightarrow \infty \quad \text{for } L \to \infty$$

AND MAKE EVERYBODY HAPPY!

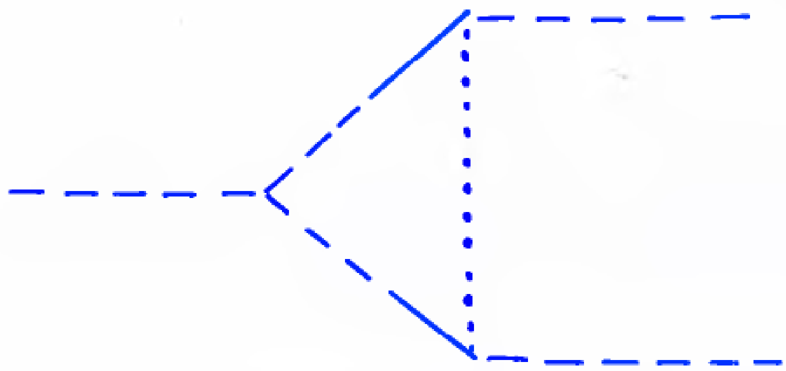
THREE-POINT FUNCTIONS

$A_i A_j A_\ell$ vertex



(m1)

$$(m1)^{abc}_{ij\ell}(q,-q,0) = \frac{1}{3}\left(2q_\ell \delta_{ij} - q_j \delta_{\ell i} - q_i \delta_{\ell j}\right)\Gamma\left(\frac{\epsilon}{2}\right) \times g^3 \pi^2 C_G f^{abc}$$



(m2a)

$$(m2a)^{abc}_{ij\ell}(q,-q,0) = \frac{1}{30}\left(17 q_\ell \delta_{ij} - 13 q_j \delta_{\ell i} - 8 q_i \delta_{j\ell}\right) \times \Gamma\left(\frac{\epsilon}{2}\right) g^3 \pi^2 C_G f^{abc}$$

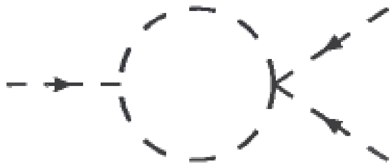


Figure 13: There are 3 graphs in this class of diagrams.

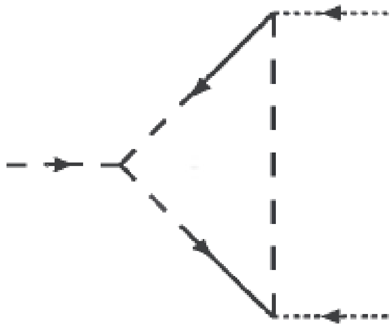


Figure 14: Graph with two external Coulomb lines (there are 3 diagrams in this class).

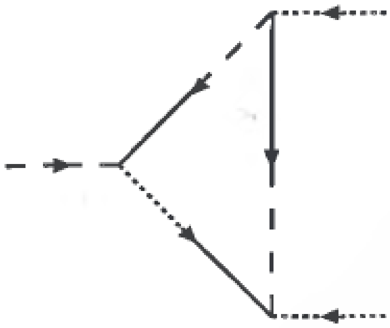


Figure 15: There are two graphs in this class.

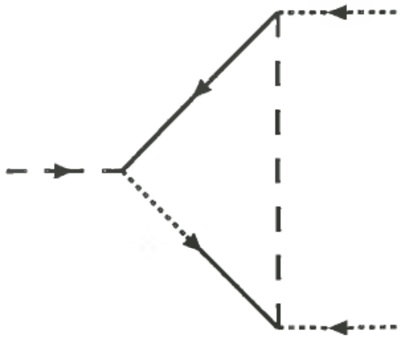


Figure 16: There are two graphs in this class.

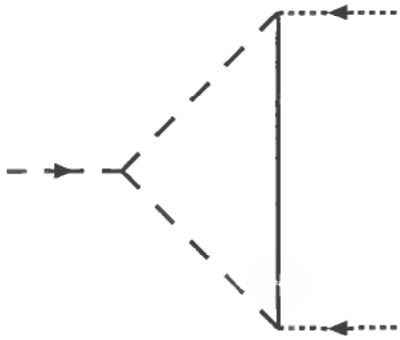


Figure 17: The graph with two external Coulomb lines and one three-gluon vertex.

Figure 18: Graphs contributing to the($\mathbf{A}_i\mathbf{A}_j\mathbf{A}_0$) three-point function.

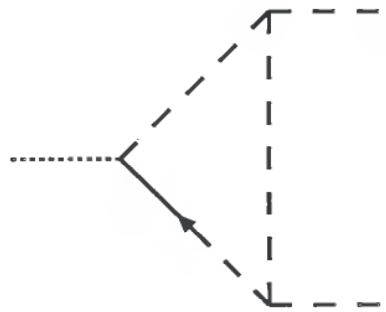


Figure 19: Graph contributing to the $(\mathbf{A}_i\mathbf{A}_j\mathbf{A}_0)$ three-point function which contains a three-gluon vertex.

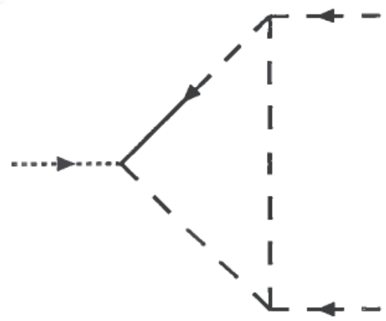


Figure 20: The $(\mathbf{A}_i\mathbf{A}_j\mathbf{A}_0)$ graph with a three-gluon vertex.

Figure 21: The $(\mathbf{A}_i\mathbf{A}_j\mathbf{A}_0)$ graph with a four-gluon vertex.

COUNTERTERMS

$$\begin{aligned}
\mathcal{L}_1 = & -\frac{11}{12} c \left(\mathbf{F}_{ij}\right)^2 - \frac{4}{3} c\, \mathbf{F}_{ij} \cdot \partial_j \mathbf{A}_i \\
& + \frac{4}{3} c g\, \mathbf{F}_{ij} \cdot \left(\mathbf{A}_i \wedge \mathbf{A}_j\right) \\
& \boxed{-\frac{1}{6} c \left(\mathbf{F}_{0i}\right)^2} + \frac{2}{3} c \left(\mathbf{E}_i\right)^2 + \frac{4}{3} c\, \mathbf{E}_i \cdot \mathbf{F}_{0i} \\
& + \frac{4}{3} c g\, \mathbf{E}_i \cdot \left(\mathbf{A}_i \wedge \mathbf{A}_0\right) - \frac{4}{3} c\, \mathbf{E}_i \cdot \partial_0 \mathbf{A}_i \\
& + \frac{4}{3} c \left(\mathbf{u}_i + \partial_i \mathbf{c}^*\right) \cdot \partial_i \mathbf{c}
\end{aligned}$$

$$c = \frac{g^2}{16\pi^2} C_G \Gamma\left(\frac{\epsilon}{2}\right)$$

$$\left(\mathbf{A}_i \wedge \mathbf{c}\right)^a = f^{abd} A_i^b c^d$$

$$\frac{4}{3} c\, \mathbf{E}_i \cdot \mathbf{F}_{0i} + \frac{4}{3} c g\, \mathbf{E}_i \cdot \left(\mathbf{A}_i \wedge \mathbf{A}_0\right) - \frac{4}{3} c\, \mathbf{E}_i \cdot \partial_0 \mathbf{A}_i$$

$$= -\frac{4}{3} \mathbf{E}_i \cdot \partial_i \mathbf{A}_0$$

$$-\frac{1}{6} + \frac{2}{3} + \frac{4}{3} = \frac{11}{6}$$

CONSISTENT RULES FOR DERIVATIVES IN $\mathcal{L}$ AND COUNTER-TERMS

(i) MOMENTA FLOWING INTO THE VERTEX

$$\overrightarrow{\partial_i} \longrightarrow -ik_i \qquad \overrightarrow{\partial_0} \longrightarrow -ik_0$$

(ii) LEFT DERIVATIVES

$$\overleftarrow{\partial_i} \longrightarrow ik_i \qquad \overleftarrow{\partial_0} \longrightarrow ik_0$$

EXTRA FACTORS

(a) $(2\pi)^4 i$ FOR EACH VERTEX

(b) $\dfrac{1}{(2\pi)^4 i}$ FOR EACH PROPAGATOR

PROPAGATORS ARE MINUS THE INVERSE OF THE QUADRATIC PART OF THE LAGRANGIAN.
PROPAGATOR MATRIX IS HERMITIAN.

$$(\underset{\sim}{A_\mu} \wedge \underset{\sim}{\omega})_a = f_{abc} A_{\mu b} \omega_c$$

$$\mathcal{L} = -\frac{1}{4}\underset{\sim}{F_{ij}}\cdot\underset{\sim}{F_{ij}} - \frac{1}{2}(\underset{\sim}{E_i})^2 + \underset{\sim}{E_i}\cdot\underset{\sim}{F_{0i}}$$
$$+\,\partial_i\underset{\sim}{c}^*\cdot\partial_i\underset{\sim}{c} + g\,\partial_i\underset{\sim}{c}^*\cdot(\underset{\sim}{A_i}\wedge\underset{\sim}{c})$$
$$+\,\underset{\sim}{u_i}\cdot[\partial_i\underset{\sim}{c} + g(\underset{\sim}{A_i}\wedge\underset{\sim}{c})]$$
$$+\,\underset{\sim}{u_0}[\partial_0\underset{\sim}{c} + g(\underset{\sim}{A_0}\wedge\underset{\sim}{c})$$
$$-\frac{1}{2}g\,\underset{\sim}{K}\cdot(\underset{\sim}{c}\wedge\underset{\sim}{c}) + g\,\underset{\sim}{v_i}\cdot(\underset{\sim}{E_i}\wedge\underset{\sim}{c})$$

u_i^a, u_0^a, v_i^a – RESPECTIVE SOURCES

v, u, c, c^* – ANTICOMMUTING OPERATORS

GENERATING FUNCTIONAL Γ FOR 1PI GREEN'S FUNCTIONS OBEYS BRST ID.

$$\frac{\delta\Gamma}{\delta\underset{\sim}{A_i}}\cdot\frac{\delta\Gamma}{\delta\underset{\sim}{u_i}} + \frac{\delta\Gamma}{\delta\underset{\sim}{A_0}}\frac{\delta\Gamma}{\delta\underset{\sim}{u_0}} + \frac{\delta\Gamma}{\delta\underset{\sim}{c}}\frac{\delta\Gamma}{\delta\underset{\sim}{K}} + \frac{\delta\Gamma}{\delta\underset{\sim}{E_i}}\cdot\frac{\delta\Gamma}{\delta\underset{\sim}{v_i}} = 0$$

Γ_0 – ORIGINAL ACTION

$$\Gamma_0 = \int d^4x\,\mathcal{L}(x)$$

LET Γ BE THE COMPLETE EFFECTIVE ACTION AND Γ_1 BE THE EFFECTIVE ACTION TO ONE-LOOP ORDER.

$$\Gamma = \Gamma_0 + \Gamma_1$$

TO ONE LOOP ORDER

$$\Gamma_1 * \Gamma_0 + \Gamma_0 * \Gamma_1 \equiv \Delta \Gamma_1 = 0$$

WHERE

$$\Delta = \frac{\partial \mathcal{L}}{\partial \underset{\sim}{A}_i} \frac{\partial}{\partial \underset{\sim}{\mu}_i} + \frac{\partial \mathcal{L}}{\partial \underset{\sim}{\mu}_i} \frac{\partial}{\partial \underset{\sim}{A}_i} + \frac{\partial \mathcal{L}}{\partial \underset{\sim}{A}_0} \frac{\partial}{\partial \underset{\sim}{\mu}_0} + \frac{\partial \mathcal{L}}{\partial \underset{\sim}{\mu}_0} \frac{\partial}{\partial \underset{\sim}{A}_0}$$
$$+ \frac{\partial \mathcal{L}}{\partial \underset{\sim}{C}} \frac{\partial}{\partial \underset{\sim}{K}} + \frac{\partial \mathcal{L}}{\partial \underset{\sim}{K}} \frac{\partial}{\partial \underset{\sim}{C}} + \frac{\partial \mathcal{L}}{\partial \underset{\sim}{E}_i} \frac{\partial}{\partial \underset{\sim}{\nu}_i} + \frac{\partial \mathcal{L}}{\partial \underset{\sim}{\nu}_i} \frac{\partial}{\partial \underset{\sim}{E}_i}$$

AND

$$\Delta^2 = 0$$

ONE CLASS OF SOLUTIONS TO THIS EQUATION IS

$$\Gamma_1^{(i)} = \Delta G$$

$$G = a_5 \underset{\sim}{A}_i \cdot (\underset{\sim}{\mu}_i + \partial_i \underset{\sim}{C}^*) + a_6 \underset{\sim}{A}_0 \cdot \underset{\sim}{\mu}_0 + a_7 \underset{\sim}{C} \cdot \underset{\sim}{K}$$
$$+ a_8 \underset{\sim}{E}_i \cdot \underset{\sim}{\nu}_i + a_9 \underset{\sim}{\nu}_i \cdot \partial_i \underset{\sim}{A}_0 + a_{10} \underset{\sim}{\nu}_i \cdot \partial_0 \underset{\sim}{A}_i$$
$$+ a_{11} \underset{\sim}{\nu}_i \cdot (\underset{\sim}{A}_0 \wedge \underset{\sim}{A}_i)$$

OTHER SOLUTIONS ARE THE EXPLICITLY GAUGE-INVARIANT TERMS

$$\Gamma_1^{(ii)} = a_1 (\underset{\sim}{F}_{ij})^2 + a_2 \underset{\sim}{E}_i \cdot \underset{\sim}{F}_{0i} + a_3 (\underset{\sim}{F}_{0i})^2 + a_4 (\underset{\sim}{E}_i)^2$$

DIFFERENTIATING BRST ID. WITH RESPECT TO g AND SPECIALIZING TO ONE-LOOP ORDER

$$\Delta \Gamma_1^{(iii)} = 0$$

(WHERE a_0 IS ANOTHER DIVERGENT CONSTANT)

$$\Gamma_1^{(iii)} = a_0 g \frac{\partial \Gamma_0}{\partial g}$$

COMBINING THESE THREE CONTRIBUTIONS

$$\Gamma_1 = \Gamma_1^{(i)} + \Gamma_1^{(ii)} + \Gamma_1^{(iii)} = \int d^4x \, \mathcal{L}_1(x)$$

NOW EVALUATE $\Gamma_1^{(i)}$ AND $\Gamma_1^{(iii)}$

$$\frac{\delta L}{\delta A_i} = -\partial_j F_{ij}^a - g(\underset{\sim}{A}_j \wedge \underset{\sim}{F}_{ij})^a - g(\partial_i \underset{\sim}{c}^* \wedge \underset{\sim}{c})^a - g(\underset{\sim}{\mu}_i \wedge \underset{\sim}{c})^a - \partial_0 E_i^a + g(\underset{\sim}{E}_i \wedge \underset{\sim}{A}_0)^a$$

$$\frac{\delta G}{\delta \mu_i^a} = a_5 A_i^a$$

etc.

⋮

$$\mathcal{L}_1 = a_1(\boldsymbol{F}_{ij})^2 + (a_2+a_8+a_9)\,\boldsymbol{E}_i\cdot\boldsymbol{F}_{0i}$$
$$+(a_3-a_9)(\boldsymbol{F}_{0i})^2 + (a_4-a_8)(\boldsymbol{E}_i)^2$$
$$+a_5\,\boldsymbol{F}_{ij}\cdot\partial_j\boldsymbol{A}_i - (a_5+\tfrac{1}{2}a_0)g\,\boldsymbol{F}_{ij}\cdot(\boldsymbol{A}_i\wedge\boldsymbol{A}_j)$$
$$-(a_0+a_5+a_6)g\,\boldsymbol{E}_i\cdot(\boldsymbol{A}_i\wedge\boldsymbol{A}_0) + \boldsymbol{E}_i\cdot(a_5\partial_0\boldsymbol{A}_i - a_6\partial_i\boldsymbol{A}_0)$$
$$-a_5(\boldsymbol{u}_i+\partial_i\boldsymbol{c}^*)\cdot\partial_i\boldsymbol{c} + a_0 g\,\partial_i\boldsymbol{c}^*\cdot(\boldsymbol{A}_i\wedge\boldsymbol{c})$$
$$-a_6\,\boldsymbol{u}_0\cdot\partial_0\boldsymbol{c} + a_0 g\,\boldsymbol{u}_0\cdot(\boldsymbol{A}_0\wedge\boldsymbol{c})$$
$$-a_7(\boldsymbol{u}_i+\partial_i\boldsymbol{c}^*)\cdot\{\partial_i\boldsymbol{c} + g(\boldsymbol{A}_i\wedge\boldsymbol{c})\}$$
$$+a_0 g\,\boldsymbol{u}_i\cdot(\boldsymbol{A}_i\wedge\boldsymbol{c}) - a_7\,\boldsymbol{u}_0\cdot\{\partial_0\boldsymbol{c}+g(\boldsymbol{A}_0\wedge\boldsymbol{c})\}$$
$$+\tfrac{1}{2}g(a_7-a_0)\,\boldsymbol{K}\cdot(\boldsymbol{c}\wedge\boldsymbol{c}) + (a_0-a_7)g\,\boldsymbol{v}_i\cdot(\boldsymbol{E}_i\wedge\boldsymbol{c})$$

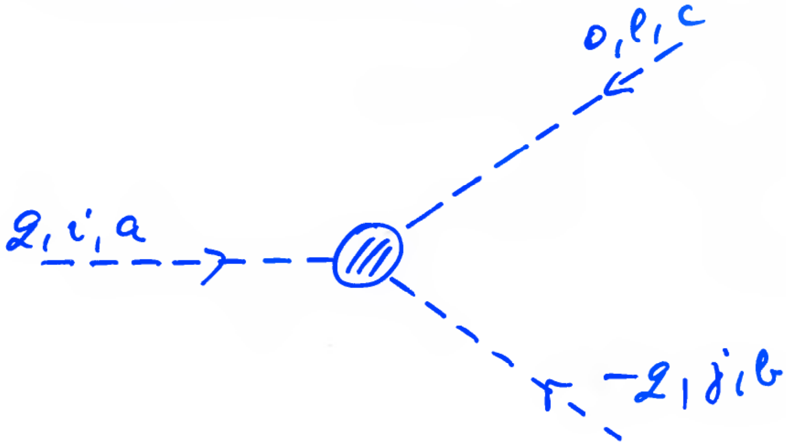
FIX THE CONSTANTS BY COMPARING WITH THE EVALUATED GRAPHS !

<u>A_iA_j</u>

$$\left[\tfrac{1}{3}k_0^2\delta_{ij} + k^2\delta_{ij} - k_ik_j\right]\frac{i\pi^2}{(2\pi)^4}g^2 C_G\,\delta_{ab}\,\Gamma(\tfrac{\epsilon}{2})$$
$$= (4a_1 - 2a_5)(k^2\delta_{ij} - k_ik_j)\,\delta_{ab}$$

$$4a_1 - 2a_5 = \frac{i\pi^2}{(2\pi)^4}g^2 C_G\,\Gamma(\tfrac{\epsilon}{2}) = -c$$

$A_i A_j A_l$ VERTEX



$$\frac{1}{3}\left[2q_l\delta_{ij} - q_i\delta_{lj} - q_j\delta_{li}\right]\Gamma\left(\frac{\epsilon}{2}\right) g^3 C_A f^{abc}\frac{\pi^2}{(2\pi)^4}$$

$$= -4igf^{abc}a_1\left[-2q_l\delta_{ij} + q_i\delta_{lj} + q_j\delta_{li}\right]$$

$$- a_5\, igf^{abc}\left[2q_l\delta_{ij} - q_i\delta_{lj} - q_j\delta_{li}\right]$$

$$+2\left(a_5 + \frac{1}{2}a_0\right) igf^{abc}\left[-2q_l\delta_{ij} + q_i\delta_{lj} + q_j\delta_{li}\right]$$

$$4a_1 - 3a_5 - a_0 = \frac{1}{3}c$$

⋮

IN TOTAL – ALL EVALUATED GRAPHS GIVE CONDITIONS

$$4a_1 - 2a_5 = -c$$
$$4a_1 - 3a_5 - a_0 = \frac{1}{3}c$$
$$a_3 - a_9 = -\frac{4}{6}c$$
$$a_6 - a_5 = \frac{4}{3}c$$
$$a_5 + a_7 = -\frac{4}{3}c$$
$$a_4 - a_8 = \frac{2}{3}c$$
$$a_2 + a_5 + a_8 + a_9 = 0$$

$$a_9 = -a_{10}$$
$$a_{11} = -g\,a_9$$
$$a_0 = a_7 = -a_6$$

THESE EQUATIONS DO NOT FIX THE CONSTANTS UNIQUELY!
WE ARE FREE TO MAKE SOME CHOICES.

THE TERM $(\underset{\sim}{F}_{0i})^2$ IN $\Gamma_1^{(ii)}$ IS NOT PRESENT IN THE ORIGINAL LAGRANGIAN, SO WE CHOOSE

$$a_3 = 0$$

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We can also arrange for the combination

$$-\frac{1}{2}(\mathbf{E}_i)^2 + \mathbf{E}_i \cdot \mathbf{F}_{0i} \tag{52}$$

to appear in $\mathcal{L}_1^{(ii)}$ as it does in $\mathcal{L}_0$. This requires (from (50))

$$\begin{aligned}
a_1 &= -\frac{1}{4}c + \frac{1}{2}a_5 \\
a_2 &= c - 2a_5 \\
a_4 &= -\frac{1}{2}c + a_5 \\
a_6 &= \frac{4}{3}c + a_5 \\
a_7 &= -\frac{4}{3}c - a_5 \\
a_8 &= -\frac{7}{6}c + a_5 \\
a_9 &= \frac{1}{6}c \\
a_0 &= -\frac{4}{3}c - a_5
\end{aligned} \tag{53}$$

and so

$$\mathcal{L}_1^{(ii)} = -4a_1[-\frac{1}{4}(\mathbf{F}_{ij})^2 - \frac{1}{2}(\mathbf{E}_i)^2 + \mathbf{E}_i \cdot \mathbf{F}_{0i}] \tag{54}$$

proportional to the non-ghost part of the original Lagrangian (3).

Equation (54) does not come from the BRST identities, it just emerges from the numerical values of the divergent integrals. It may be a consequence of some hidden Lorentz invariance.

The constants $a_0, a_1, \ldots$ are still not uniquely fixed. There are two particularly simple choices.

(i) Choose $a_0 = 0$ with $a_5 = -\frac{4}{3}c$. Then we find

$$\begin{aligned}
a_1 &= -\frac{11}{12}c \\
a_2 &= \frac{11}{3}c \\
a_4 &= -\frac{11}{6}c \\
a_6 &= a_7 = 0 \\
a_8 &= -\frac{5}{2}c \\
a_9 &= \frac{1}{6}c.
\end{aligned} \tag{55}$$

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(ii) The second choice is $a_1 = 0$ with $a_5 = \frac{1}{2}c$. Then

$$
\begin{aligned}
a_0 &= -\frac{11}{6}c \\
a_2 &= 0 \\
a_4 &= 0 \\
a_6 &= \frac{11}{6}c \\
a_7 &= -\frac{11}{6}c \\
a_8 &= -\frac{2}{3}c \\
a_9 &= \frac{1}{6}c.
\end{aligned}
\tag{56}
$$

Note that a_0 has the expected value for coupling constant renormalization.

The counter-terms in either case are

$$
\begin{aligned}
\mathcal{L}_1 = -\frac{11}{12}c(\mathbf{F}_{ij})^2 - \frac{4}{3}c\mathbf{F}_{ij}\cdot\partial_j\mathbf{A}_i + \frac{4}{3}cg\mathbf{F}_{ij}\cdot(\mathbf{A}_i\wedge\mathbf{A}_j) \\
-\frac{1}{6}c(\mathbf{F}_{0i})^2 + \frac{2}{3}c(\mathbf{E}_i)^2 + \frac{4}{3}c\mathbf{E}_i\cdot\mathbf{F}_{0i} \\
+\frac{4}{3}cg\mathbf{E}_i\cdot(\mathbf{A}_i\wedge\mathbf{A}_0) - \frac{4}{3}c\mathbf{E}_i\cdot\partial_0\mathbf{A}_i + \frac{4}{3}c(\mathbf{u}_i+\partial_i\mathbf{c}^*)\cdot\partial_i\mathbf{c}.
\end{aligned}
\tag{57}
$$

The counter-terms in a_5, a_6, a_7, a_8 and a_9 are involved in a rescaling of the fields. Defining

$$
\begin{aligned}
\mathbf{A}'_i &= (1+a_5)\mathbf{A}_i \\
\mathbf{A}'_0 &= (1+a_6)\mathbf{A}_0 \\
\mathbf{E}'_m &= (1+a_8)\mathbf{E}_m - a_9\mathbf{F}_{0m} \\
\mathbf{u}'_i &= (1-a_5)\mathbf{u}_i \\
\mathbf{u}'_0 &= (1-a_6)\mathbf{u}_0 \\
\mathbf{c}' &= (1-a_7)\mathbf{c} \\
\mathbf{K}' &= (1+a_7)\mathbf{K} \\
g' &= (1+a_0)g \\
\mathbf{c}'^* &= (1-a_5)\mathbf{c}^* \\
\mathbf{v}' &= (1-a_8)\mathbf{v},
\end{aligned}
\tag{58}
$$

we have from (48) that

$$
\mathcal{L}_0 + \mathcal{L}_1 = (1-4a_1)\mathcal{L}_0(g', \mathbf{A}'_i, \mathbf{A}'_0, \mathbf{E}', \mathbf{c}', \mathbf{c}'^*, \mathbf{u}'_i, \mathbf{u}'_0, \mathbf{K}'). \tag{59}
$$

Note that a_6 which determines the renormalization of the Coulomb field A_0^a has the same numerical value as a_0.

We have not calculated the divergences in graphs with four external lines. We assume they will be cancelled by the same counter-terms.

CONCLUSIONS

1. KNOWLEDGE OF COVARIANT GAUGES DOES NOT HELP WITH PHYSICAL GAUGES

2. QUESTION OF PRINCIPLE: DOES AN EXPLICITLY UNITARY GAUGE EXIST AT ALL?

3. BRST IDENTITIES ARE NOT SUFFICIENT TO FIX THE COUNTER-TERMS

4. WE ARE FREE TO MAKE A CHOICE $a_3 = 0$ AND STAY WITHIN THE HAMILTONIAN FORMALISM

5. CHOICE $a_3 \neq 0$ TAKES US OUT OF THE HAMILTONIAN FORMALISM

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