PRESELECTED SUB–POISSONIAN CORRELATIONS

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Abstract

The simplest possible photon–number–squeezed states containing only two photons and exhibiting sub–Poissonian statistics with the Fano factor approaching 0.5 have been used for a proposal of a loophole–free Bell experiment requiring only 67% of detection efficiency. The states are obtained by the fourth order interference first of two downconverted photons at an asymmetrical beam splitter and thereupon of two photons from two independent singlets at an asymmetrical beam splitter. In the latter set–up the other two photons which nowhere interacted and whose paths never crossed appear entangled in a singlet–like correlated state.

1 Introduction

In 1985 Chubarov and Nikolayev [1] showed that quantum states with sub–poissonian statistics of photons interfering at a beam splitter (in a polarization experiment) violate the Bell inequality. Analyzing their result Ou, Hong, and Mandel [2] showed in 1987 that a pair of downconverted photons interfering in the fourth order at a symmetrical beam splitter should violate the inequality to the same extent although they exhibit poissoinan statistics. In 1988 Ou and Mandel [3] carried out the experiment and gave, together with Hong, its correct theoretical description in Ref.[4]. (The description of Ref. [3] was erroneous.[5, 6]) Pavičić and Summhammer provided in 1994 a theoretical description of two pair spin entanglement at a symmetrical beam splitter which would enable a loophole–free Bell experiment with 83% detection efficiency. On the other hand, in 1989 Campos, Saleh, and Teich [7], in effect, pointed out that not only two (or more) photons incoming to the beam splitter from the same side (as with Chubarov and Nikolayev) but also two photons incoming from the opposite sides (as with Ou and Mandel) and interferening in the fourth order at an *asymmetrical* beam splitter (the simplest photon–number–sqeezed state) exhibit sub–poissonian statistics with the *Fano factor* (the ratio between the variance and the mean of the photocounts) changing from 1 to 0.5 as the ratio between reflection and transmission coefficients changes from 1 to 0. A theoretical description of the interference at an asymmetrical beam splitter was given in 1994 by Pavičić [6]. In Sec. 2 we show how one can use such a beam splitter to devise a loophole–free Bell experiment with a detection efficiency as low as 67%. In 1995 it was pointed out by Pavičić [8] that two pair spin entanglement at an asymmetrical beam splitter enables a preselected loophole–free Bell 67% experiment. In Sec. 3 we present such an experiment.

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2 Simple sub–poissonian correlations

To describe the behavior of the photons at a beam splitter in the spin space we follow the results obtained in Pavičić [6, 8]. The signal and idler downconverted photons emerging from a nonlinear crystal of type–I (see Fig. 1) are parallelly polarized [3]. Because of this a 90^0 rotator is introduced. Since the signal and idler photons have random relative phases, we will have no interference of the second order but only of the fourth order which we describe in the second quantization formalism following Reff. [6, 8]. The actions of beam–splitter BS, polarizer Pj, and detector Dj ($j = 1, 2$) are taken into account by the outgoing electric fields as given in Ref. [8]. For a realistic elaboration by means of wave packets we refer to Reff. [5, 8]. We only stress here that these equations remain unchanged insomuch that all experimental paramaters are absorbed by η and r below.

The probability of joint detection of two ordinary photons by detectors D1 and D2 is

$$P(\theta_1, \theta_2) = \langle\Psi|\hat{E}_2^\dagger \hat{E}_1^\dagger \hat{E}_1 \hat{E}_2|\Psi\rangle = \eta^2 s^2 (\cos\theta_1 \sin\theta_2 - r \sin\theta_1 \cos\theta_2)^2 \,, \tag{1}$$

for $z_1 = z_2$ in Fig. 1, where $\hat{E}_j$ ($j = 1, 2$) are as given in Reff. [5, 8], $s = t_x t_y$, $r = \frac{r_x r_y}{t_x t_y}$, r_x and r_y are reflection coefficients, t_x and t_y are transmission coefficients, and η is detection efficiency. The probability tells us that the photons appear to be in a nonmaximally correlated state whenever they emerge from two different sides of BS. The singles–probability of detecting one photon by, e.g., D1 and the other going through P2 and through either D2 or D2$^\perp$ without necessarily being detected by either of them is

$$P(\theta_1, \infty) = \eta s^2 (\cos^2\theta_1 + r^2 \sin^2\theta_1) \,. \tag{2}$$

The singles–probability of detecting one photon by D1 and the other going through P1 and D1 (without necessarily being detected by it) is (assuming $t_x = t_y$)

$$P(\theta_1 \times \theta_1) = \frac{\eta s^2 r}{2} \sin^2(2\theta_1) \,. \tag{3}$$

Let us see the effect of these results on the violations of the Bell inequality $B \le 0$ where B is defined by

$$\eta s^2 B \equiv P(\theta_1, \theta_2) - P(\theta_1, \theta_2') + P(\theta_1', \theta_2') + P(\theta_1', \theta_2) - P(\theta_1') - P(\theta_2) \,, \tag{4}$$

where $P(\theta_1') = P(\theta_1', \infty)$ [as given by Eq. (2)] and $P(\theta_2) = P(\infty, \theta_2)$. To be able to use Eq. (4) we have to have a perfect "control" of all photons at BS. If we do not have it, we have to subtract Eqs. (3) (for appropriate angles) from Eq. (4) in order to take into account that detectors cannot tell one from two photons when they both emerge from the same side of BS.

By a computer optimization of angles we obtain $Max[B](r, \eta)$ surfaces as shown in Fig. 2. The values above the $B = 0$ plane mean violations of the Bell inequality. As shown by the lower curve in Fig. 3, for controlled photons, for $r = 1$ $Max[B] = 0$ yields $\eta = 0.828427$ and for $r \to 0$ we get a violation of the Bell inequality for any efficiency greater then 66.75%. The efficiencies for uncontrolled photons are shown as the upper curve in Fig. 3. We see that uncontrolled photons, i.e., the ones that also may emerge from the same sides of BS as well, violate the Bell inequality—starting with 85.8% efficiency—in opposition to the widespread belief that "unless the detector can differentiate one photon from two... no indisputable test of Bell's inequalities is possible." [9]

The afore–mentioned "control" of all photons can be achieved best if photons never emerge from the same side of a beam splitter and this is what Kwiat *et al.* [9] aimed at. We obtain their set–up by substituting the nonlinear crystal in Fig. 1 with two type–II crystals (MZ–II inset in Fig. 1) which downconvert two collinear and orthogonally polarized photons of the same average frequencies (half of the pumping beam frequency). The crystals are pumped by a 50:50 split laser beam (filtered out before reaching detectors) whose intensity is accommodated so as to give only one downconversion at a chosen time–window. Since one cannot tell which crystal a downconverted pair is coming from, the state of the photons incoming at the beam splitter must be described by the following superposition

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|1_x\rangle_1|1_y\rangle_1 + f|1_x\rangle_2\,|1_y\rangle_2\right)\,, \tag{5}$$

where $0 \le f \le 1$ describes attenuation of the lower incoming beam.

The probability of both photons emerging from the same sides of BS is

$$P(\infty\times\infty) = (t_x t_y \pm f r_x r_y)^2 + (r_x r_y \pm f t_x t_y)^2\,, \tag{6}$$

where '$-$' stands for $z_1 = z_2$ and '+' for $z_2 - z_1 = L/2$ where L is the spacing of the interference fringes.

The probability of both photons emerging from the opposite sides of BS is

$$P(\theta_1,\theta_2) = \eta^2(\cos\theta_1\sin\theta_2 \mp f\sin\theta_1\cos\theta_2)^2\,, \tag{7}$$

where '+' stands for $z_1 = z_2$ and '$-$' for $z_2 - z_1 = L/2$. This gives the same η curve as shown in Fig. 3 but, in order to collect data for the probabilities in B in Eq. (4), we must be able to "control" single pairs of photons so as to prevent them to emerge both from the same side of BS. This means that the conditions $r_x r_y = f t_x t_y$ and $t_x t_y = f r_x r_y$ from Eq. (6) should be simultaneously satisfied what is however clearly impossible for $f < 1$. Thus, contrary to the claims of Kwiat *et al.* [9], the only way to make use of $f < 1$ is the crosstalk $t_y = r_x = 0$ for either $z_1 = z_2$ or $z_2 - z_1 = L/2$ and this is apparently difficult to control within a measurement.[9] It therefore turns out that the set–up is ideal for a loophole–free experiment with maximal singlet–like states, i.e., with $f = 1$ and $\eta > 83\%$ but that attenuation ($f < 1$) is not the best candidate for Bell's *event–ready* [10] preselector. We therefore propose another "event–ready set–up" which dispenses with variable f and offers a more fundamental insight into the whole issue.

3 Preselected sub–poissonian correlations

Schematic of the proposed experiment is given in Fig. 4. Two afore–discussed set–ups MZ–II 1 and MZ–II 2, fed by a split laser beam act as two independent sources of two independent singlet pairs. As shown above, for $z_2 - z_1 = L/2$ photons appear only from the opposite sides of the beam splitters of MZ–II 1 and MZ–II 2. Two photons from each pair interfere at the beam splitter of the *event–ready preselector* and as a result the other two photons appear to be in a nonmaximal singlet state although the latter photons are completely independent and nowhere interacted. The state of the four photons immediately after leaving MZ–II 1 and MZ–II 2 is

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|1_x\rangle_{1'}|1_y\rangle_1 \;-\; |1_y\rangle_{1'}|1_x\rangle_1)\otimes\frac{1}{\sqrt{2}}(|1_x\rangle_{2'}|1_y\rangle_2 \;-\; |1_y\rangle_{2'}|1_x\rangle_2)\,. \tag{8}$$

The probability of detecting all four photons by detectors D1, D2, D1', and D2' is thus

$$P(\theta_{1'},\theta_{2'},\theta_1,\theta_2)=\langle\Psi|\hat{E}_{2'}^{\dagger}\hat{E}_{1'}^{\dagger}\hat{E}_{2}^{\dagger}\hat{E}_{1}^{\dagger}\hat{E}_{1}\hat{E}_{2}\hat{E}_{1'}\hat{E}_{2'}|\Psi\rangle=\frac{1}{4}(A-B)^2\,, \qquad (9)$$

for $z_1=z_2$ where $\hat{E}_j$ are as given in Reff. [5, 8], $A=Q(t)_{1'1}Q(t)_{2'2}$ and $B=Q(r)_{1'2}Q(r)_{2'1}$; here $Q(q)_{ij}=q_x\sin\theta_i\cos\theta_j-q_y\cos\theta_i\sin\theta_j$.

For $\theta_1=90^\circ$ and $\theta_2=0^\circ$ Eq. (9) yields (non)maximal singlet–like probability $P(\theta_{1'},\theta_{2'})$ given by Eq. (1) which permits a perfect control of photons $1'$ and $2'$ and which is much more appropriate for the whole issue than Eq. (7), because the former reflects total spin conservation and quantum mechanical nonlocality while the latter satisfies the Bell inequality only inasmuch as it belongs to a non–product state [11]. This means that D1 and D2—while detecting coincidences—act as event–ready preselectors [10] and with the help of a gate (see Fig. 4) we can extract those $1'$ and $2'$ photons that are in a non–maximal singlet state, take them miles away and carry out a loophole–free Bell experiment by means of P1', D1', D1'$^\perp$, P2', D2', and D2'$^\perp$ with only 67% efficiency in the limit $r\to 0$. Thus, one might also view the experiment as a realistic device for *teleportation* of Bennett *et al.* [12]

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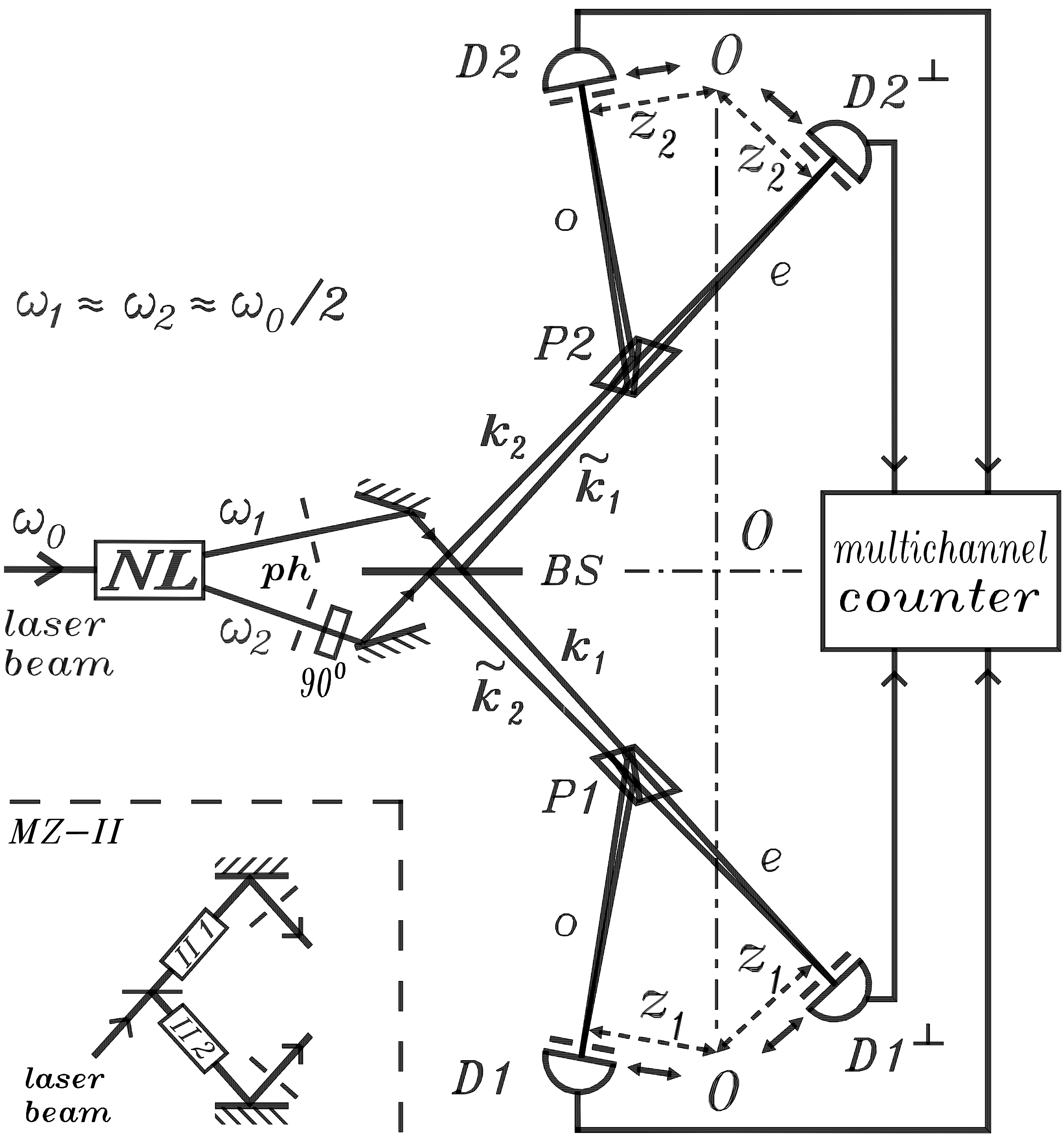
FIG. 1. Beam splitter set–up and MZ–II set–up (when inset MZ–II is put in place of NL; according to Kwiat *et al.* [9]). As birefringent polarizers P1 and P2 may serve Nicol or Wollaston prisms (which at the same time filter out the uv pumping beam in case of MZ–II). Pinholes *ph* determining the frequency ($\omega_0/2$) of signal and idler coming to the beam splitter BS and assuring that only one downconverted pair appears at a time are positioned as far away from the crystal as possible.

FIG. 2. The surface showing maximal violation of the Bell inequality for the optimal angles of the polarizers. All the values above the $B = 0$ plane violate the Bell inequality $B \leq 0$, where B is given by Eq. (4).

FIG. 3. Minimal efficiencies. Lower plot: η's as obtained for $B = 0$ from Eq. (4). Upper plot: η's as obtained for $B = r[\sin^2(2\theta_1') + \sin^2(2\theta_2)]/2$ from Eqs. (1–4).

FIG. 4. Proposed experiment. As the event–ready preselector serves a beam splitter with detectors D1, $D1^\perp$, D2 and $D2^\perp$ as shown in Fig. 1. MZ–II 1 and MZ–II 2 are devices as shown in Fig. 1 with MZ–II from the inset substituted for NL; they serve as sources of singlet pairs. As birefringent polarizers P1' and P2' may serve Wollaston prisms (which at the same time filter out the uv pumping beam).



D2
O
D2⊥
z2
z2
o
e
$\omega_1 \approx \omega_2 \approx \omega_0/2$
P2
k2
k̃1
ω0
ω1
NL
ph
BS
O
multichannel
counter
laser
beam
ω2
90⁰
k1
k̃2
P1
MZ–II
e
o
II 1
II 2
z1
z1
laser
beam
D1
O
D1⊥

Figure 1:

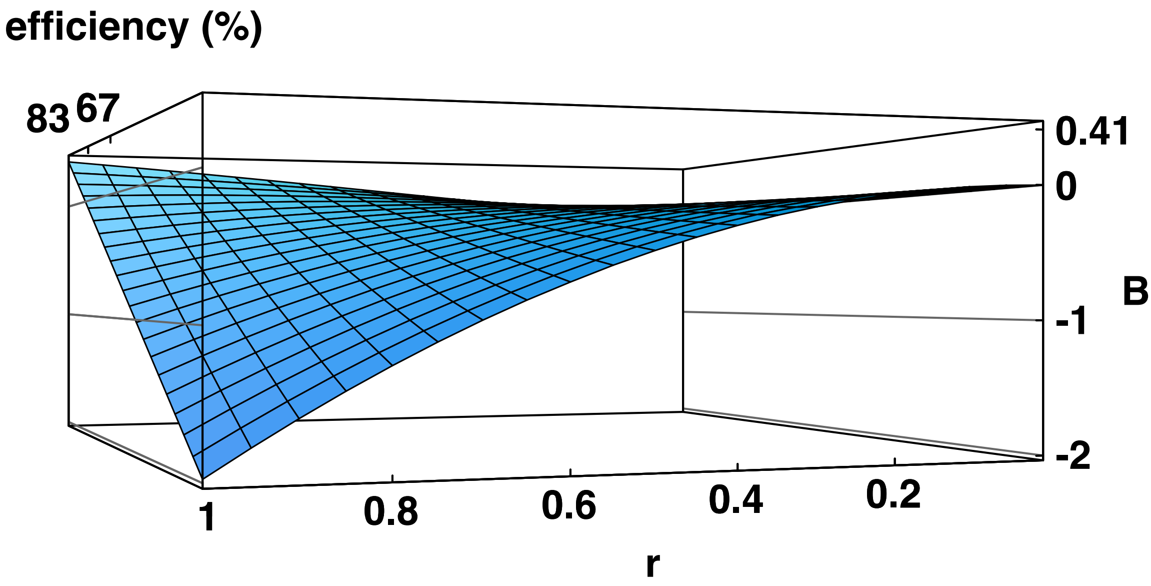


Figure 2:

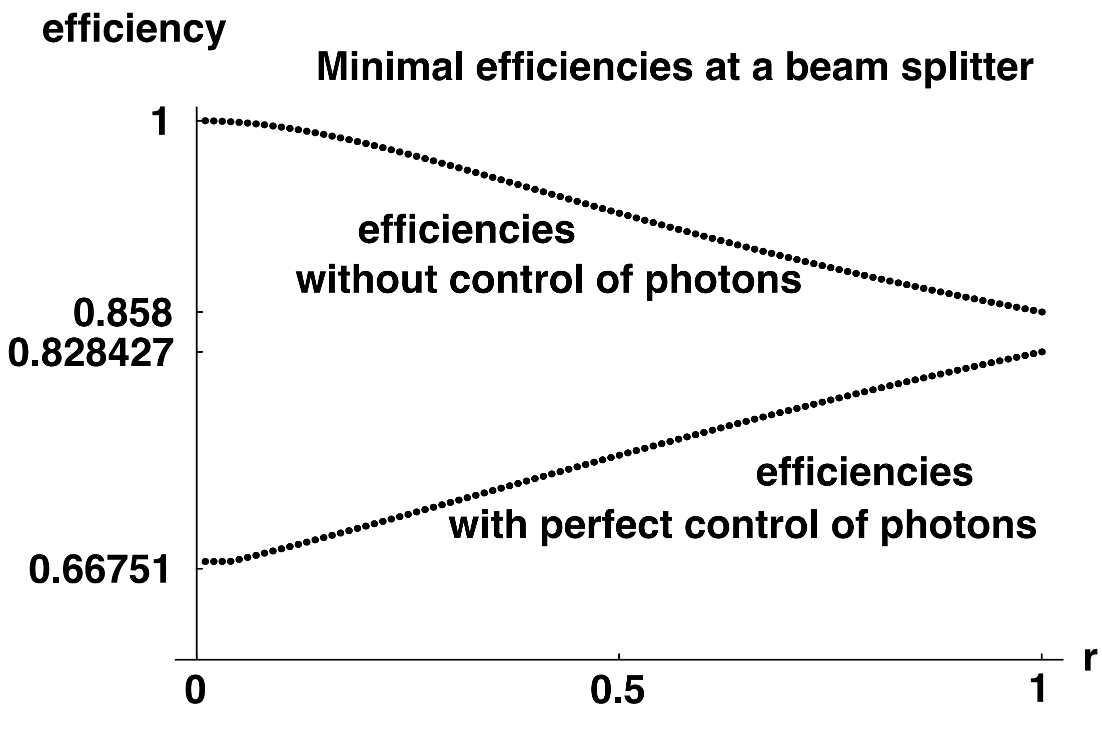


Figure 3:

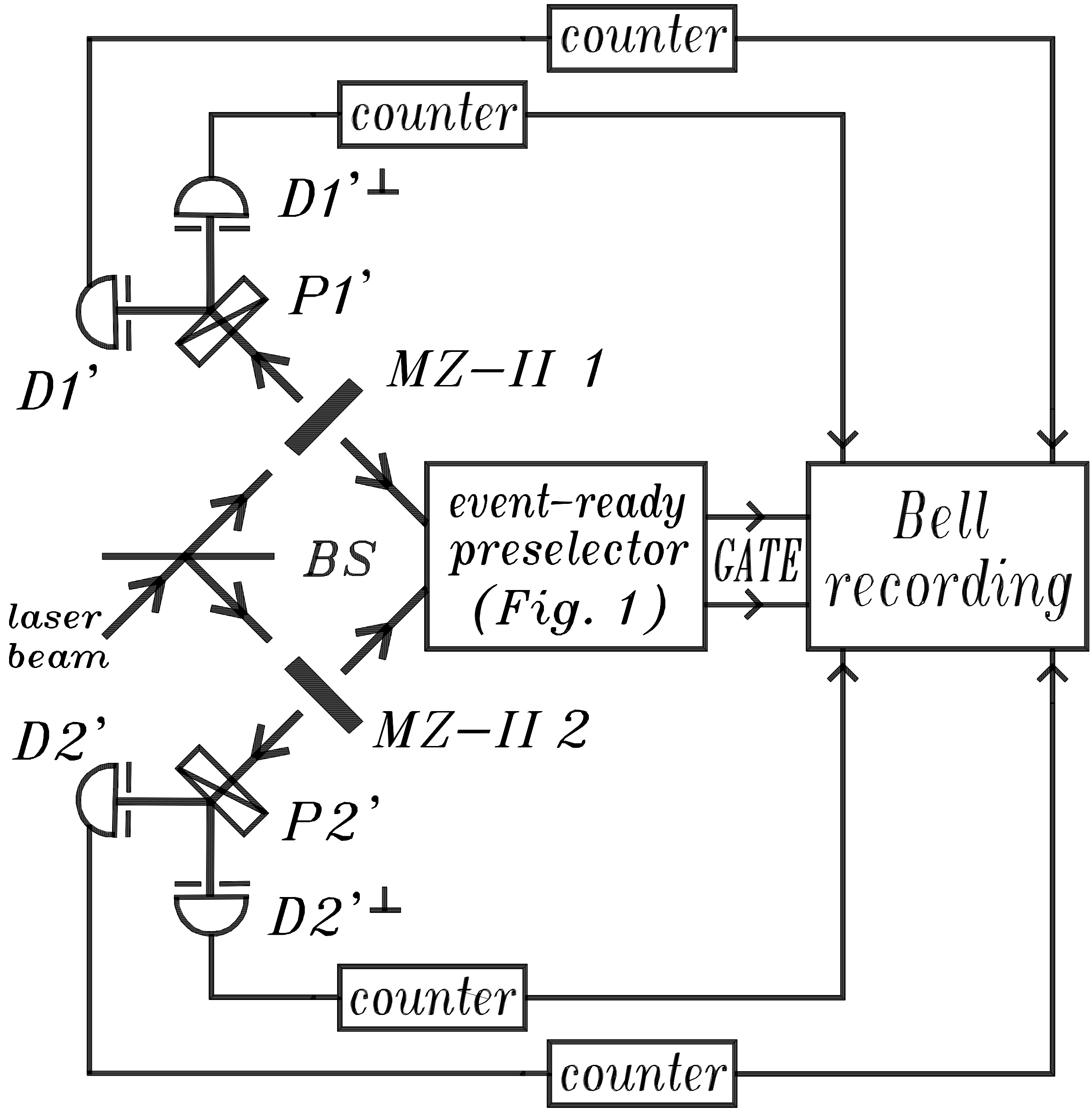

counter
counter
D1'⊥
P1'
D1'
MZ–II 1
event–ready
preselector
(Fig. 1)
GATE
Bell
recording
BS
laser
beam
MZ–II 2
D2'
P2'
D2'⊥
counter
counter

Figure 4: