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A method for reaching detection efficiencies necessary for optical loophole–free Bell experiments

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Abstract

A method for preparing a loophole–free four–photon Bell experiments which requires a detection efficiency of 67% is proposed. It enables realistic detection efficiencies of 75% at a visibility of 85%. Two type–II crystals each down convert one correlated photon pair and we entangle one photon from one pair with one photon from the other pair on a highly transparent beam splitter. The entanglement selects two other conjugate photons into a Bell state. Wide solid angles for the conjugate photons then enable us to collect close to 100% of them. The cases when both photon pairs come from only one of the two crystals are successfully taken into account. Hardy's equalities are discussed.

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I. INTRODUCTION

Two–photon interferometry using down converted photons has recently been extensively used for demonstrating violations of local [1–8] as well as non–local [9] hidden–variables theories. However, in spite of the recent improvement in the efficiencies of single–photon detectors (close to 80%) [10], all experiments carried out to date have had ten or more times fewer coincidence counts than singles counts and this, in effect, meant a detection efficiency under 10%. The reason for this lies in a directional uncertainty of signal with respect to idler photons. On every ten or more detected signal (idler) photons only one of conjugate idler (signal) photons finds its way to the other detector. (Detectors in two–photon experiments must have the same openings.) Therefore, all experiments carried out so far relied only on coincidence counts and herewith on additional assumptions—the *no enhancement assumption* was the most important one—which were considered very plausible. Then Santos devised [11–13] local hidden–variable models which violate not only the low detection loophole but also the *no enhancement assumption.* These models, as well as improvements in techniques resulted in an interest into *loophole–free* experiments. In the past two years several sophisticated proposals appeared which rely on detailed elaborations of all experimental details. [4,7,8,14–16] The last three proposals make use of maximal superpositions and require detection efficiency of at least 83% [17], while the first three references make proposals for nonmaximal superpositions relying on recent results [18–20] which require only 67% detection efficiency. In this paper we dispense with all supplementary assumptions by proposing a feasible selection method of doing a loophole–free Bell experiment which ideally requires only 67% detection efficiency and can reach a realistic detection efficiency as high as 75%. It is shown that this means a feasible conclusive experiment with a realistic visibility of 85%. The method employs solid angles of signal and idler photons (in a type–II down conversion process) which differ five times from each other. This enables a detection of more than 90% of conjugate photons. We also consider a method of combining unpolarized independent photons into spin correlated pairs by means of non–spin observables. The physics of such a preparation of spin states by means of non–spin observables can be paralleled with the polarization correlation between unpolarized photons discovered by Pavičić [6] and formulated for classical light by Paul and Wegmann [21]. The main difference is that in the latter experiments photons cross each other's paths while in the present proposal they do not. In the end we compare Hardy's equalities with the Bell inequalities.

II. THE EXPERIMENT

A schematic representation of the experiment is shown in Fig. 1. An ultra–short laser beam (of frequency ω_p), split by a beam splitter, simultaneously pumps up two type–II crystals. We assume that they are beta barium borate (BBO) crystals. In each type–II crystal the parametric down conversion produces intersecting cones of mutually perpendicularly polarized signal (extraordinary linearly polarized within e–ray plane of the BBO) and idler (ordinary linearly polarized within o–ray plane of the BBO) photons (of frequencies $\omega_e + \omega_o = \omega_p$). [22] Signal and idler photons can be of the same frequencies ($\omega_e = \omega_o = \omega_p/2$) in which case the cones are tangent to each other. When we tilt the crystal so as to increase the angle between the pumping beam and the crystal axis of the BBO (increasing ω_o and decreasing ω_e slightly) the cones start to intersect each other [see inlet in Fig. 1]. Looking only at polarization, we see the photons at the intersections of the cones as entangled, because one cannot know which cone which photon comes from. [23] We then entangle one photon from one pair with one photon from the other pair by an interference of the fourth order at a beam splitter. Each successful entanglement (coincidence firing of detectors D1' and D2') selects (opens the computer gates for their counts) the other two conjugate photons into a Bell state.

In a real experiment one first has to make photons at the cone intersections of each BBO indistinguishable, which means one has to compensate for the finite bandwidths and different group velocities inside the crystal, i.e., for transversal and longitudinal (e–photon pulls ahead) walkoff effects. Half–wave plates (exchanging retardation of e and o photons) and quartz plates (being positive uniaxial crystal—BBO is negative) do the job. [4,24–26] Then, by rotating the crystal, one can entangle the photons in a (non)maximal singlet–like or triplet–like state. [26] In our proposal we assume both, crystals and plates, prepared so as to produce maximal singlet–like states. (It is interesting that starting with two maximal triplet–like states we arrive at the same final expressions for the probabilities; Cf. Ref. [5].) We also assume that the intensity of the laser pumping beam is reduced so that the probability of having more than two down converted singlets in chosen solid angles within the pumping time is negligible. [4] We stress here that we choose a subpicosecond laser since without such an ultra–short pumping one would not be able to collect valid coincidence counts of D1' and D2' simply because there are no detectors which could react in a time short enough [5] to confirm the intensity interference between two *independent* down converted photons (from two crystals) whose coherence time lies in a subpicosecond region. Two successive pumping can take place within several nanoseconds as determined by the lowest available detector resolution (recovering) time. For the feasibility of the experiment it is crucial that the probability of both photon pairs coming from only one of the two crystals can be made sufficiently small in comparison with the probability of photon pairs coming from both crystals by using more and more asymmetric beam splitter which at the same time lowers the required detection efficiency more and more towards 67%. We show this at the end of this section.

As we mentioned in Sec. I, the main detection efficiency problem in two photon interferometry is that signal and idler photons have to be in equal solid angles and that therefore less than 10% of conjugate photons reach a detector. The present set–up enables us to use different solid angles for selecting photons (those which interfere at the beam splitter) and their conjugate photons (whose counts are passed by the gates). For the purpose, one has to evaluate the angular width of the conjugate photons once we know central directions (cone intersections) of both photons. One can show that the angular width of a conjugate photon depends on the frequencies of the pump, signal, and idler photons, on the band widths, on the pump, signal, and idler group refraction indexes, and on the directions of signal and idler photons with respect to the pumping beam, but for all combinations of these terms, the ratio of 1:5 between the solid angle of the photons we detect by detectors D1' or D2' and the solid angle centered around central direction of the conjugate photons (*ph* in Fig. 1) assures that over 90% of conjugate photons will be found in the latter solid angle [27] and that the probability of detecting "third party" photons will be negligible. Let us now dwell on deriving our probabilities.

We can have three input states, depending on whether the two pairs come from different crystals or both of them from only one of the crystals. The probabilities of the pairs being emitted in any of these three possible ways are equal. The singlets coming from the crystals are mutually independent and we therefore formally describe them by tensor products. The former one, coming from different crystals is given by

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|1_x\rangle_1|1_y\rangle_{1'} \; - \; |1_y\rangle_1|1_x\rangle_{1'}) \otimes \frac{1}{\sqrt{2}}(|1_x\rangle_2|1_y\rangle_{2'} \; - \; |1_y\rangle_2|1_x\rangle_{2'})\,, \tag{1}$$

where x corresponds to the e–ray planes of the BBO's and y to the o–ray planes. The latter ones, coming from the same crystals are given by

$$|\Psi_{20}\rangle = \frac{1}{2}(|1_x\rangle_1|1_y\rangle_{1'} - |1_y\rangle_1|1_x\rangle_{1'})^2|0\rangle_{2'}\,, \tag{2}$$

$$|\Psi_{02}\rangle = \frac{1}{2}(|1_x\rangle_2|1_y\rangle_{2'} - |1_y\rangle_2|1_x\rangle_{2'})^2|0\rangle_{1'}\,. \tag{3}$$

To obtain the four photon coincidence probabilities we cannot superpose these three input states upon one another because that would violate the principle of indistinguishability. To see this let us for the moment assume that our detection efficiency be ideal (100%) and that the polarizers P1, P1$^\perp$, P2, and P2$^\perp$ be removed. Then, with the help of the responses of the detectors D1 and D2 we could tell $|\Psi\rangle$ (both D1 and D2 would fire), from $|\Psi_{02}\rangle$ (only D1 would fire) or from $|\Psi_{20}\rangle$ (only D2 would fire). If detections in reality had been ideal we would have used only $|\Psi\rangle$. Since they are not, we have to take $|\Psi_{02}\rangle$ and $|\Psi_{20}\rangle$ into account as well, but helpfully it turns out that the Bell inequality containing their corresponding counts (in addition to $|\Psi\rangle$ counts) is still violated. Therefore, we start with $|\Psi\rangle$, i.e., with two pairs coming out from different crystals, and discuss $|\Psi_{02}\rangle$ and $|\Psi_{20}\rangle$ later on. A multi–mode representation of the input state we give later on.

The outgoing electric–field operators describing photons—we call them *selector photons*—which pass through beam splitter BS, through polarizers P1' and P2' (oriented at angles $\theta_{1'}$ and $\theta_{2'}$, respectively), and are detected by detectors D1' and D2', read (see Fig. 1)

$$\begin{aligned}\hat{E}_{1'} &= (\hat{a}_{1'x}t_x\cos\theta_{1'} + \hat{a}_{1'y}t_y\sin\theta_{1'})\, e^{i\mathbf{k}_{1'}\cdot\mathbf{r}_{1'}-i\omega_1(t-t_{1'}-\tau_{1'})} \\ &\quad + i\,(\hat{a}_{2'x}r_x\cos\theta_{1'} + \hat{a}_{2'y}r_y\sin\theta_{1'})\, e^{i\tilde{\mathbf{k}}_{2'}\cdot\mathbf{r}_{1'}-i\omega_2(t-t_{2'}-\tau_{1'})}\,, \end{aligned} \tag{4}$$

$$\begin{aligned}\hat{E}_{2'} &= (\hat{a}_{2'x}t_x\cos\theta_{2'} + \hat{a}_{2'y}t_y\sin\theta_{2'})\, e^{i\mathbf{k}_{2'}\cdot\mathbf{r}_{2'}-i\omega_2(t-t_{2'}-\tau_{2'})} \\ &\quad + i\,(\hat{a}_{1'x}r_x\cos\theta_{2'} + \hat{a}_{1'y}r_y\sin\theta_{2'})\, e^{i\tilde{\mathbf{k}}_{1'}\cdot\mathbf{r}_{2'}-i\omega_1(t-t_{1'}-\tau_{2'})}\,, \end{aligned} \tag{5}$$

where t_x^2, t_y^2 are transmittances, r_x^2, r_y^2 are reflectances, t_j' is time delay after which photon j' reaches BS, $\tau_{j'}$ is time delay between BS and Dj', $\omega_{j'}$ is the frequency of photon j', $\mathbf{k}_{j'}$ is the wave vector of photon j', and $\tilde{\mathbf{k}}_{j'}$ is the wave vector corresponding to $\mathbf{k}_{j'}$ after reflection at BS. The annihilation operators act as follows: $\hat{a}_{jx}|1_x\rangle_{j'} = |0_x\rangle_{j'}$, $\hat{a}_{jx}|0_x\rangle_{j'} = 0$, $j' = 1,2$.

Operators describing photons—we call them *Bell photons*—which pass through polarizers P1 and P2 (oriented at angles θ_1 and θ_2, respectively) and are detected by detectors D1 and D2 read

$$\hat{E}_1 = (\hat{a}_{1x}\cos\theta_1 + \hat{a}_{1y}\sin\theta_1)e^{-i\omega_1 t_1}\,, \tag{6}$$

$$\hat{E}_2 = (\hat{a}_{2x}\cos\theta_2 + \hat{a}_{2y}\sin\theta_2)e^{-i\omega_2 t_2}\,. \tag{7}$$

The probability of detecting all four photons by detectors D1, D2, D1', and D2' is thus

$$\begin{aligned}P(\theta_{1'},\theta_{2'},\theta_1,\theta_2) &= \eta^2\langle\Psi|\hat{E}_{2'}^\dagger\hat{E}_{1'}^\dagger\hat{E}_2^\dagger\hat{E}_1^\dagger\hat{E}_1\hat{E}_2\hat{E}_{1'}\hat{E}_{2'}|\Psi\rangle \\ &= \frac{\eta^2}{4}(A^2 + B^2 - 2AB\cos\phi)\,, \end{aligned} \tag{8}$$

where η is detection efficiency; $A = Q(t)_{11'}Q(t)_{22'}$ and $B = Q(r)_{12'}Q(r)_{21'}$; here $Q(q)_{ij} = q_x\sin\theta_i\cos\theta_j - q_y\cos\theta_i\sin\theta_j$; $\phi = (\boldsymbol{k}_1 - \tilde{\boldsymbol{k}}_2)\cdot\boldsymbol{r}_1 + (\boldsymbol{k}_2 - \tilde{\boldsymbol{k}}_1)\cdot\boldsymbol{r}_2 + (\omega_1 - \omega_2)(\tau_{1'} - \tau_{2'})$.

To obtain a realistic estimation of the above result we start with the multi–mode input states

$$|1\rangle_{1'}|1\rangle_{2'} = \int\int d\omega_1' d\omega_1' \psi_1(\omega_1)\psi_2(\omega_2)|\omega_1\rangle_{1'}|\omega_2\rangle_{2'}\,, \tag{9}$$

which we introduce into Eq. (1); $\psi_i(\omega_i)$ $(i = 1, 2)$ are both peaked at $\omega = \frac{1}{2}\omega_p$: $\omega_i' = \omega - \omega_i$ $(i = 1, 2)$. We can keep the singlet state description as given by Eq. (1) because it has recently been proved by Keller and Rubin [28]—as we briefly present below—that a subpicosecond pulse would give practically the same output as the continuous pumping beams provided a group velocity condition is matched. In doing so we rely on the experimental and theoretical results obtained by Kwiat *et al.* [26]. We then make a Fourier decomposition of the electric–field operators [Eqs. (4) and (5)] and obtain the mean value for $P(\theta_{1'}, \theta_{2'}, \theta_1, \theta_2)$. By integrating the latter probability over $\tau_{1'}$, $\tau_{2'}$, ω_1', and ω_2' and using

$$\frac{1}{T}\int_{-T/2}^{T/2}\cos(\omega\tau + a)\,d\tau = \frac{\sin(\omega T/2)}{\omega T/2}\cos a\,, \qquad \int_{-\infty}^{\infty}\frac{\sin a\omega}{\omega}\sin b\omega\,d\omega = 0\,, \tag{10}$$

$$\int_{-\infty}^{\infty}\frac{\sin a\omega}{\omega}\cos b\omega\,d\omega = \begin{cases} \pi & \text{for } b < a \\ \pi/2 & \text{for } b = a \\ 0 & \text{for } b > a \end{cases} \tag{11}$$

we obtain

$$P(\theta_{1'}, \theta_{2'}, \theta_1, \theta_2) = \frac{\eta^2}{4}(A^2 + B^2 - 2ABv_e\cos\Phi)\,, \tag{12}$$

where

$$v_e = \frac{\int_{-T/2}^{T/2} f_1(\tau - \tau_1) f_2(\tau - \tau_2)\,d\tau}{\int_{-T/2}^{T/2} f_1^2(\tau - \tau_1)\,d\tau + \int_{-T/2}^{T/2} f_2^2(\tau - \tau_2)\,d\tau}\,, \tag{13}$$

where $f_i(\tau) = \int_{-\infty}^{\infty}\psi_i(\omega)\cos\omega\tau\,d\omega$, $(i = 0, 1)$, where T is the detection time, and where $\Phi = 2\pi(z_2 - z_1)/L$; here L is the spacing of the interference fringes, z_j are the coordinates of detectors Dj' along $\boldsymbol{k}_1 - \tilde{\boldsymbol{k}}_2$ and $\boldsymbol{k}_2 - \tilde{\boldsymbol{k}}_1$ (see Fig. 1 in reference [7]); we dropped primes from $\tau_{1'}$ and $\tau_{2'}$ for simplicity. We see that Φ can be changed by moving the detectors transversally to the incident beams.

By numerical calculation one can easily show that v_e is not susceptible to the variation of detection time T provided $|\tau_1 - \tau_2| < |\omega_1 - \omega_2|^{-1}$ (for $|\tau_1 - \tau_2| \ll |\omega_1 - \omega_2|$ even when $T \gg |\omega_1 - \omega_2|^{-1}$). For $|\tau_1 - \tau_2| \ll |\omega_1 - \omega_2|^{-1}$ we have $v_e \to 1$, i.e., the sharpest fringes, and for $|\tau_1 - \tau_2| \gg |\omega_1 - \omega_2|^{-1}$ we have $v_e \to 0$ and the fringes disappear. With the experimentally reachable frequency passband $\Delta\omega$ of the order of magnitude of THz within a single parametric down conversion with a continuous pumping beam reaching the condition $|\tau_1 - \tau_2| \ll |\Delta\omega|^{-1}$ is not a problem because the time interval between the idler and signal photons is of the order of femtoseconds. In our case, when dealing with two simultaneous down conversions we have to resort to an ultra–short pumping beam to satisfy the condition. Pulse pumping beam shorter than 1 ps would in general "make it possible to distinguish pairs of photons born at sufficiently different depths inside the crystal with a consequent decrease in two photon interference" as recently shown by Keller and Rubin. [28] This happens when the center of momentum of signal and idler photons and the center of pump pulse does not leave the crystal simultaneously. When they do, i.e., when, by choosing appropriate material conditions and pump frequency for a down conversion within a type–II crystal, we make "the inverse of the pump group velocity equals the mean of the idler and signal inverse group velocity" [28] and therewith we make the photons indistinguishable again. Singlets appearing from such "compensated" crystal therefore keep their description given in Kwiat *et al.* [26] and that is what we rely on in the afore–given calculation.

Another realistic detail of the experiment is that the pinholes of detectors D1' and D2' are not points but have a certain width Δz. Therefore, in order to obtain a realistic probability we integrate Eq. (8) over z_1 and z_2 over Δz to obtain

$$\begin{aligned} P(\theta_{1'}, \theta_{2'}, \theta_1, \theta_2) &= \frac{\eta^2}{4} \int\limits_{z_1-\frac{\Delta z}{2}}^{z_1+\frac{\Delta z}{2}} \int\limits_{z_2-\frac{\Delta z}{2}}^{z_2+\frac{\Delta z}{2}} [A^2 + B^2 - 2ABv_e \cos\frac{2\pi(z_2 - z_1)}{L}] dz_1 dz_2 \\ &= \frac{\eta^2}{4}(A^2 + B^2 - v2AB\cos\Phi)\,, \end{aligned} \qquad (14)$$

where $v = v_e[\sin(\pi\Delta z/L)/(\pi\Delta z/L)]^2$ is the *visibility* of the coincidence counting.

An analysis of Eq. (14) shows that triggering of D1' and D2' by selector photons means that their conjugate Bell photons appear entangled in spite of the fact that they stem from two independent sources (two crystals) and that they do not interact in any way (e.g., they do not cross each other's paths). In general, Bell photons are only partially entangled as in the case of classical intensity interferometry. For special cases, however, one can achieve full quantum nonmaximal entanglement, i.e., one can obtain probability zero for certain orientations of polarizers P1 and P2. In order to obtain such an entangled state, which would at the same time enable a violation of the Bell inequality with only 67% detection efficiency, it is necessary to use an asymmetrical beam splitter, to orient polarizers P1' and P2', e.g., along $\theta_{1'} = 90^\circ$ and $\theta_{2'} = 0^\circ$, and to put detectors D1' and D2' in a symmetric position with respect to BS and with respect to the photons paths from the middle of the crystals so as to obtain $\Phi = 0$. Eq. (8) then projects out the following nonmaximal singlet–like probability:

$$\begin{aligned} P(\theta_1,\theta_2) &= \eta^2 s(\cos^2\theta_1\sin^2\theta_2 - 2v\rho\sin\theta_1\cos\theta_1\sin\theta_2\cos\theta_2\cos\Phi + \rho^2\sin^2\theta_1\cos^2\theta_2)\,, \\ &\equiv \eta^2 p(\theta_1,\theta_2) \end{aligned} \tag{15}$$

where we assumed the near normal incidence at BS so as to have $r_x^2 = r_y^2 = R$ and $t_x^2 = t_y^2 = T = 1 - R$, where we used $s \equiv T^2/(R^2+T^2)$, $\rho \equiv R/T$, and where we multiplied Eq. (14) by 4 for other three possible coincidence detections [i.e., for $(\theta_{1'},\theta_{2'}^\perp)$, $(\theta_{1'}^\perp,\theta_{2'})$, and $(\theta_{1'}^\perp,\theta_{2'}^\perp)$ which we do not take into account because only $(\theta_{1'},\theta_{2'})$–triggering opens the gates] and by $(R^2+T^2)^{-1}$ for photons emerging from the same side of BS (which also do not open the gates).

The singles–probability of detecting a photon by D1 is

$$P(\theta_1) = \eta s(\cos^2\theta_1 + \rho^2\sin^2\theta_1) \equiv \eta p(\theta_1)\,. \tag{16}$$

Analogously, the singles–probability of detecting a photon by D2 is

$$P(\theta_2) = \eta s(\sin^2\theta_2 + \rho^2\cos^2\theta_2) \equiv \eta p(\theta_2)\,. \tag{17}$$

Introducing the above obtained probabilities into the Clauser–Horne [29] form of the Bell inequality

$$B_{CH} \equiv P(\theta_1,\theta_2) - P(\theta_1,\theta_2') + P(\theta_1',\theta_2') + P(\theta_1',\theta_2) - P(\theta_1') - P(\theta_2) \le 0\,, \tag{18}$$

we obtain the following minimal efficiency for its violation

$$\eta = \frac{p(\theta_1') + p(\theta_2)}{p(\theta_1,\theta_2) - p(\theta_1,\theta_2') + p(\theta_1',\theta_2') + p(\theta_1',\theta_2)}\,. \tag{19}$$

We stress here that the probabilities in Eq. (18) are proper probabilities—not the ratios of coincidence counts as in the experiments carried out so far. For example, $P(\theta_2) = \eta p(\theta_2)$ is a total number of counts detected by detector D2 with the polarizer P2 oriented along θ_2—it is *not* either $\eta^2 p(\infty, a_2)$ or $\eta^2 p(\infty, a_2)/p(\infty,\infty)$.

This efficiency is a function of visibility v and by looking at Eqs. (15), (16), and (17) we see that for each particular v a different set of angles should minimize it. A computer optimization of angles—presented in Fig. 2—shows that the lower the reflectivity is, the lower is the minimal detection efficiency. Also, we see a rather unexpected property that a low visibility does not have a significant impact on the violation of the Bell inequality. For example, with 70% visibility and 0.2 reflectivity of the beam splitter we obtain a violation of Eqs. (18) with a lower detection efficiency than with 100% visibility and 0.5 ($\rho = 1$) reflectivity.

In Ref. [7] we have shown that one can select fully quantum entangled Bell photons even without polarizers P1’ and P2’; i.e., whenever unpolarized selector photons trigger detectors D1’ and D2’ they open the gates for maximally entangled singlet–like state of Bell photons. Now, it is of interest to find out whether we can use such non–polarization preparation to prepare full non–maximal polarization–entangled states. To this aim, we calculate

$$P_\infty(\theta_1,\theta_2) = P(\theta_{1'},\theta_{2'},\theta_1,\theta_2) + P(\theta_{1'}^\perp,\theta_{2'},\theta_1,\theta_2) + P(\theta_{1'},\theta_{2'}^\perp,\theta_1,\theta_2) + P(\theta_{1'}^\perp,\theta_{2'}^\perp,\theta_1,\theta_2) \tag{20}$$

where we obtain the last three probabilities by analogy with the first one [Eq. (8)]; e.g., in order to obtain $P(\theta_{1'}^\perp,\theta_{2'},\theta_1,\theta_2)$, we introduce $E_{2'}^\perp$ instead of $E_{2'}$ into Eq. (8), where we get $E_{2'}^\perp$ from Eq. (5) upon substituting $-\sin\theta_{2'}$ for $\cos\theta_{2'}$ and $\cos\theta_{2'}$ for $\sin\theta_{2'}$. Eq. (20) yields

$$P_\infty(\theta_1,\theta_2) = \eta^2 \frac{(1-2r_x^2t_x^2)\sin^2\theta_1\sin^2\theta_2 + (1-2r_y^2t_y^2)\cos^2\theta_1\cos^2\theta_2 + S - 2vW\cos\Phi}{2(1-2r_x^2t_x^2 - 2r_y^2t_y^2 + t_x^2t_y^2 + r_x^2r_y^2)} \tag{21}$$

where

$$S = (t_x^2 t_y^2 + r_x^2 r_y^2)(\sin^2\theta_1\cos^2\theta_2 + \cos^2\theta_1\sin^2\theta_2)\,, \tag{22}$$

$$W = (t_x r_x \sin\theta_1\sin\theta_2 + t_y r_y \cos\theta_1\cos\theta_2)^2\,. \tag{23}$$

A computer calculation shows that this probability can violate the Bell inequalities only for a detection efficiency 83% or higher. It also shows that the probability cannot be used for obtaining Hardy's equalities [30]. On the other hand, an analysis of Eq. (21) shows that the only way to obtain a non–partial, i.e., full quantum (non–classical) entanglement is to use a symmetric beam splitter ($r_x^2 = r_y^2 = 1/2$) and a symmetric position of detectors D1' and D2' with respect to BS and with respect to the photons paths from the middle of the crystals so as to obtain $\Phi = 0$. Under these conditions Eq. (21) yields $P_\infty(\theta_1,\theta_2) = \frac{1}{2}\sin^2(\theta_1-\theta_2)$, i.e., a maximal singlet–like state. Thus, by means of non–spin preparation we can prepare only "symmetric" (maximal) non–classical spin correlated states.

In the end we show that other down conversions which may occur in the crystals and enter our statistics do not significantly influence the obtained probabilities. The probability of both photon pairs coming from only one of the two crystals and the probability of their coming from both crystals are of course equal, but for ρ close to 0 the influence of photon pairs coming from only one of the two crystals can be made small enough for a conclusive Bell experiment. Let us see this in detail.

Choosing $\theta_{1'} = 90^\circ$, $\theta_{2'} = 0^\circ$, $\Phi = 0$, and rewriting the electric–field operators [Eqs. (4) and (5)] accordingly, we obtain the following probabilities of detecting the "intruder" counts (corresponding to both photons coming from the same crystal and being both detected by D1 and D2, respectively) while collecting the singles–probabilities [Eqs. (16) and (17)]

$$P_{20}(90^\circ,0^\circ,\theta_1,-) = P_{20}(\theta_1) = \eta s\rho(1+v)\sin^2(2\theta_1)\,, \tag{24}$$

$$P_{02}(90^\circ,0^\circ,-,\theta_2) = P_{02}(\theta_2) = \eta s\rho(1+v)\sin^2(2\theta_2)\,. \tag{25}$$

We could dispense with these counts only if detectors D1 and D2 could tell one photon from two. It is therefore important to see whether the Bell inequality Eqs. (18) is still violated when we have them in our statistics. In order to include them into the Bell inequality we should add them to the singles–probabilities given by Eqs. (16) and (17). By comparing $P_{20}(\theta_1)$ and $P_{02}(\theta_2)$ with $P(\theta_1)$ and $P(\theta_2)$, respectively, we see that for the angles close to $\frac{\pi}{2}$ and π, for which the asymmetrical states violate the Bell inequality, the following inequalities hold: $P_{20}(\theta_1) \ll P(\theta_1)$ and $P_{02}(\theta_2) \ll P(\theta_1)$. For example, for $\rho = 0.1$, $\eta = 0.75$, $v = 0.9$, $\theta_1 = 104°$, $\theta_1' = 89°$, $\theta_2 = 181°$, and $\theta_2' = 161°$ we obtain the violation $B_{CH} > 0$. For the same parameters we also obtain $B_{CH} - P_{20} - P_{02} > 0$. However, this reduces the value of B_{CH} for which the Bell inequality is violated by 2/3. On the other hand, we have to use birefringent polarizers P1 and P2 to be able to discard counts which fire both D1 and D1$^\perp$ when collecting data for the singles–probability $P(\theta_1)$ by D1 and those which fire both D2 and D2$^\perp$ when collecting data for the singles–probability $P(\theta_2)$ by D2. Therefore, in doing a real experiment we should better split unwanted two–photon wave packets across additional polarized beam splitters [3] or, even better, by applying *photon chopping* [31] when collecting counts for singles–probabilities. We stress here that by this method we do not affect the conclusiveness of the Bell experiment but only pick out *valid* Bell pairs from all those ones already selected by the D1'–D2' coincidence gates. That is, we do not discard any counts corresponding to firing of D1 and/or D2 by photons coming from different crystals.

III. CONCLUSION

Our elaboration shows that the proposed loophole–free Bell experiment which selects two out of four photons into nonmaximal singlet–like states can be carried out with the present technique. The proposal makes use of an asymmetrical preparation of two *input* photon singlets generated by two nonlinear type–II crystals. The asymmetry consists in the fact that we first let one photon from the first singlet interfere in the fourth order with one photon from the other singlet at a highly transparent beam splitter. Coincidental detections of the photons interfering at the beam splitter (we call them *selector photons*) open gates for a selection of the remaining two conjugate photons, one from each singlet, into a new correlated state: nonmaximal *Bell singlet*. In other words, since no coincidence detection between signal and idler photons of the input singlets is needed we can use several times wider solid angles for the *Bell photons* than for the *selector photons*. With five times wider solid angle (determined by pinholes *ph* in Fig. 1) we collect practically all Bell companions of those *selector photons* which trigger detectors D1' and D2' in coincidence. In this way we eliminate the main cause of the low detection in all two–photon experiments so far: loosing of the conjugate photons (in most cases they "miss" the detector opening). An apparent draw–back to our set–up is that the probabilities of two pairs coming from both crystals and of both pairs coming from only one of the crystals are equal. However, the above calculations show that for reflectivity 0.1, realizable visibility of 85–90%, and achievable detector efficiency of 75% [4,10] the Bell inequality is violated even when the counts corresponding to photons emerging from only one of the two crystals are included into the statistics by which the inequality is fed.

We should mention here that although 67% efficiency result for Hardy's equalities has been obtained recently as well [32,33] their low marginal violations (of maximal value 0.09 as opposed to 0.41 for the Bell inequalities) make a loophole–free "Hardy experiment" more demanding. However, it would be worth trying to collect data for it within the proposed set–up because of its conceptual clarity and because our results add to the physics of the Hardy experiment. In particular, an analysis of Eq. (21) shows that Hardy's equalities, as opposed to the Bell inequalities, cannot be formulated for a system which is not fully non–classical. Thus, our set–up reveals nonlocality of quantum systems as a property of selection of their subsystems and Hardy's equalities as a test (ideally, some detectors should always react and some never) of whether the system is fully quantum or not. It may turn out that quantum nonlocality is only operationally defined in the same way in which quantum phase might turn out to be only operationally defined. [34] On the other hand, since Hardy's equalities reach their maximal violation for $R = 0.32$ and not for $R = 0.5$, it might turn out the unwanted effect of both photon pairs coming from the same crystal on the marginal probabilities can be compensated sufficiently well to make the experiment feasible.

In the end we mention that the set–up may find its application in quantum cryptography and quantum computation for its property to deliver Einstein–Podolsky–Rosen singlets [35] whose "coherence ... [is] retained over considerable distances and for long times" [36]; actually, our *Bell singlets* stay coherent for ever, i.e., until we make use of them and collapse them.

Note. Some preliminary results to these paper have been presented within an invited talk at the *Adriatico Research Conference on Quantum Interferometry II*, held in Trieste, Italy, 4–8 March 1996. [37]

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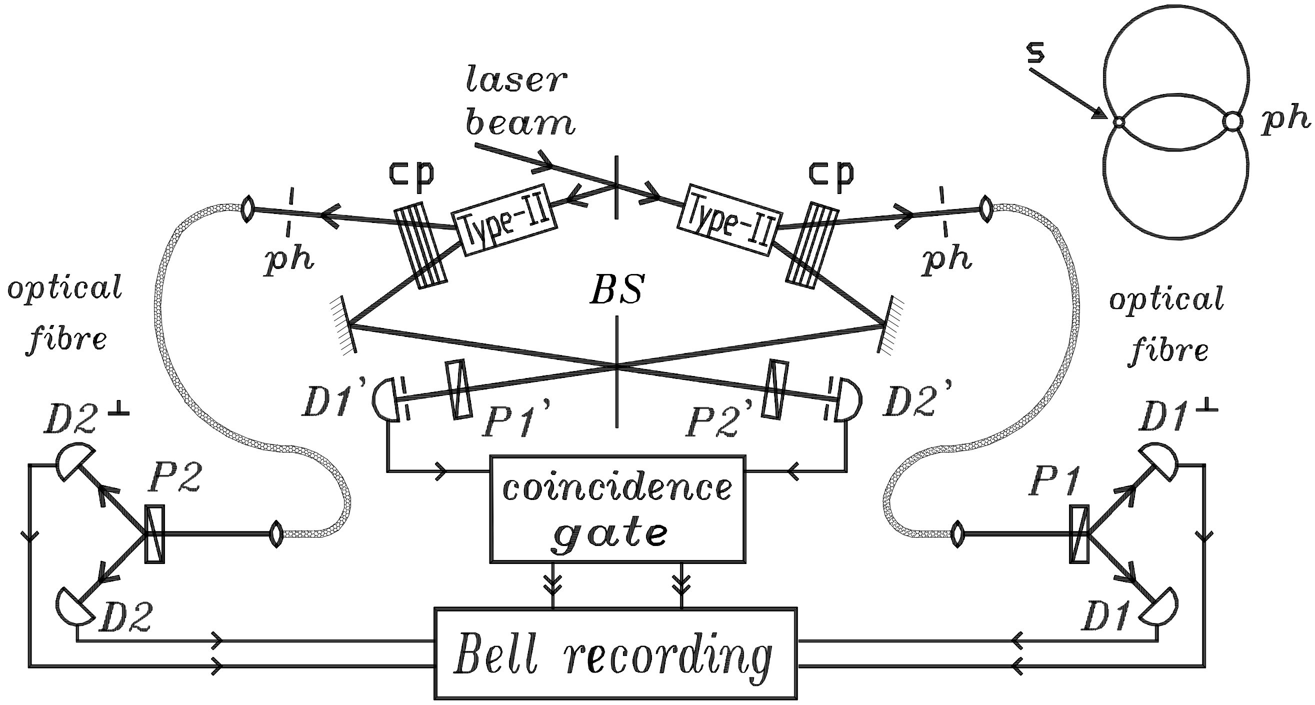
FIGURES

FIG. 1. Lay–out of the proposed experiment. Beam splitter BS and detectors D1' and D2', and their counters [which open the gates for Bell photon singlets] serve as an event–ready [29] selection device. *cp*'s (compensation plates) represent half–wave plates and quartz plates. Inlet in the upper right corner shows intersecting cones of down converted photons emerging from type–II crystals. s represents solid angles (along the intersection of the cones) determined by the pinholes of detectors D1' or D2'.

FIG. 2. Minimal detection efficiencies η necessary for a violation of the Bell–Clauser–Horne inequality as functions of visibility v and of $\rho = R/(1-R)$, where R is the reflectivity of the beam splitter.



laser
beam
cp
Type-II
Type-II
cp
s
ph
ph
ph
BS
optical
fibre
optical
fibre
D1'
P1'
P2'
D2'
D2⊥
D1⊥
P2
P1
coincidence
gate
D2
D1
Bell recording

Fig. 1

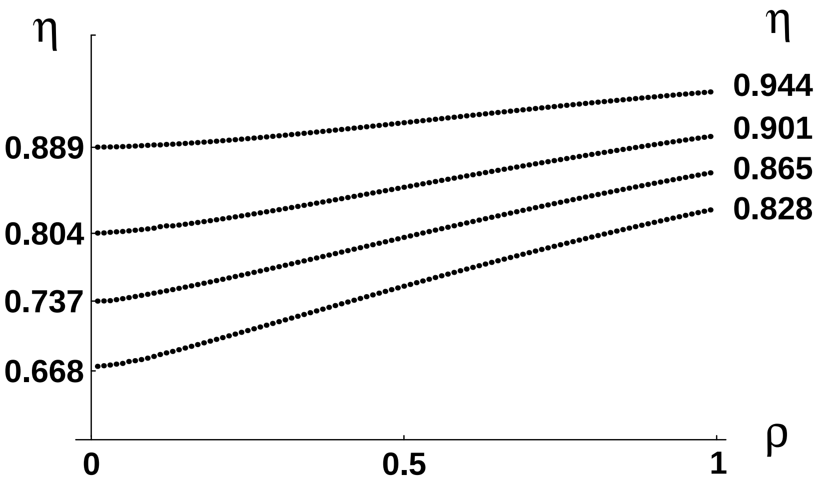


Fig. 2