Energy-Exchange-Free Quantum Gates

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Early Interaction-Free Experiments

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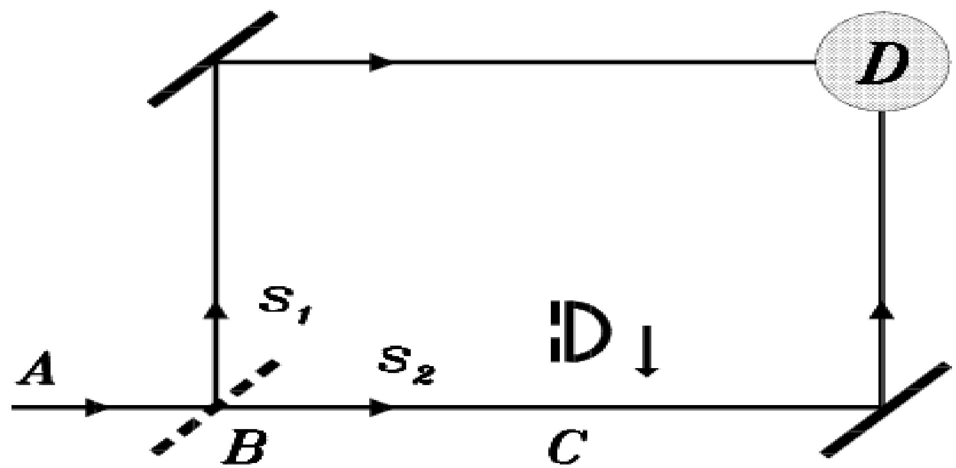

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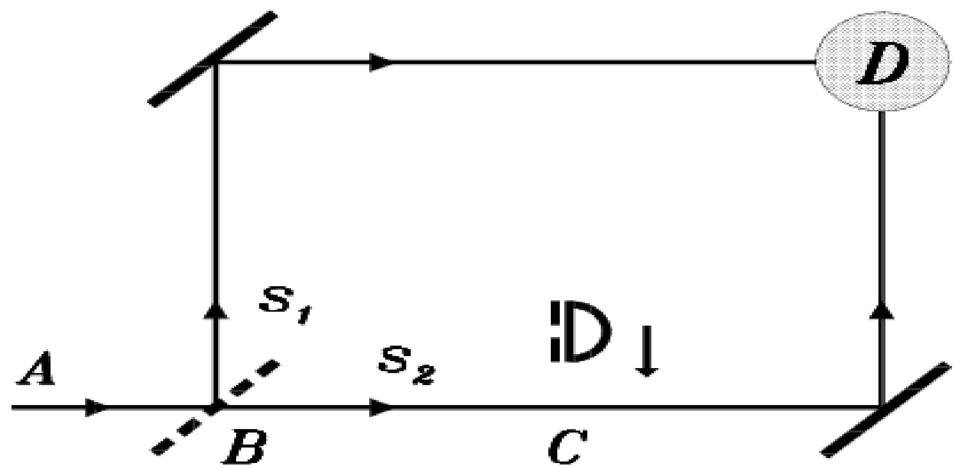

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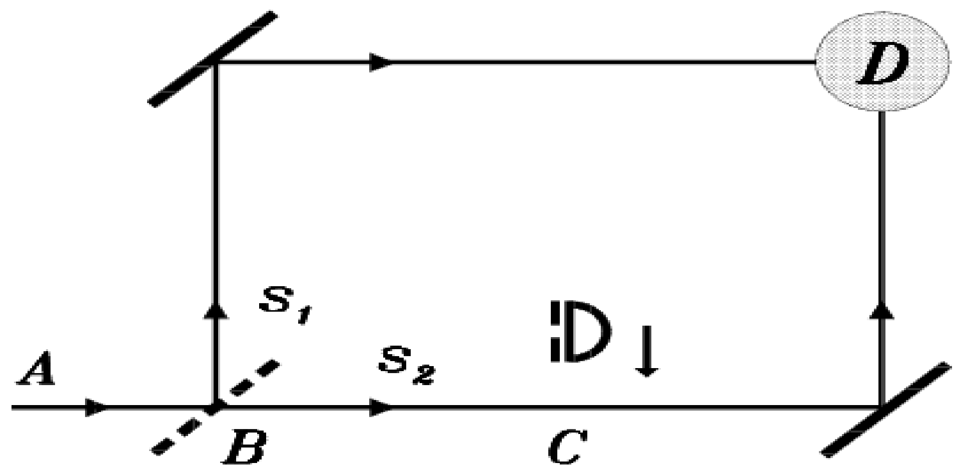

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“Measurements might be useful”

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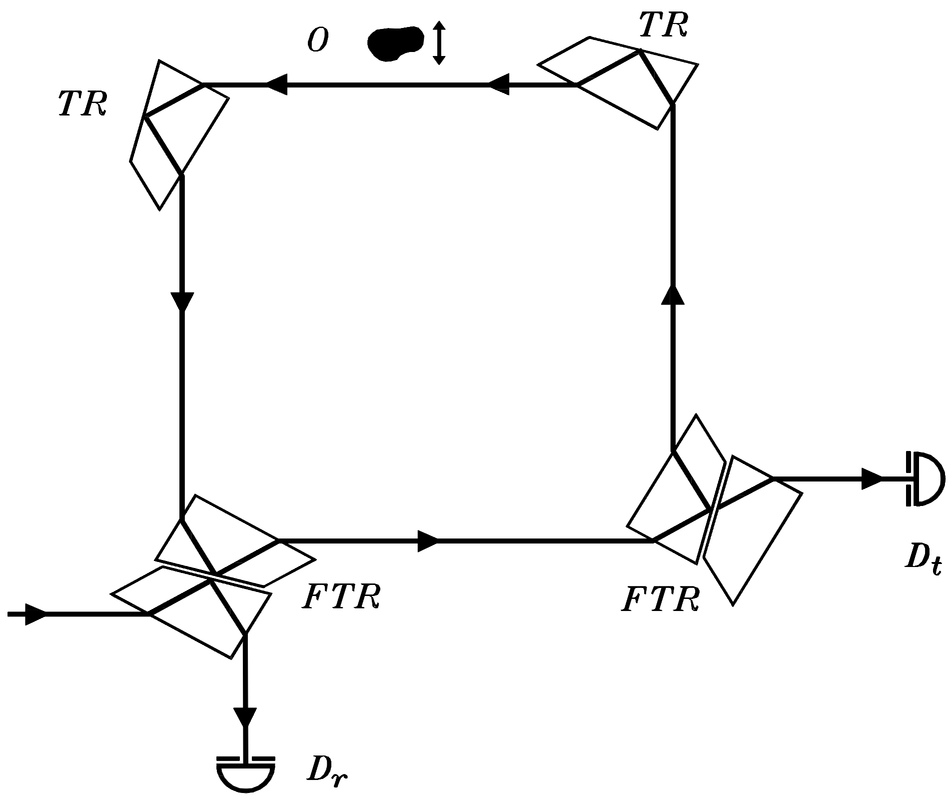

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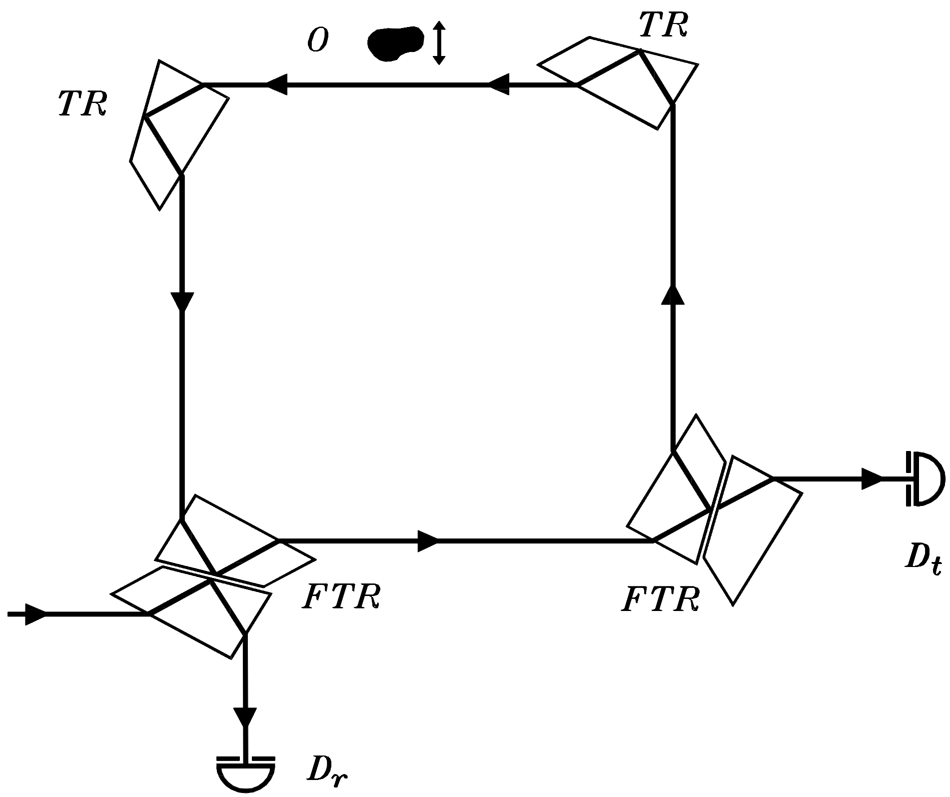

Let us calculate what we get at D_r:

Interference

Reflected portion of the incoming beam:

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"All" round trips: interference (a geometric progression) — the total amplitude (D_r):

$$B = \sum_{i=0}^{\infty} B_i = -A\sqrt{R}\frac{1-e^{i\psi}}{1-R\, e^{i\psi}}$$

Resonator Int.-Free Experiments

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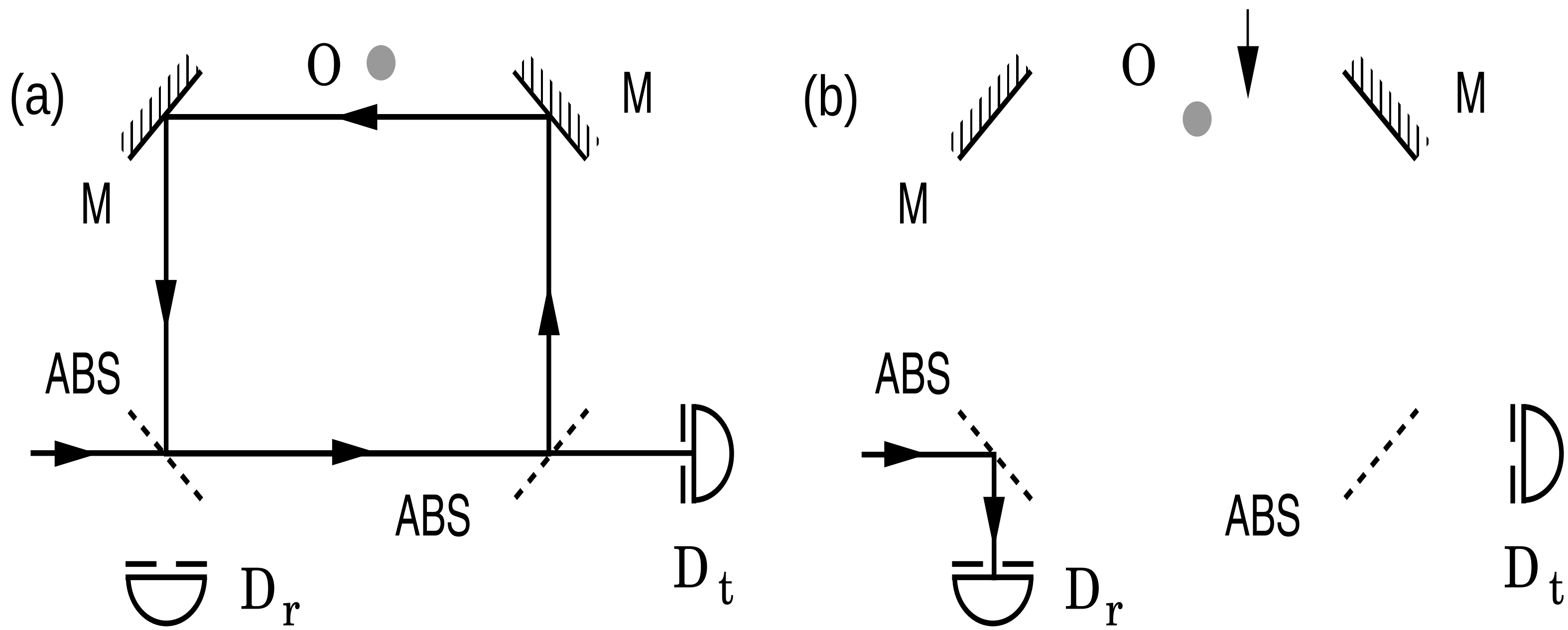


Wave-packet calculations

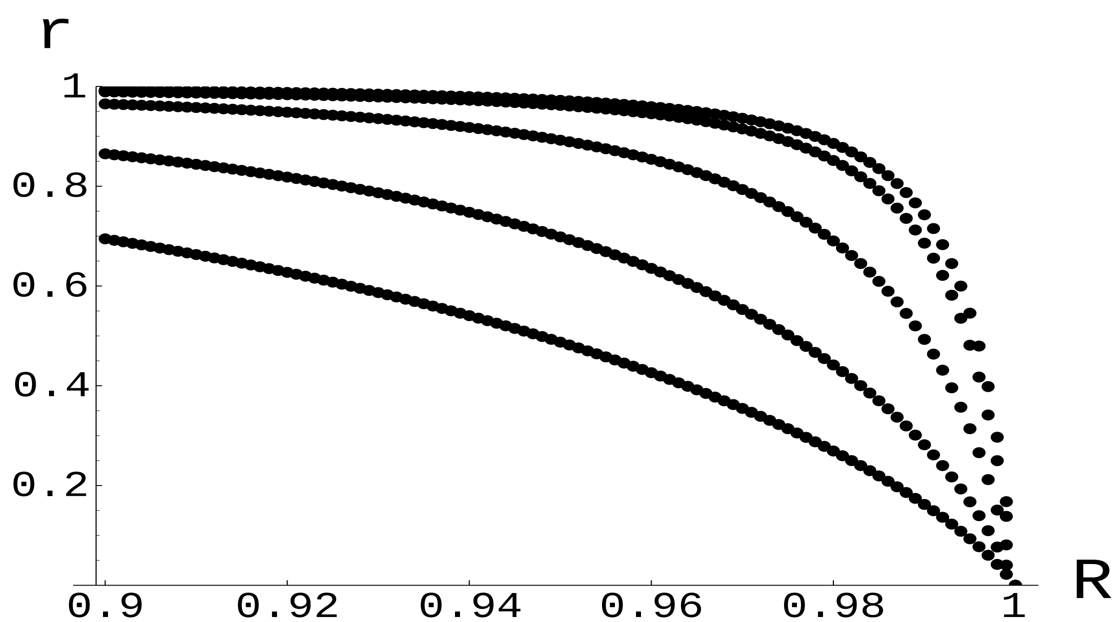
$$r = (1-R)(1-\rho^2 R)\,\Phi, \qquad\qquad t = (1-R)^2\,\Phi$$

where $\rho \leq 1$ is a measure of overall losses and

$$\Phi = \frac{\displaystyle\int_0^\infty \frac{\exp[-\mathcal{T}^2(\omega-\omega_{res})^2/2]d\omega}{1-2\rho R\cos[(\omega-\omega_{res})T]+\rho^2R^2}}{\displaystyle\int_0^\infty \exp[-\mathcal{T}^2(\omega-\omega_{res})^2]d\omega}$$

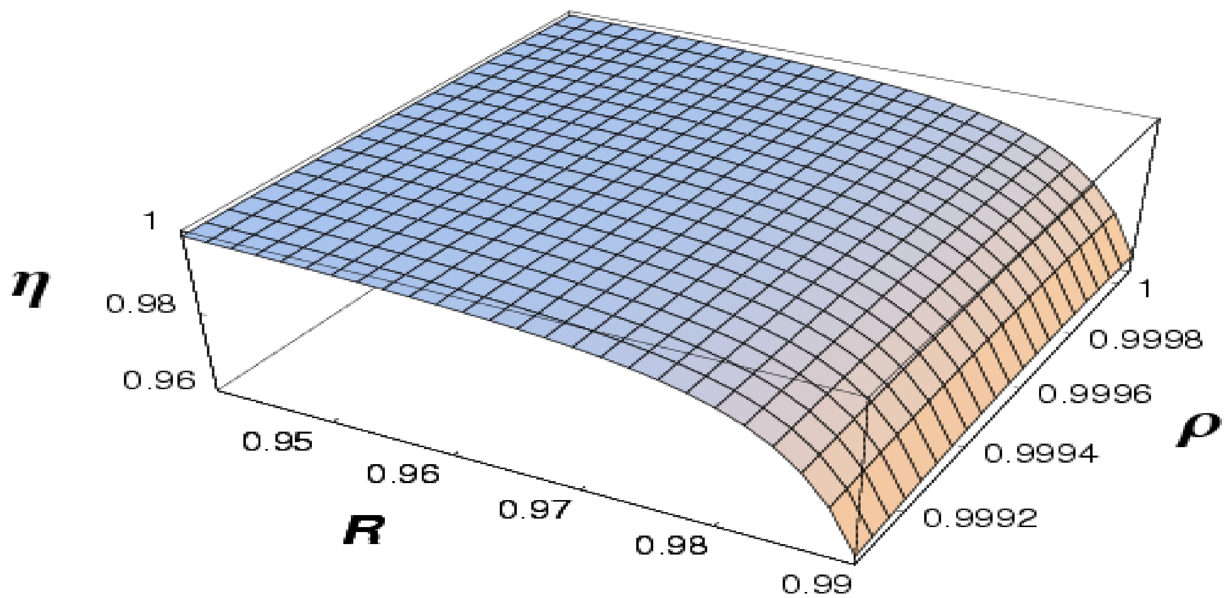
where $\mathcal{T}$ is the coherence time and T is the round-trip time.

Efficiency



r as a function of $\mathcal{T}/T$ for $\rho = 0.99$ and $0.9 \leq R \leq 1$: $\mathcal{T}/T = 500$ (top), 150, 50, 20, and 10 (bottom). The differences in the shapes stem from the amount of losses.

Efficiency



The efficiency of the suppression of the reflection into D_r when there is no object in the resonator; ρ is the measure of losses

Let object be atom

Pavicic, M., Nondestructive Interaction-Free Atom-Photon Controlled-NOT Gate, Physical Review A, 75, 032342-1-8 (2007)

Pavicic, M., Quantum Computation and Quantum Communication: Theory and Experiments, Springer, New York (2005)

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External magnetic field $\mathbf{B}$ splits the levels into magnetic Zeeman sublevels:
$m = -F, -F+1, \ldots, F$.   (See Fig. below.)

Atom vs. photon

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When a photon is emitted, the same selection rules must be observed.

Atom vs. photon (ctnd.)

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we arrive at the Hamiltonian

$$\hat{H} = \frac{\hbar}{2}\begin{bmatrix} 0 & \Omega_1(t) & 0 \\ \Omega_1(t) & 2\Delta & \Omega_2(t) \\ 0 & \Omega_2(t) & 0 \end{bmatrix}$$

Ω_1 and Ω_2 are Rabi frequencies

Excited state drops out

One of the eigenstates of the Hamiltonian is

$$|\Psi^0\rangle = \frac{1}{\sqrt{\Omega_1^2(t) + \Omega_2^2(t)}}(\Omega_2(t)|g_1\rangle - \Omega_1(t)|g_2\rangle)$$

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We can use this to obtain a direct transfer of electrons from $|g_1\rangle$ to $|g_2\rangle$ without either emitting or absorbing photons on the part of atom in the following way—*Stimulated Raman adiabatic passage* (STIRAP).

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Experimentally, let photons be laser beams.

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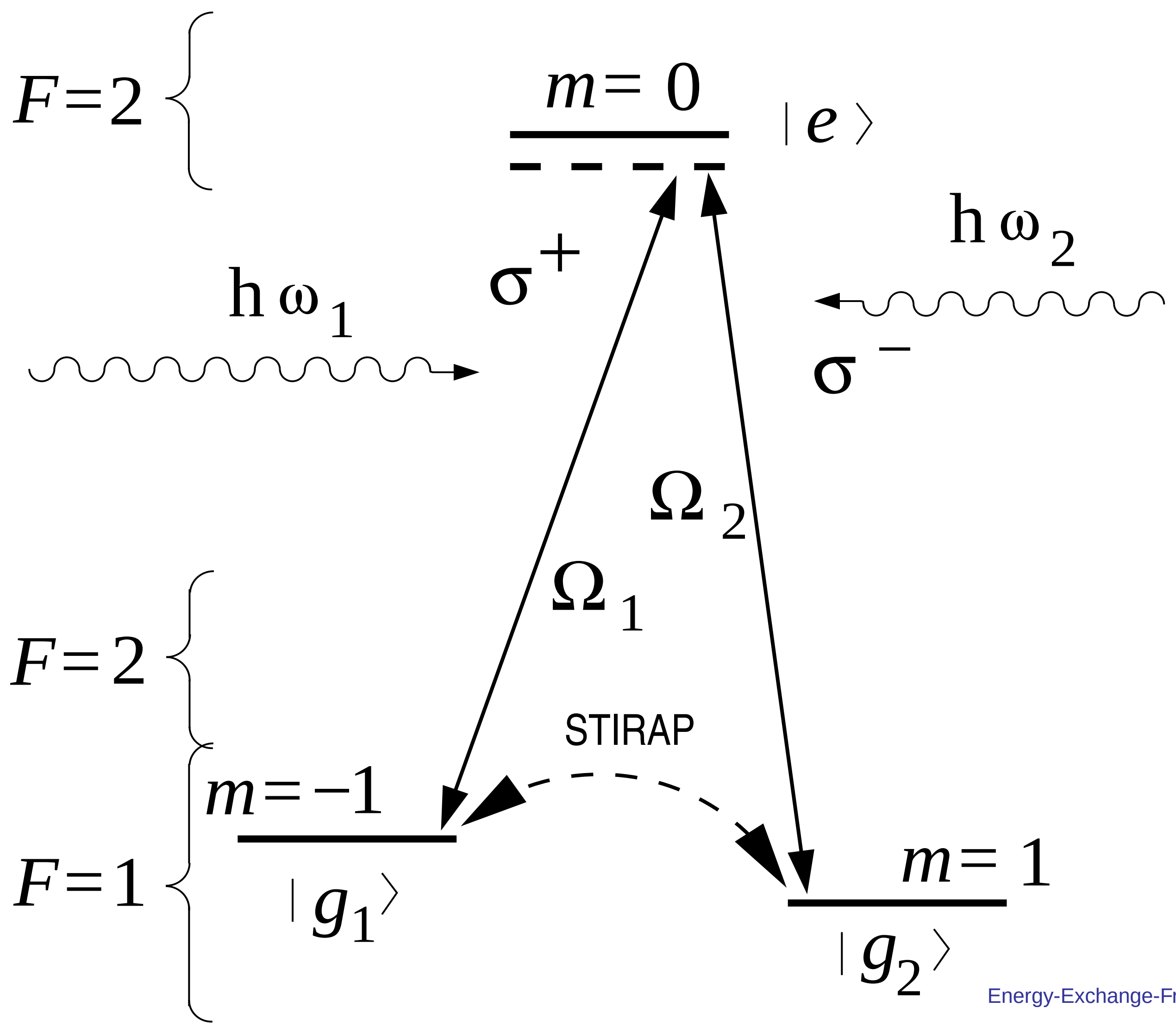
This can be described by

$$\left|\langle g_1|\Psi^0\rangle\right|^2 = 1 \quad \text{for} \quad t \to -\infty$$

$$\left|\langle g_2|\Psi^0\rangle\right|^2 = 1 \quad \text{for} \quad t \to +\infty$$

Adiabatic complete population transfer $|g_1\rangle \to |g_2\rangle$ is STIRAP:

STIRAP $|g_1\rangle \leftrightarrow |g_2\rangle$



Interaction-free "excitation"

A left-hand circularly polarized photon *could* excite an atom from its ground state $|g_1\rangle$ to its excited state $|e\rangle$ and a right-hand circularly polarized photon *could* excite the atom from $|g_2\rangle$ to $|e\rangle$.

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We can induce a change of the atom from $|g_1\rangle$ to $|g_2\rangle$ and back by a STIRAP process, with two additional external laser beams

State notation

We feed our resonator with $+45^\circ$ and -45° linearly polarized photons.

In front of an atom we place a quarter-wave plate (QWP) to turn a 45°-photon into an R-photon and a -45°-photon into an L-photon.

Behind the atom we place a half-wave plate and then another QWP to transform the polarization back into the original linear polarization.

State notation (ctnd.)

We denote the atom states as follows:

$$|0\rangle = |g_1\rangle, \qquad |1\rangle = |g_2\rangle$$

They are control states; atom is control qubit.

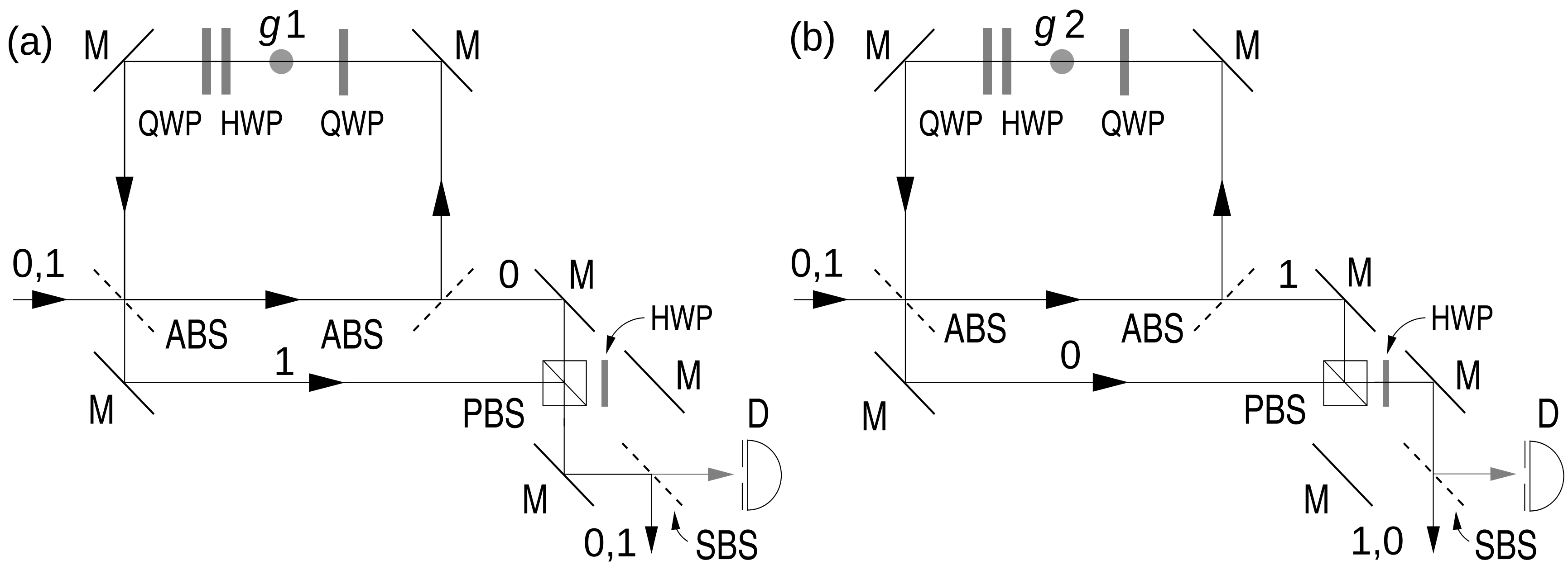
We denote the photon states as follows:

$$|0\rangle = |45^\circ\rangle, \qquad |1\rangle = |-45^\circ\rangle$$

They are target states; photons are target qubits.

For example, $|01\rangle$ means that the atom is in state $|g_1\rangle$ and the photon is polarized along -45°.

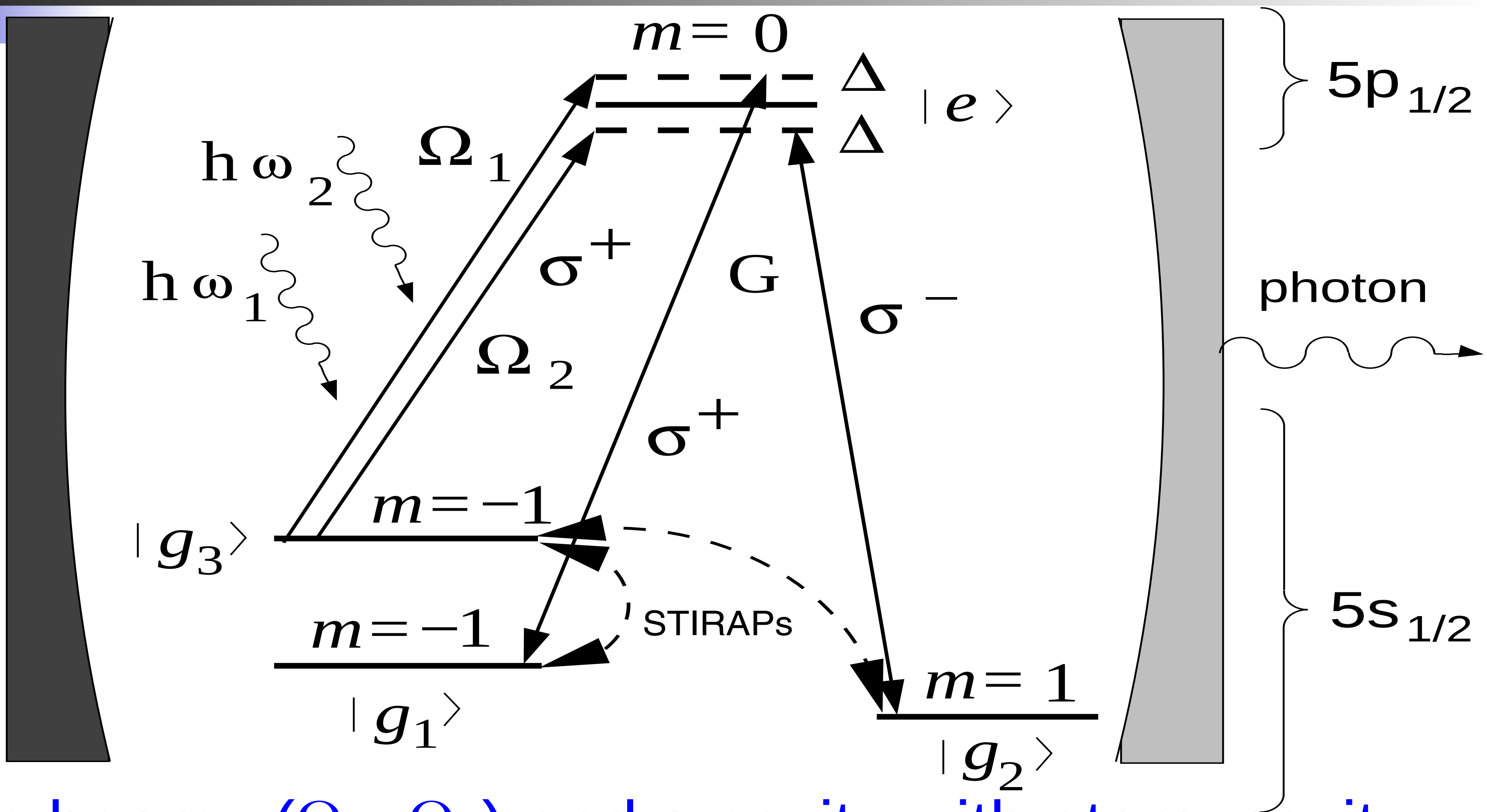
Interaction-free CNOT gate



(a) The atom is in state $|g_1\rangle$ and can absorb $|1\rangle$;
(b) The atom is in state $|g_2\rangle$ and can absorb $|0\rangle$;

$$|00\rangle \rightarrow |00\rangle,\ |01\rangle \rightarrow |01\rangle,\ |10\rangle \rightarrow |11\rangle,\ |11\rangle \rightarrow |10\rangle$$

Superposition STIRAP



Two pump beams (Ω_1, Ω_2) and a cavity with atom–cavity coupling (G) instead of the Stokes laser beams produce superposition $\alpha|g_1\rangle + \beta|g_2\rangle$.

Superposition (ctnd.)

The corresponding Hamiltonian is

$$\hat{H} = \frac{\hbar}{2}\begin{bmatrix} 0 & \Omega_1(t) & 0 \\ \Omega_1(t) & 2\Delta & 2G \\ 0 & 2G & 0 \end{bmatrix}$$

$G = \sqrt{\hbar\omega/(2\varepsilon_0 V_{\text{cavity}})}$ is the *atom-cavity coupling constant* (V_{cavity} is the cavity mode volume).

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The photon which supports the cavity modes and the population of g_1 and g_2 levels eventually leaks from the cavity.

Interaction-Free Preparation of Superposition

$$|\Psi(t)\rangle = \frac{\alpha}{\sqrt{4G^2 + \Omega_1(t)}}(2G|g_3, \emptyset\rangle - \Omega_1(t)|g_1, R\rangle)$$

$$+ \quad \frac{\beta}{\sqrt{4G^2 + \Omega_2(t)}}(2G|g_3, \emptyset\rangle - \Omega_2(t)|g_2, L\rangle).$$

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Superposition manipulations