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Resonance Interaction–Free Measurement

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We show that one can use a single optical cavity as a simplest possible resonator in order to ascertain the presence of an object with an arbitrarily low portion of an incoming laser beam. In terms of individual photons, each photon non–repeatedly tests the object with an arbitrary high probability of detecting its presence without interacting with it.

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1 INTRODUCTION

Ever since Born formulated his probabilistic interpretation of the quantum wave function it was obvious that a *void*, i.e., interaction–free registration of an individual quantum system can provide us with the same amount of information as a direct interaction registration with a full transfer of energy. For example, if in a double slit experiment we put a detector behind one of the slits and take care only for those photons which did not trigger the detector then we are sure that they will arrive at different points at the screen in the long run than they would without such a "control." In the latter case they would form the interference fringes. One can apply the same reasoning to the *Heisenberg microscope* as done by Dicke who has "shown that momentum [transferred to the particle by the scattered photon] is also transferred when the *lack* of a scattered photon is used to discover that the particle is *absent* from the field of view of the microscope" (Dicke, 1981), or to a Mach–Zehnder interferometer as recently done by Elitzur and Vaidman. (Elitzur and Vaidman, 1993; Vaidman, 1994). However, they changed the question which is asked and proposed to use the interferometer for testing the presence of an object, in effect, in the following way. One can adjust a Mach–Zehnder interferometer so that a detector placed in one output port will almost never detect a photon. If it does, then we are certain that an object blocked one path of the interferometer. To dramatize the effect Elitzur and Vaidman assumed that the object is a bomb and showed that it will explode in 50% of the tests with asymmetrical beam splitters. The tests, of course, always have to be carried out with single photons.

When one thinks of possible applications of the afore–mentioned "devices," e.g., of being X–rayed without being exposed to X–rays, then one first wants to improve their low efficiency of at most 50%. Therefore Kwiat *et al.* (1995) proposed a set–up which uses single photons and which is based on "weak repeated tests" carried out by each employed photon. The set–up should reduce the above probability of exploding the bomb to as close to 0% as chosen. The proposal boils down to two identical cavities weakly coupled by a highly reflective beam splitter. Due to the interference the probability for a photon inserted into the first one to be located there approaches 0, and to be found in the second one approaches 1, at a certain time T_N. However, if there is an absorber (a bomb) in the second cavity it is the other way round, i.e., the probabilities reverse. So, if we insert a detector in the first cavity at time T_N we almost never get a click if there is no absorber in the second cavity, and almost always if there is one. On the other hand, the probability of exploding the bomb in the latter case approaches zero. The only drawback of the proposal is that it is apparently hard to carry it out. Apart from cavity losses and the problem of inserting a detector into a cavity at a given time, the introducing of a single photon into the first cavity is by itself a difficult task.

In this paper we propose a very simple and easily feasible interaction–free experiment—with an arbitrary high probability of *detecting the bomb without exploding it*—which is based on the resonance in a single cavity. The proposal assumes a pulse laser beam or a gated continuous–wave laser beam and a properly cut isotropic crystal. So any optical laboratory should be able to carry it out with an efficiency close to 100%.

2 EXPERIMENT

The lay–out of the experiment is shown in Fig. 1. The experiment uses a crystal as an optical cavity for an incoming beam. The cavity behaves as a transmitting resonator when no object is in the way of the round–trip of the beam within the crystal. However, when an object is inserted the beam is almost totally reflected. Let us first consider a plane wave presentation of the experiment. Our aim is to determine the intensity of the beam arriving at detector D_r. The portion of the incoming beam of amplitude A reflected at the entrance surface is described by the amplitude $B_0 = -A\sqrt{R}$, where R is reflectivity and the minus sign is for the reflection at an optically denser medium. The transmitted part will travel around the crystal guided by a reflection at the exit surface and by two total internal reflections. After a full round–trip the following portion of this beam joins the directly reflected portion of the beam: $B_1 = A\sqrt{1-R}\sqrt{R}\sqrt{1-R}e^{i\psi}$. Each subsequent round–trip contributes to a geometric progression whose infinite sum yields the total amplitude of the reflected beam:

$$B = \sum_{i=0}^{\infty} B_i = -A\sqrt{R}\frac{1 - e^{i\psi}}{1 - R\,e^{i\psi}}\,, \tag{1}$$

where $\psi = (\omega - \omega_{res})T$ is the phase added by each round–trip; here ω is the frequency of the incoming beam, T is the round–trip time, and ω_{res} is the resonance frequency corresponding to a wavelength which satisfies $\lambda/2 = L/k$, where L is the round–trip length of the cavity and k is an integer. We see that, in the long run, for any $R < 1$ and $\omega = \omega_{res}$ we get no reflection at all—i.e., no response from D_r (see Fig. 2)—if nothing obstructs the round–trip, and almost a total reflection when the bomb blocks the round–trip and R is close to one. In terms of single photons (that we can obtain by attenuating the intensity of a laser until the chance of having more than one photon at a time becomes negligible) the probability of detector D_r reacting when there is no bomb in the system is zero. A response from D_r means an interaction–free detection of a bomb in the system. The probability of the response is R, the probability of making a bomb explode by our device is $R(1-R)$, and the probability of photon exiting into D_t detector is $(1-R)^2$.

We consider more realistic experimental conditions by looking at two possible sources of individual photons: a continuous wave laser and a pulse laser. (E.g., Nd:YAG laser can work in both regimes with optimal expected properties.)

A continuous wave laser (oscillating on a single transverse mode) has the advantage of an excellent frequency stability (down to 10 kHz in the visible range) and therefore also a very long (up to 300 km) coherence length (Svelto, 1993). This yields almost zero intensity at detector D_r as with plane waves above. The only disadvantage of a continuous laser is that we have to modify the set–up by adding a gate in the following way. The intensity of the beam should be lowered so as to make probable for only one photon to appear within an appropriate time window (1 ms – 1 μs $<$ coherence time) determined by the gate through which the input beam arrives at the crystal and allows the intensity in the cavity to build up. We start each testing by opening the gate, and when either D_r or D_t fires, or the bomb explodes, the testing is over.

It should be emphasized that we get information on the presence or absence of the bomb in any case from a detector click. Hence we need no additional information that a photon has actually arrived at the entrance surface. This is a great advantage over the above–mentioned proposal by Kwiat *et al.* (1995) in which the absence of the bomb is, in fact, inferred from the absence of a detector click. When nothing happens during the exposition time (due either to the absence of a photon or to detector inefficiency), the test has to be repeated. Since the gate selects pulses of given duration, the physical situation is similar as with pulse lasers. A realistic discussion of this case is given below. Before, we would like to mention that our the set–up can enable us to test the inverse frustrated–photon–problem. The frustrated–photon–problem was recently discussed by Fearn, Cook, and Milonni (1995) and can be rephrased as follows. By imposing particular boundary conditions one can suppress a photon emission from an atom or from a crystal (in a down–conversion process). The problem reads: Would a photon appear at the boundary immediately, if we suddenly changed the conditions? The inverse problem would in our case read: Does a photon arriving at the resonator immediately upon opening the gate *"see"* its own "round–trip possibility"? Or: Would a photon which happened to be at the beginning of a coherent rectangular wave–packet behave differently from one which happened to be at the end of the packet? In any case, possible 300 km coherence length does not leave any doubt that a real experiment can be carried out successfully. Time–resolved measurements that would answer the above questions are feasible. From the viewpoint of classical optics it is clear that the probability of a photon to be reflected is initially high and decreases with time (see below).

Pulse lasers have mean frequency dependent on the working conditions of the laser and this is their main disadvantage because each repetition of the experiment takes a considerable time necessary to stabilize the frequency. Their advantage is that they do not require any gates. We describe the input beam coming from such a laser by means of a Gaussian wave packet $A(\omega) = A\exp[-\tau^2(\omega-\omega_{res})^2/2]$, where τ is the coherence time which obviously must be significantly longer than the round–trip time T. We therefore define $a \equiv$

τ/T. The following ratio of intensities of the reflected and the incoming beam which describes the efficiency of the device for free round–trips one obtains straightforwardly:

$$\eta = \frac{\int_0^\infty B(\omega)B^*(\omega)d\omega}{\int_0^\infty A(\omega)A^*(\omega)d\omega} = 1-(1-R)^2\frac{\displaystyle\int_0^\infty \frac{\exp[-\tau^2(\omega-\omega_{res})^2]d\omega}{1-2R\cos[(\omega-\omega_{res})\tau/a]+R^2}}{\displaystyle\int_0^\infty \exp[-\tau^2(\omega-\omega_{res})^2]d\omega}\,, \quad (2)$$

where an infinite number of round–trips is assumed. We see that within the region where the exponential function is significantly different from zero the cosine in the denominator of the integral in the numerator is comparatively constant and can, to a good approximation, be replaced by unity for each $a > 200$. This yields $\eta \to 0$ and at the same time shows that a should be large enough to allow sufficiently many round–trips. We can see that if we express η as a function of a and n (the number of round–trips):

$$\eta = R\Big\{1-\frac{1-R}{1+R}[R^{2n}-1+2\sum_{j=1}^{n}(1+R^{2n-2j+1})R^{j-1}\Phi(j)]\Big\} \quad (3)$$

where $\Phi(j) = 1$ for continuous wave lasers and $\Phi(j) = \exp(-j^2a^{-2}4^{-1})$ for pulse lasers. This expression is obtained by mathematical induction from the geometric progression of the amplitudes and a subsequent integration over wave–packets. The series with $\Phi(j) = \exp(-j^2a^{-2}4^{-1})$ converges because the series with $\Phi(j) = 1$ obviously converges, as follows from Eq. (1). Both series, of course, converge to the values obtained above as one can see from Fig. 2, where three upper curves representing three sums—obtained for $a = 100$, $a = 200$, and $a = 400$, respectively—converge to values (shown as big dots) which one also obtains directly from Eq. (2). The figure shows that a and n are closely related in the sense that the coherence length should always be long enough ($a > 200$) to allow at least 200 round trips. As for the inverse frustrated–photon–problem, for a Gaussian wave packet one can always assume that its tail *"sees"* the "round–trip possibility." Moreover, the dependence of η on n indicates roughly the time dependence of the detection probability at detector D_r, in agreement with the above statement.

In the end, let us give a numerical example by taking $R = 0.98$ and the round–trip length $L = 1$ cm and assuming an attenuation of the incoming laser beam which would give only single photons within the appropriate time window. For both types of lasers we have to use beams with the coherence length of 2 m or longer. For a pulse laser with a coherence length of 5 m ($a = 500$) we have $\eta = 0.005$, i.e., 99.5% probability of *not* having a click at D_r when the bomb is *not* in the system. The probability of *not* having a click at D_r when the bomb *is* in the system is 2%. The probability of exploding the bomb in the latter case is approximately as low as in the ideal case: 1.96%. On the other hand, for a continuous wave laser we obtain $\eta = 1.7 \times 10^{-9}$ for $n = 500$ from Eq. (3) [with $\Phi(j) = 1$; as enabled by a chosen a, e.g., $a = 600$].

3 CONCLUSION

It becomes obvious from our study that one can treat the so–called interaction–free measurement as a basically classical interference effect in order to arrive at the required intensities, i.e., probabilities. However, one must say that, in realistic conditions, there is no strict absence of interaction in the classical picture. There will always be a finite, however extremely small, exchange of energy. The seeming paradox in the sense that one can get information without interaction hints at the basically statistical character of the behavior of the microcosmos, as it is correctly described by quantum theory. In fact, there is a formal correspondence between the classical and the quantum description in the sense that classical quantities, e.g., the amount of energy absorbed by a bomb in any individual case, are identical to quantum mechanical *ensemble averages*. So, when ensembles are considered, there is actually no difference between the quantum and the classical picture. The difference in the individual case, however, is drastic: According to quantum theory, in an overwhelming number of cases no photon will be absorbed so that there is no energy exchange. In some rare cases, however, the full energy quantum $h\nu$ will be transferred to the bomb.

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FIGURES

Fig. 1. Lay–out of the proposed experiment. In the shown free round–trips the intensity of the reflected beam is ideally zero, i.e., detector D_r does not react. However, for reflectivity $R = 0.98$, when the bomb is immersed in the liquid (whose reflectivity is the same as that of the crystal in order to prevent losses of the free round–trips) 98% of the incoming beam reflects into D_r, 0.04% goes into D_t, and 1.96% activates the bomb.

Fig. 2. Realistic values of η for $R = 0.98$. For pulse lasers—3 upper curves represent sums given by Eq. (3) [with $\Phi(j) = \exp(-j^2 a^{-2} 4^{-1})$] as a function of n for $a = 100$, $a = 200$, and $a = 400$; dots represent the corresponding values of η obtained from Eq. (2). For continuous wave lasers—the lowest curve represents the sum given by Eq. (3) [with $\Phi(j) = 1$] as a function of n.

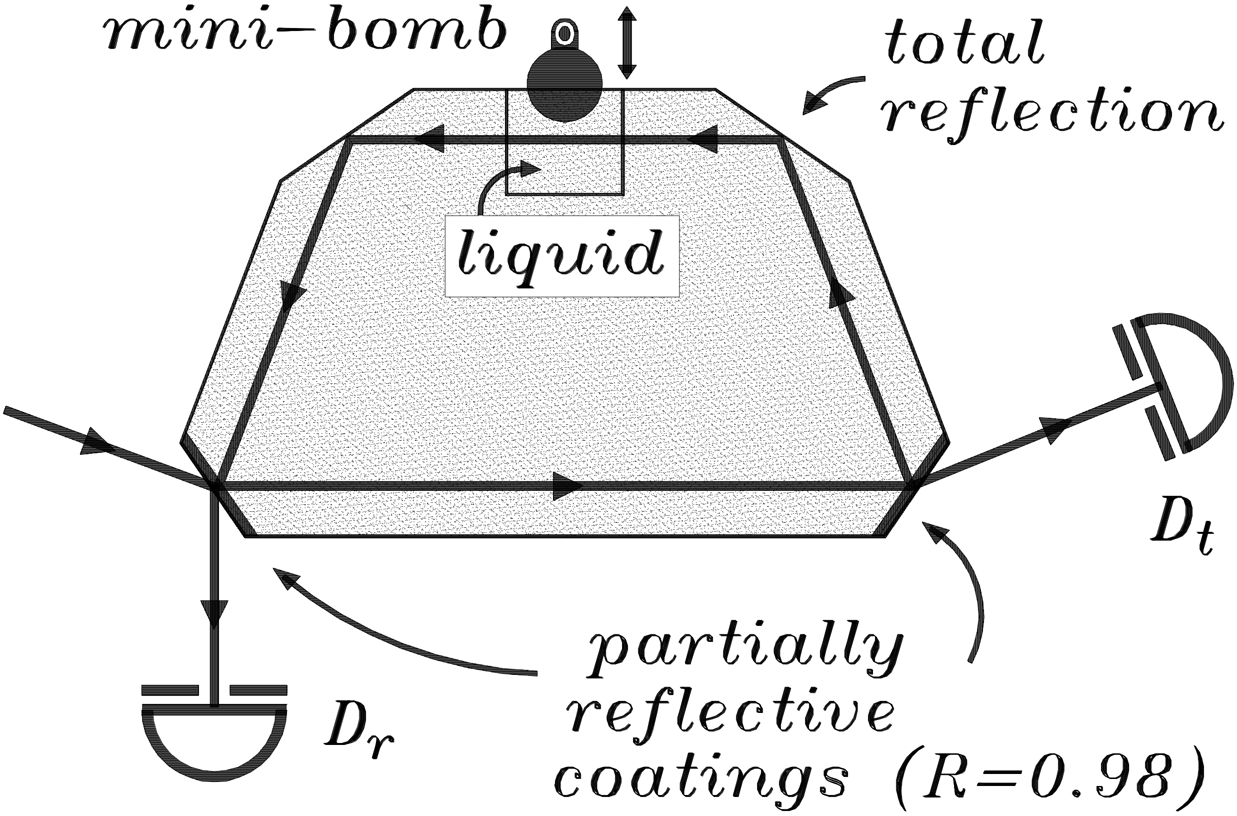
mini–bomb
total
reflection
liquid
D_t
D_r
partially
reflective
coatings (R=0.98)

Fig. 1

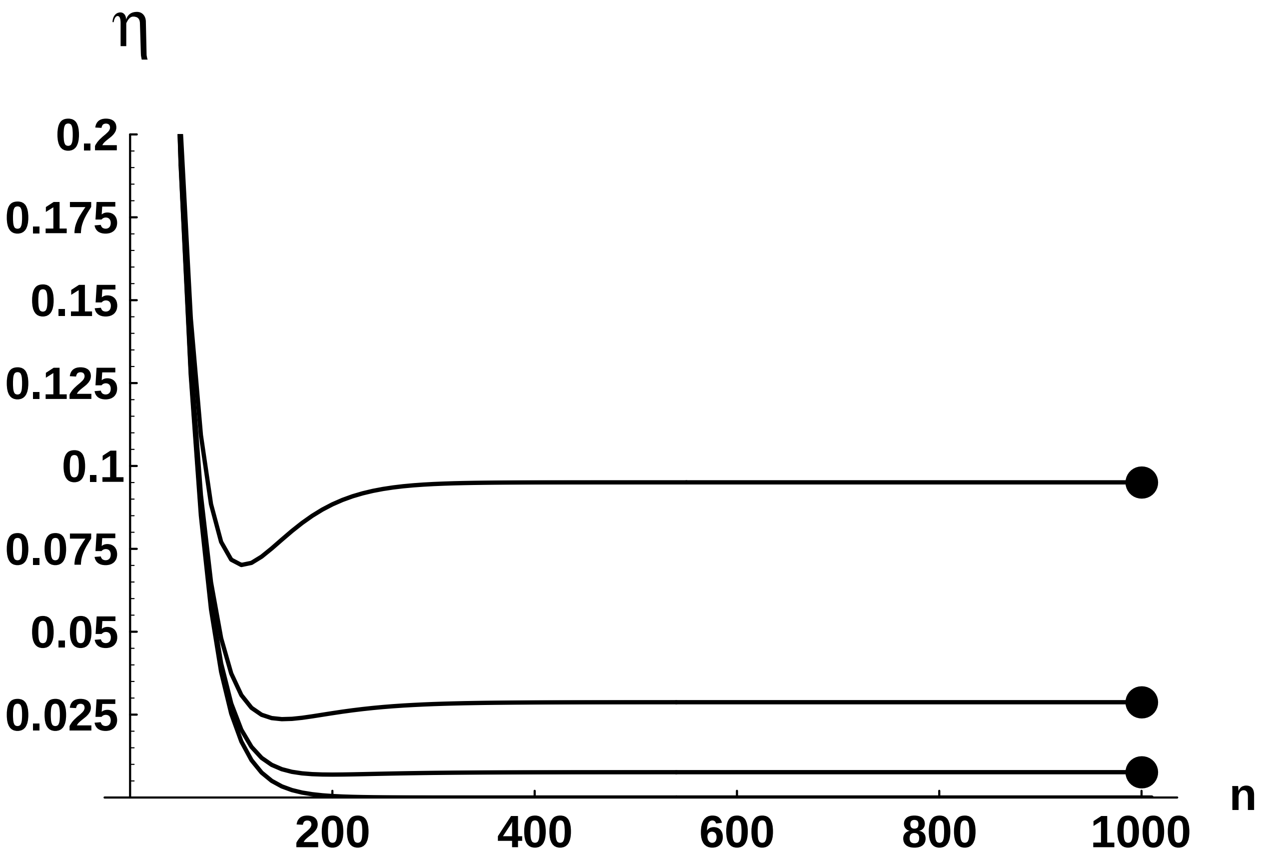

Fig. 2