PRESELECTED QUANTUM OPTICAL CORRELATIONS

Mladen Pavičić

Max–Planck–AG Nichtklassische Strahlung,

Humboldt University of Berlin, D–12484 Berlin, Germany,

and Department of Mathematics, GF, University of Zagreb,

Kačićeva 26, POB 217, HR–41001 Zagreb, Croatia

E-mail: mpavicic@faust.irb.hr. Web page: http://m3k.grad.hr/pavicic.

Two previously discovered effects in the intensity interference: a spin–correlation between formerly unpolarized photons and a spin entanglement *at–a–distance* between photons that nowhere interacted, have been used for a proposal of a new *preselection experiment*. The experiment puts together two photons from two independent singlets and makes them interfere at an asymmetrical beam splitter. A coincidental detection of two photons emerging from different sides of the beam splitter *preselect* their pair–companion photons (which nowhere cross each other's path) into a nonmaximal singlet state. The quantum–mechanical nonlocality thus proves to be essentially a property of selection. This enables a loophole–free experimental disproof of local hidden–variable theories requiring detection efficiency as low as 67% and an exclusion of all nonlocal hidden–variable theories that rely on some kind of a physical entanglement by means of a common medium.

S. Jeffers, S. Roy, J.–P. Vigier, and G. Hunter (eds.), *The Present Status of the Quantum Theory of Light* (Kluwer Academic Publishers, Holland, 1996), pp. 311–322.

I. INTRODUCTION

In the past ten years the fourth order interference of photons has been given a considerable attention. [1–18] It proved to be a powerful tool for testing both local [2–12] and nonlocal [13,14] hidden variable theories and it also revealed several new features of quantum phenomena. The interference turned out to be a genuine *quantum* phenomenon which does not have a proper classical counterpart as opposed to the interference of the 2nd order. [3] In particular, its visibility reaches 100% (in contradistinction to the classical 50%) and does not depend on the relative intensity of the incoming beams. [4]

The interference served us to recognize a beam splitter as a source of singlet photon states and as a device for spin correlated interferometry even when incoming photons are unpolarized. [8] We elaborate on this properties and introduce general beam splitter spin formalism for different input states in Sec. II. In Sec. III we introduce the Bell inequalities in such a way to enable us to close the remaining loopholes in the Bell theorem. These properties and elaborations open the way for the preselection experiment presented in Sec. IV. The experiment is the first realization of the spin entanglement of independent photons which do not have any common history.

II. INTERFEROMETRY WITH ASYMMETRICAL BEAM SPLITTERS

In the next subsection we use the second quantization formalism in order to describe the fourth order interference of two incoming photons at a beam splitter without specifying their input state. In the subsequent two subsections we consider three kinds of input states and their outputs.

A. Formalism of a beam splitter and its output

In this subsection the only assumption we make about the input of a beam splitter is that it consists of two input beams falling on the beam splitter. The beams contain altogether two photons but one of them might be empty while the other might contain two photons (see Fig. 1). Let the state of incoming photons be $|\Psi\rangle$. Two particular special states will be specified in the next two subsections. The actions of beam–splitter BS, polarizers P1,P2 and detection D1,D2,D1$^\perp$,D2$^\perp$ are taken into account by the outgoing electric field operators which in the second quantization formalism one obtains in the following way.

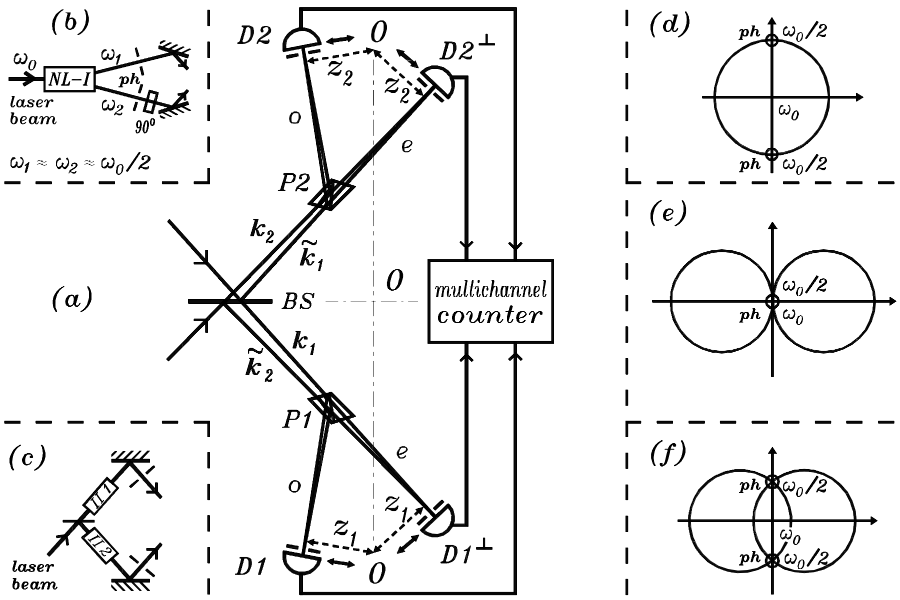

FIG. 1. (a) Beam splitter fed by a down–converted photon pair coming out from a non–linear crystal of type I (b) (Sec. II B) and by two superposed down–converted photon pairs (c) (Sec. II D). Photon wave vectors form cones in space (d-f). For type I the same frequency cones coincide and with pinholes *ph* as in (d) we ideally always get either both photons together and parallelly polarized or none. Type II of the first kind (e) emit perpendicularly polarized photons; *ph* has them together with the pump beam (ω_0) which must be eliminated by a filter. Type II of the second kind (f), which serves as a source for the main experiment, emits a superposition of perpendicularly polarized photons through the shown *ph* because one cannot know which cones they belong to.

The polarization is described by means of two orthogonal scalar field components. The scalar component of the stationary electric field operator in the plane wave interpretations will read: $\hat{E}_j(\mathbf{r}_j, t_j) = \hat{a}(\omega)e^{i\mathbf{k}_j\cdot\mathbf{r}_j - i\omega t_j}$, where ω_j is the frequency of incoming photons, $\mathbf{k}$ is the wave vector ($k = \omega/c$), $\mathbf{r}$ is the radius vector pointing at detector D, t is the time at which a photon is detected, $j = 1, 2$ refer to a particular outcoming photon in question, and the annihilation operators describe actions of detectors so as to act on the states as follows: $\hat{a}_{1x}|1_x\rangle_1 = |0_x\rangle_1$, $\hat{a}^\dagger_{1x}|0_x\rangle_1 = |1_x\rangle_1$, $\hat{a}_{1x}|0_x\rangle_1 = 0$, etc.

One can easily obtain [10] the electric outgoing field operators describing photons which pass through beam splitter BS and polarizers P1,P2 and are detected by detectors D1,D2:

$$\hat{E}_1 = (\hat{a}_{1x} t_x \cos\theta_1 + \hat{a}_{1y} t_y \sin\theta_1)\, e^{i\mathbf{k}_1\cdot\mathbf{r}_1 - i\omega(t-t_1-\tau_1)}$$
$$+ i\,(\hat{a}_{2x} r_x \cos\theta_1 + \hat{a}_{2y} r_y \sin\theta_1)\, e^{i\tilde{\mathbf{k}}_2\cdot\mathbf{r}_1 - i\omega(t-t_2-\tau_1)}\,, \tag{1}$$

$$\hat{E}_2 = (\hat{a}_{2x} t_x \cos\theta_2 + \hat{a}_{2y} t_y \sin\theta_2)\, e^{i\mathbf{k}_2\cdot\mathbf{r}_2 - i\omega(t-t_2-\tau_2)}$$
$$+ i\,(\hat{a}_{1x} r_x \cos\theta_2 + \hat{a}_{1y} r_y \sin\theta_2)\, e^{i\tilde{\mathbf{k}}_1\cdot\mathbf{r}_2 - i\omega(t-t_1-\tau_2)}\,, \tag{2}$$

where t_j is time delay after which photon $j = 1, 2$ reaches BS and τ_j is time delay between BS and Dj.

The electric outgoing field operator describing photons which pass through beam splitter BS and polarizers P1,P2 and are detected by detector D2$^\perp$ reads

$$\hat{E}_2^\perp = (-\hat{a}_{2x} t_x \sin\theta_2 + \hat{a}_{2y} t_y \cos\theta_2)\, e^{i\mathbf{k}_2\cdot\mathbf{r}_2 - i\omega(t-t_2-\tau_2^\perp)}$$
$$+ i\,(-\hat{a}_{1x} r_x \sin\theta_2 + \hat{a}_{1y} r_y \cos\theta_2)\, e^{i\tilde{\mathbf{k}}_1\cdot\mathbf{r}_2 - i\omega(t-t_1-\tau_2^\perp)}\,, \tag{3}$$

where $\tau_2^\perp$ is time delay between BS and D2$^\perp$.

The probability of joint detection of two ordinary photons (see Fig. 1: denoted by o) coming out from opposite sides of the beam splitter by detectors D1 and D2 is

$$P(\theta_1, \theta_2, \phi) = \langle\Psi|\hat{E}_2^\dagger \hat{E}_1^\dagger \hat{E}_1 \hat{E}_2|\Psi\rangle, \tag{4}$$

where $\phi = (\tilde{\mathbf{k}}_2 - \mathbf{k}_1)\cdot\mathbf{r}_1 + (\tilde{\mathbf{k}}_1 - \mathbf{k}_2)\cdot\mathbf{r}_2 = 2\pi(z_2 - z_1)/L$, where L is the spacing of the interference fringes. ϕ can be changed by moving the detectors transversely to the incident beams as indicated by '$\leftrightarrow$' in Fig. 1. θ_1 and θ_2 are the angles along which polarizers P1 and P2 are oriented with respect to a chosen fixed direction.

The probability of joint detection of two ordinary photons coming out from the same side of the beam splitter, e.g., the lower one, by the detector D1 (in the experiment we dispense with such a detection, but we do need the expression for the calculation) is

$$P(\theta_1 \times \theta_1) = \langle\Psi|\hat{E}_1^{\dagger 2}\hat{E}_1^2|\Psi\rangle. \tag{5}$$

And the probability of one ordinary and one extraordinary photon being detected by D1 and D2$^\perp$ (as enabled by birefringent polarizer P2) is given by

$$P(\theta_1, \theta_2^\perp, \phi) = \langle\Psi|\hat{E}_2^{\perp\dagger}\hat{E}_1^\dagger\hat{E}_1\hat{E}_2^\perp|\Psi\rangle. \tag{6}$$

B. Beam splitter fed by a down–converted photon pair

A laser beam of frequency ω_0 pumps a nonlinear crystal of the first type NL–I producing in it a down–converted pair of signal and idler photons of frequencies ω_1 and ω_2, respectively, which satisfy the energy conservation condition: $\omega_0 = \omega_1 + \omega_2$. By means of appropriately symmetrically (and as far away from the crystal as possible) positioned pinholes we select half–frequency sidebands so as to have $\omega_2 = \omega_1$ [see Fig. 1(d)]. Idler and signal photons coming out from the crystals do not have definite phases [15] with respect to each other and consequently one cannot have a second order interference. Photons emerging from a nonlinear crystal of the first type are parallelly polarized [16] and because we aim at a spin correlated photon state we use a polarization rotator for one of the beams. One achieves the best correlation with 90^0 rotation although for an asymmetrical beam splitter even 0^0 yields a nonvanishing probability of photons emerging from opposite sides of the beam splitter. [8] The state of the incoming polarized photons is thus given by $|\Psi\rangle = |1_x\rangle_1|1_y\rangle_2$.

Assuming $\phi = 0$ (we can modify ϕ by moving detectors perpendicular to the light path as indicated by '↔' in Fig. 1) and introducing $s = t_x t_y$ and $r = \frac{r_x r_y}{t_x t_y}$ the probability of joint detection of two ordinary [see Fig. 1(a)] photons by detectors D1 and D2 reads

$$P(\theta_1, \theta_2) = \langle\hat{E}_2^\dagger\hat{E}_1^\dagger\hat{E}_1\hat{E}_2\rangle = \eta^2 s^2(\cos\theta_1\sin\theta_2 - r\sin\theta_1\cos\theta_2)^2 \tag{7}$$

where η is the (detection) efficiency. This probability tells us that the photons emerge from the beam splitter correlated in polarization whenever they emerge from two different sides of it.

The probability of one ordinary and one extraordinary photon being detected by D1 and $D2^{\perp}$ (as enabled by birefringent polarizer P2) is given by

$$P(\theta_1, \theta_2^{\perp}) = \eta^2 s^2 (\cos\theta_1 \cos\theta_2 + r \sin\theta_1 \sin\theta_2)^2 \,. \tag{8}$$

On the other hand in case of a symmetrical beam splitter photons emerge from it unpolarized. To convince ourselves let us look at the singles–probability of detecting one photon by D1 and the other going through P2 and through either D2 or $D2^{\perp}$ without necessarily being detected by either of them [obtained by summing up Eqs. (7) and (8) and dividing them by η for only D1 detection] is

$$P(\theta_1, \infty) = \eta s^2 (\cos^2\theta_1 + r^2 \sin^2\theta_1) \,, \tag{9}$$

and analogously:

$$P(\infty, \theta_2) = \eta s^2 (\sin^2\theta_2 + r^2 \cos^2\theta_2) \,. \tag{10}$$

We see that for $r = 1$, i.e., for a symmetrical beam splitter, Eq. (9) gives $P(\theta_1, \infty) = \eta/4$, i.e., the outgoing photon is unpolarized. In other words whenever photons emerge from different sides of a symmetrical beam splitter they emerge anticorrelated in polarization and unpolarized, i.e., they appear in a *singlet state*. It is therefore to be expected that unpolarized balanced incoming photons also appear correlated in polarization. That this is indeed the case we show in the next subsection.

And for a later use we give here the probability of both photons being detected at the same side of the beam splitter by, e.g., D1 assuming $t_y = t_x$

$$P(\theta_1 \times \theta_1) = \frac{\eta s^2}{2} sin^2(2\theta_1) \,. \tag{11}$$

C. Beam splitter fed by two unpolarized photons

Sources of unpolarized light can be cascade atom processes, but even better, photons emerging from the opposite sides of a symmetrical beam splitter (as we learned in the previous subsection) or photons emerging from two type II crystals of the second order [17] [from pinholes *ph* in Fig. 1(f)].

We obtain the general probability for unpolarized light, by calculating the mean value given by Eq. (4) for four input states $|1_x\rangle_1|1_x\rangle_2$, $|1_x\rangle_1|1_y\rangle_2$, $|1_y\rangle_1|1_x\rangle_2$, and $|1_y\rangle_1|1_y\rangle_2$ and adding them together. For $t_y = t_x$ they sum up to the following correlation probability:

$$P(\infty,\infty;\theta_1,\theta_2,\phi) = \frac{\eta^2 s^2}{4}[1 + r^2 - 2r\cos^2(\theta_1-\theta_2)\cos\phi]\,. \tag{12}$$

The correlation is maximal for a symmetric beam splitter for $\phi = 0$ in which case we obtain:

$$P(\infty,\infty,\theta_1,\theta_2) = \frac{1}{8}\sin^2(\theta_2-\theta_1)\,. \tag{13}$$

D. Beam splitter fed by two superposed down–converted photon pairs

The main disadvantage of feeding a beam splitter by a down–converted photon pair is that 50% of photons emerge from it from the same side [8] [see Eq. (11)]. Namely, detectors still cannot (at least not efficiently enough) tell two photons from one and we therefore cannot *control* photons. Such a *control* of photons would however be possible if photons never appeared together from the same side of a beam splitter. This was achieved by Kwiat *et al.* [18] Their scheme is shown in Fig. 1(c). Two type II crystals of the first order down–convert two collinear and orthogonally polarized photons of the same average frequencies (half of the pumping beam frequency). The crystals are pumped by a 50:50 split laser beam (filtered out before reaching detectors) whose intensity is accommodated so as to give only one down–conversion at a chosen time–window. Since one cannot tell which crystal a down–converted pair is coming from, the state of the photons incoming at the beam splitter must be described by the following superposition

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|1_x\rangle_1|1_y\rangle_1 + f|1_x\rangle_2\,|1_y\rangle_2\right) , \tag{14}$$

where $0 \le f \le 1$ describes attenuation of the lower incoming beam.

The joint D1–D2 probability is given [as follows from Eq. (4)] by

$$\begin{aligned} P(\theta_1,\theta_2) = \frac{1}{2}[&\cos\theta_1\sin\theta_2(t_x r_y + f\,t_y r_x\cos\phi) \\ &+\sin\theta_1\cos\theta_2(t_y r_x + f\,t_x r_y\cos\phi)]^2 . \end{aligned} \tag{15}$$

The probability of both photons emerging from either the upper or the lower side of BS is for $\phi = 180^\circ$ (+) and $\phi = 0^\circ$ (−) given, respectively, by

$$P(\infty\times\infty) = (t_x t_y \pm f r_x r_y)^2 + (r_x r_y \pm f t_x t_y)^2 . \tag{16}$$

We see that for the crosstalk $t_y = r_x = 0$ and $\phi = 180^\circ$ we obtain:

$$P(\theta_1,\theta_2,180^\circ) = \eta^2(\cos\theta_1\sin\theta_2 - f\sin\theta_1\cos\theta_2)^2 , \tag{17}$$

which is functionally equivalent to Eq. (7) and has the advantage of giving a perfect *control* of photon as follows from Eq. (16) which yields $P(\infty\times\infty) = 0$. The main disadvantage of this solution, however, is that the crosstalk is very difficult to control in the laboratory [18]. Kwiat *et al.* wanted to go around this problem by choosing $\phi = 0^\circ$ but this does not help either, as we show in the next section.

III. BELL INEQUALITIES

No Bell experiment carried out so far could conclusively disprove hidden–variable theories without additional assumptions. [19] Cascade photon pair experiments have to rely on the *no enhancement* assumption (made by Clauser and Horne [20]: *a subset of a total set of events gives the same statistics as the set itself*) because the directions of photons in the process (which is a three–body decay) are uncontrollable. The fourth order interference, on the other hand, provides directional photon correlation but it was believed that one has to discard 50% of counts which correspond to photons emerging from the same sides of a beam splitter. Therefore Kwiat *et al.* [18] and we [9,11] devised two different schemes that, on the one hand, do not require discarding of counts and, on the other, reduce the required efficiency to (ideally) 67% which is 16% lower than the previously required efficiency of 83%. [21] We devised the preselection experiment which we present in Sec. IV but which is, in effect, based on a recognition of a possibility to obtain nonmaximally entangled photons by means of coefficients of transmission and reflection as elaborated in Secs. II A and II B. Kwiat *et al.*, on the other hand, used Eberhard's result [12] in order to make a proposal for a loophole–free Bell inequality experiment which we present below and in Sec. II D. Eberhard considered nonmaximally entangled photons and showed that for a violation of the Bell inequality only 67% efficiency is required. He obtained his result by employing an asymmetrical form of the Bell inequality for which he found angles of polarizers that violated this Bell inequality only after he set efficiency η to particular values and connected it to the background level.

To compare the two approaches let us first look at the usual Clauser–Horne form and then at Eberhard's form of the Bell inequality: $B \leq 0$. In the Clauser–Horne form B is defined so as to satisfy [see Eqs. (7), (9), and (10)]

$$\eta s^2 B \equiv P(\theta_1, \theta_2) - P(\theta_1, \theta_2') + P(\theta_1', \theta_2') + P(\theta_1', \theta_2) - P(\theta_1') - P(\theta_2)\,, \tag{18}$$

where $P(\theta_1') = P(\theta_1', \infty)$ and $P(\theta_2) = P(\infty, \theta_2)$, as given by Eqs. (9) and (10). B of the Eberhard's form is, in effect, defined so as to satisfy [see Eqs. (17) and the equations corresponding to Eqs. (8) and (9)—with $s = 1$ and $r = f$]

$$\begin{aligned}\eta s^2 B \equiv P(\theta_1, \theta_2) - P(\theta_1', \theta_2') - P(\theta_1, \theta_2'^{\perp}) - P(\theta_1'^{\perp}, \theta_2)\\ -(1-\eta)[P(\theta_1) + P(\theta_2)]\,,\end{aligned} \tag{19}$$

where $(1-\eta)P(\theta_1)$ is the probability of one photon being detected by D1 and the other reaching either D2 or D2$^{\perp}$ but not being detected by them due to their inefficiency. In other words, to be able to use either of the above forms we have to have a perfect *control* of all photons at BS. As we stressed in Sec. II D, Kwiat *et al.* do achieve this *control* but at the expense of the crosstalk. They also claim that the choice $\phi = 0°$ would preserve the *control* and at the same time dispense with the crosstalk. Unfortunately, this does not work what can be easily seen from Eq. (16). To obtain $P(\infty \times \infty) = 0$ one has to satisfy $r_x r_y = f t_x t_y$ and $t_x t_y = f r_x r_y$ what is however clearly impossible for $f < 1$. Thus, contrary to the claims of Kwiat *et al*. [18], the only way to make use of $f < 1$ for either $\phi = 0°$ or $\phi = 180°$ is the crosstalk $t_y = r_x = 0$ and this is apparently difficult to achive within a measurement. [18] It seems that the set–up is ideal for a loophole–free experiment with maximal non–product states, i.e., with $f = 1$ and $\eta > 83\%$ but that attenuation ($f < 1$) is not the best candidate for Bell's event–ready preselector [20]. We therefore propose another "event–ready set–up" which dispenses with variable f, enables a full *control* of photons, and offers a fundamental insight into the whole issue.

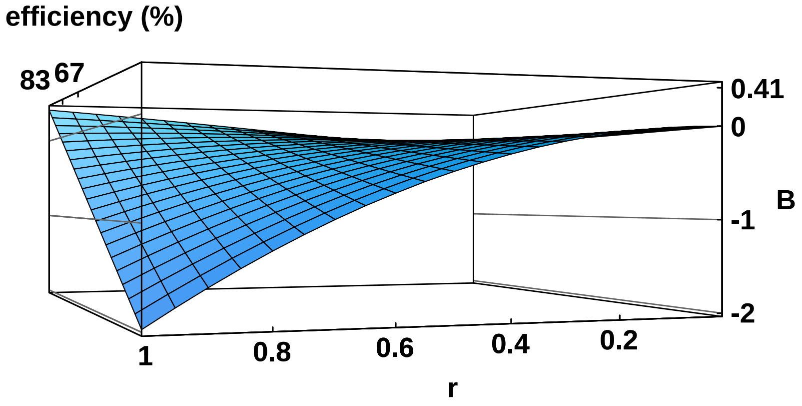

FIG. 2. The surface $Max[B]$ [Eqs. (18) and (19)] for the optimal angles of the polarizers. Values above the $B = 0$ plane violate the Bell inequality $B \leq 0$.

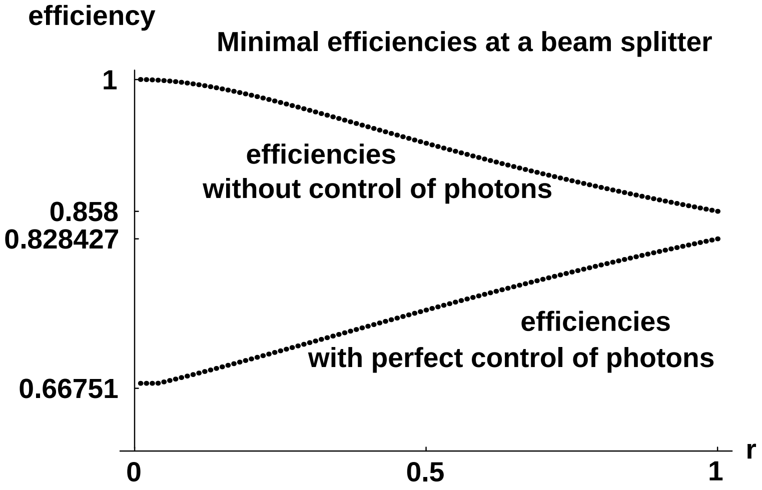

FIG. 3. Lower plot: η's as obtained for $B = 0$ from Eqs. (18) and (19). Upper plot: η's as obtained for $B = r[\sin^2(2\theta_1') + \sin^2(2\theta_2)]/2$ from Eqs. (7), (9), (10), and (18), and for $B = r[\sin^2(2\theta_1) + \sin^2(2\theta_2)]/2$ from Eqs. (7), (8), (9), (10), and (19).

But before we dwell on the experiment itself let us first compare the two afore–introduced forms of the Bell inequality in two ways. First, we obtain $Max[B](r, \eta)$ surfaces (by a computer optimization of angles) for both forms (18) and (19). As we can see in Fig. 2, there is no difference between them although the maxima were achieved for different angles. (The differences are 10^{-5} in average, for 100 iterations used in numerical calculations of maxima.) The values above the $B = 0$ plane mean violations of the Bell inequality. For $r = 1$ we obtain $Max[B] = 0$ for $\eta = 0.828427$ in accordance with the result of Garg and Mermin. [21] For $r \to 0$ we obtain a violation of the Bell inequality for any efficiency greater then 66.75%. Secondly, we calculate η by setting $B = 0$, first, from Eq. (18) and then from Eq. (19). Again, [see Fig. 3] there is no difference between the two forms. We can also see that there is no need to optimize the angles only *after* values for η have been fixed, contrary to the claim from Ref. [12]. The efficiencies for uncontrolled photons are shown as the upper curve in Fig. 3. Once again, there is no difference between the forms. On the other hand, we see that uncontrolled photons, i.e., the ones that may emerge from the same sides of BS as well, violate the Bell inequality—starting with 85.8% efficiency—in opposition to the widespread belief that "unless the detector can differentiate one photon from two...

no indisputable test of Bell's inequalities is possible." [18]

IV. PRESELECTION EXPERIMENT

A schematic representation of the experiment is shown in Fig. 4. Two independent sources $S1$ and $S2$ simultaneously emit two independent entangled pairs. Left photons from each pair fly towards detectors D1' or D1'$^{\perp}$ and D2' or D2'$^{\perp}$. Right photons from each pair interfere at an asymmetrical beam splitter which acts as an *event–ready* preselector and as a result the so preselected left photons, under particular conditions elaborated below, appear to be in a nonmaximal singlet state although the latter photons are completely independent and nowhere cross each other's path. There are several possible sources for such an entangled state of photons.

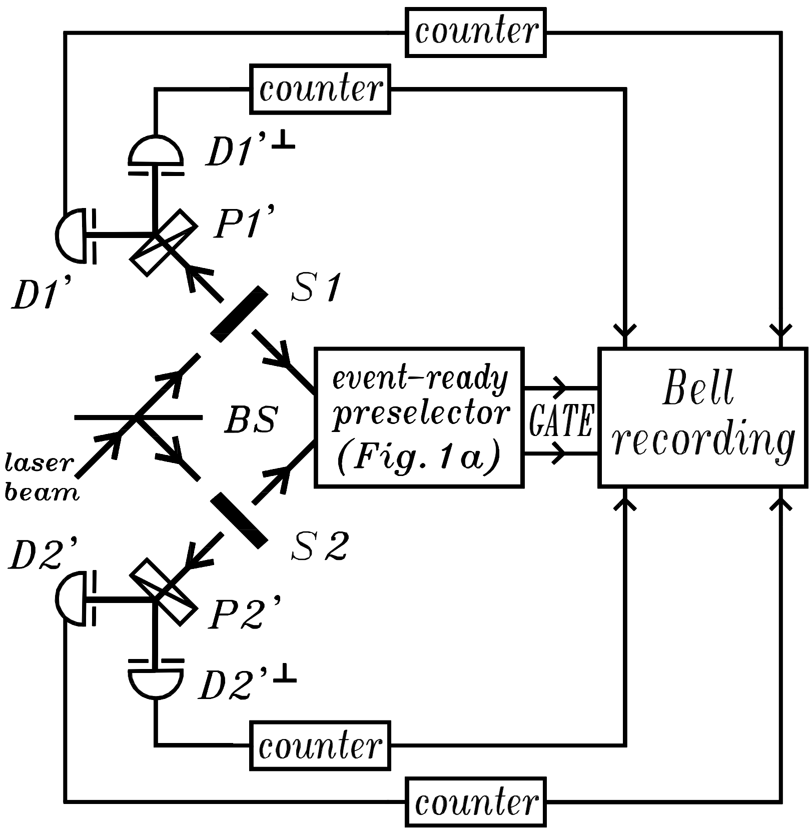

FIG. 4. Proposed experiment. As the event–ready preselector, serves a beam splitter with detectors D1, D1$^{\perp}$, D2 and D2$^{\perp}$ as shown in Fig. 1. $S1$ and $S2$ are sources (type II crystals as explained in the text) emitting singlet–like photon pairs. As birefringent polarizers P1' and P2' may serve Wollaston prisms (which at the same time filter out the uv pumping beam).

Atoms exhibiting cascade emission. The atoms of the two sources could be pumped to an upper level by two independent lasers. This level would decay by emitting two photons correlated in polarization in a triplet–like state. The independence of the two sources can be assured by slight differences in central frequency and drift of the two pump lasers. The sources have been elaborated by Pavičić and Summhammer and an experiment with such sources was estimated to be very difficult to carry out. [7]

Symmetrical beam splitters fed by two nonlinear crystals of type I. The experiment with such sources relies on quadruple recording obtained in the following way: whenever exactly two of the *preselection detectors* D1–D2$^{\perp}$ fire in coincidence (see Fig. 4) a gate for counters D1'–D2'$^{\perp}$ opens. In case only one or none of the so preselected D1'–D2'$^{\perp}$ detectors fires we discard the records. In case exactly two of four D1'–D2'$^{\perp}$ detectors fire, the corresponding counts contribute to our statistics. The sources have been elaborated by Pavičić [9] and an experiment with such sources was found to be rather demanding. Besides, the procedure of discarding data is a *no enhancement* assumption and the experiment cannot be considered a *loophole–free* one,

Symmetrical beam splitters in a crosstalk fed by two "superposed" crystals of type II. The experiment with such sources, emitting photons in a singlet like state, relies only on the *preselection detectors* D1–D2$^{\perp}$. When they fire in coincidence (see Fig. 4) a gate for counters D1'–D2'$^{\perp}$ opens. In case only one of the so preselected D1'–D2'$^{\perp}$ detectors fires we do *not* discard the records but use them to form frequencies approximating one–photon probabilities in the Bell inequalities formed by Eq. (18) and Eq. (19). The whole set–up would apparently be difficult to tune in because of the crosstalk [18] but this nevertheless seem to be a feasible loophole–free Bell experiment.

Crystals of type II of the second order. These crystals emit perpendicularly polarized photons into two intersecting cones as shown in Fig. 1(f). If we position pinholes *ph* at the intersections we shall obtain superposition of perpendicularly polarized photons, i.e., photons in a singlet–like state because one cannot know which cones they belong to. The sources seem to enable a rather feasible loophole–free Bell experiment. [17]

In what follows we shall adopt the latter two kinds of sources. Thus the state of photons immediately after leaving the sources $S1$ and $S2$ is given by a tensor product of two singlet–like states:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|1_x\rangle_{1'}|1_y\rangle_1 \; - \; |1_y\rangle_{1'}|1_x\rangle_1) \otimes \frac{1}{\sqrt{2}}(|1_x\rangle_{2'}|1_y\rangle_2 \; - \; |1_y\rangle_{2'}|1_x\rangle_2)\,. \tag{20}$$

The probability of detecting all four photons by detectors D1, D2, D1', and D2' is thus

$$\begin{aligned} P(\theta_{1'},\theta_{2'},\theta_1,\theta_2) &= \langle\Psi|\hat{E}^\dagger_{2'}\hat{E}^\dagger_{1'}\hat{E}^\dagger_2\hat{E}^\dagger_1\hat{E}_1\hat{E}_2\hat{E}_{1'}\hat{E}_{2'}|\Psi\rangle \\ &= \frac{1}{4}(A^2+B^2-2AB\cos\phi)\,, \end{aligned} \tag{21}$$

where $\hat{E}_1$, $\hat{E}_2$, and ϕ are as given above, $\hat{E}_{j'} = (\hat{a}_{j'x}\cos\theta_{j'} + \hat{a}_{j'y}\sin\theta_{j'})\exp(-i\omega'_j t_{j'});\ j=1,2;$ $A = Q(t)_{1'1}Q(t)_{2'2}$ and $B = Q(r)_{1'2}Q(r)_{2'1}$; here $Q(q)_{ij} = q_x\sin\theta_i\cos\theta_j - q_y\cos\theta_i\sin\theta_j$.

For $\phi = 0°$, $\theta_1 = 90°$, and $\theta_2 = 0°$ Eq. (21) yields (non)maximal singlet–like probability $P(\theta_{1'},\theta_{2'})$ given by Eq. (7) which permits a perfect control of photons $1'$ and $2'$. This means that D1 and D2—while detecting coincidences—act as event–ready preselectors and with the help of a gate (see Fig. 4) we can extract those $1'$ and $2'$ photons that are in a non–maximal singlet state, take them miles away and carry out a loophole–free Bell experiment by means of P1', D1', D1'$^\perp$, P2', D2', and D2'$^\perp$ with only 67% efficiency in the limit $r \to 0$.

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