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Closure of the Enhancement and Detection Loopholes in the Bell Theorem by the Fourth Order Interference with Photons of Different Colours

Mladen Pavičić

Humboldt University of Berlin, Max-Planck-AG Nichtklassische Strahlung, D-12484 Berlin, Germany

Atominstitute of the Austrian Universities, Schüttelstraße 115, A–1020 Wien, Austria

and University of Zagreb, Department of mathematics, GF, Kačićeva 26, Pošt. pret. 217, HR–10001 Zagreb, Croatia[*]

Abstract

A loophole–free Bell experiment requiring detection efficiency as low as 67% is proposed. The experiment uses different frequency photon interference of the fourth order at an asymmetrical beam splitter and a resulting non–maximal entanglement.

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[*] Permanent address. Internet: mpavicic@faust.irb.hr; Web page: http://m3k.grad.hr/pavicic

No Bell experiment carried out so far could conclusively disprove hidden–variable theories without additional assumptions. [1] Cascade photon pair experiments have to rely on the *no enhancement* assumption (made by Clauser and Horne [2,3]: *a subset of a total set of events gives the same statistics as the set itself*) because the directions of photons in the process (which is a three–body decay) are uncontrollable. The fourth order interference, on the other hand, provides directional photon correlation [4–11] but it was believed that one has to discard 50% of counts which correspond to photons emerging from the same sides of a beam splitter. We have recently shown that this is not the case for a polarization experiment which would use birefringent polarizers but also that such an experiment at a single beam splitter would require 85.8% overall efficiency, i.e., in effect, over 92% detector efficiency which is still not available. [10] Therefore we devised a preselection set-up which permits lowering of the required efficiency down to 67% in the ideal limit. [10] The latter experiment reveals the nonlocality as a property of selection but for its primary purpose of enabling a conclusive Bell experiment a simplified version would be welcome and this is what we aim at in this paper. Kwiat *et al*. [7] also aimed at 67% efficiency by means of three type–II crystals that down–convert onto a beam splitter but they failed to recognize (as shown in Ref. [10]) that with the attenuation of one of the incident beams (as required in the proposal) photons start to emerge from the same sides of the second beam splitter as opposed to the symmetrical case.

In this paper we use a nonclassical property of the fourth–order interference of photons of different colours (obtained by Larchuk *et al*. [12] and by Ou and Mandel [13]) in order to devise an experiment which would close the last two loopholes in the Bell theorem proof: the *no enhancement* one and the insufficient detection efficiency one. The proposal is physically simpler and apparently easier to carry out than the previous two proposals put forward by Kwiat *et al*. [7] and Pavičić [5,8].

In the experiment we make use of spin (polarization) interference of polarized photons at a low–reflectivity beam splitter a theory of which we recently elaborated in Ref. [6]. What gave us the clue is Eberhard's lowering of efficiency to 67% for a non–maximal entanglement. [14] We achieved a non–maximal entanglement by means of $r < 1$ ratio function of transmission coefficients of an asymmetrical beam splitter and not by attenuating one of the incident beams. We make use neither of Eberhard's special form of the Bell inequality nor of his special condition (*"choose the efficiency first"*) imposed on optimization procedure for finding the angles which violate the inequality. Both of them are thus—as a *by–product* of our result—shown as unnecessary. On the other hand, in Ref. [10] we have explicitly shown that the Eberhard's form of the Bell inequality gives exactly the same results as its usual form.

A schematic representation of the experiment is shown in Fig. 1. A subpicosecond laser beam of frequency ω_0 pumps up a nonlinear crystal NL of type I at a repetition rate of 100 MHz (corresponding to nanoseconds time windows, i.e., computer gates) and down–converts into pairs of signal and idler photons of frequencies ω_1 and ω_2, respectively, which satisfy the following energy and momentum conservation conditions: $\omega_0 = \omega_1 + \omega_2$ and $\boldsymbol{k}_0 = \boldsymbol{k}_1 + \boldsymbol{k}_2$. [15] By means of two pairs of asymmetrically positioned pinholes—as shown in Fig. 1—we select sidebands containing idler and signal photons of frequencies ω_1 and $\omega_2 = \omega_0 - \omega_1$. Down–converted pairs coming out from the crystals do not have definite phases [16] with respect to each other and consequently interference of the second order does not occur. Signal and idler photons from each pair are parallelly polarized and isolated they would not emerge from a beam splitter entangled [6]. However, if the pairs are prepared in superposition so that effectively only two photons appear from the crystal—from which pair we of course cannot know—they emerge non–maximally entangled from an asymmetrical beam splitter if the pairs are perpendicularly polarized to each other by a 90^0 polarization rotator. [4] Thanks to the ultrashort pumping beam which by appropriate lowering in intensity ensure an average appearance of one down–converted pair of photons at a time, we are able to effectively control coincidences each of which occurs as a property of down–conversion within a few fs, i.e, well within our time windows of 10 ns. On the other hand, the intensity corresponds to the coherence time of two pairs. In other words, two pairs of signals and idlers appear in superposition in the sense that we never now which one of the two appeared while the probability of having four photons as a result of one pumping is practically equal to zero.

Upon emerging from the crystal we direct the photon pairs to a low–reflectivity beam splitter, with unequal transmission and reflection coefficients in particular directions, from its opposite sides. Photons coming out from the beam splitter pass through the Wollaston prisms P1,P2 (which are at the same time birefringent polarizers and frequency selectors) to detectors $\mathrm{D1}_{\omega_1}, \mathrm{D1}^{\perp}_{\omega_1}, \mathrm{D1}_{\omega_2}, \mathrm{D1}^{\perp}_{\omega_2}, \mathrm{D2}_{\omega_1}, \mathrm{D2}^{\perp}_{\omega_1}, \mathrm{D2}_{\omega_2}, \mathrm{D2}^{\perp}_{\omega_2}$. We record the intensity (not amplitude) correlation (beating) between photons of frequencies ω_1 and ω_2 which do not overlap [13] in the following way. Firing of three or all four counters discard the corresponding data because they do not belong to our set of pair events. We also discard data upon firing of two detectors on the same side (e.g., $\mathrm{D1}_{\omega_1}, \mathrm{D1}_{\omega_2}$) because such data correspond to emerging of both photons from the same side of the beam splitter and therefore do not belong to our entangled state.

Since, ideally, no photon escapes a detection — thus satisfying Santos' [1] demand — the probability of coincidental firing of either $\mathrm{D1}_{\omega_1}$ and $\mathrm{D2}_{\omega_2}$ or $\mathrm{D1}_{\omega_2}$ and $\mathrm{D2}_{\omega_1}$, given by Eq. (6), is then approximated by the following ratio between the numbers of coincidence counts:

$$f(\theta_1,\theta_2) = \frac{n[(\mathrm{D1}_{\omega_1} \cap \mathrm{D2}_{\omega_2}) \cup (\mathrm{D1}_{\omega_2} \cap \mathrm{D2}_{\omega_1})]}{n(\mathrm{D1}_{\omega_1} \cup \mathrm{D1}^{\perp}_{\omega_1} \cup \mathrm{D1}_{\omega_2} \cup \mathrm{D1}^{\perp}_{\omega_2} \cup \mathrm{D2}_{\omega_1} \cup \mathrm{D2}^{\perp}_{\omega_1} \cup \mathrm{D2}_{\omega_2} \cup \mathrm{D2}^{\perp}_{\omega_2})}\,. \qquad (1)$$

where θ_1 and θ_2 are the orientation angles of polarizers P1 and P2, respectively.

The state of our two photons incoming to the beam splitter is in the ideal case of monochromatic photons described by: $|\Psi\rangle = \frac{1}{\sqrt{2}}(|1_x\rangle_1|1_x\rangle_2 + |1_y\rangle_1|1_y\rangle_2)$, where $|1_x\rangle$ and $|1_y\rangle$ denote the mutually orthogonal photon states in the sense that if the beam splitter were removed a response to an incoming photon in state $|1_x\rangle$ would be a *"click"* at the detector D1 and no *"click"* at the detector $\mathrm{D1}^{\perp}$ provided the birefringent polarizer P1 is oriented along x. We shall use the second quantization formalism following closely the results obtained in Pavičić [6,8] where we described the action of the beam splitters, polarizers, and detectors on photons by means of outgoing electric field operators which acted on $|\Psi\rangle$ reducing it to a Fock vacuum state. We find that photons always emerge from the opposite sides of the beam splitter and never from the same sides of it. The photons emerge in a non–maximal singlet state.

To describe such an action of the polarizers and detectors ($D1_{\omega_1}$ and $D2_{\omega_2}$) we start with the following electric field operators (cf. Eqs. (7–10) of [6])

$$\hat{E}_1 = (\hat{a}_{1x} t_x \cos\theta_1 + \hat{a}_{1y} t_y \sin\theta_1)\, e^{-i\omega_1(t-\tau_1)} + i\,(\hat{a}_{2x} r_x \cos\theta_1 + \hat{a}_{2y} r_y \sin\theta_1)\, e^{-i\omega_1(t-\tau_1+\delta\tau)}\,, \quad (2)$$

$$\hat{E}_2 = (\hat{a}_{2x} t_x \cos\theta_2 + \hat{a}_{2y} t_y \sin\theta_2)\, e^{-i\omega_2(t-\tau_1)} + i\,(\hat{a}_{1x} r_x \cos\theta_2 + \hat{a}_{1y} r_y \sin\theta_2)\, e^{-i\omega_2(t-\tau_1-\delta\tau)}\,, \quad (3)$$

where the annihilation operators $\hat{a}$ describe the action of detectors and on the photon states they act as follows: $\hat{a}_{1x}|1_x\rangle_1 = |0_x\rangle_1$, $\hat{a}^\dagger_{1x}|0_x\rangle_1 = |1_x\rangle_1$, $\hat{a}_{1x}|0_x\rangle_1 = 0$, etc., where $\delta\tau$ corresponds to possible small displacements $\pm c\delta\tau$ of the beam splitter BS towards D1 or D2 detectors, and where τ_1 is the propagation time between the beam splitter and detectors (equal paths between the middle of the crystal and the beam splitter for all photons are assumed so as to assure overlapping of photons at the beam splitter).

In a realistic experiment photons however cannot be taken as monochromatic and, following Hong *et al.* [16], Larchuk *et al.* [12], and Pavičić and Summhammer [5], we assume them Gaussian–distributed around ω_1 and ω_2:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \int_0^\infty \int_0^\infty \zeta(\omega_1,\omega_2)(|1_x\rangle_{\omega_1}|1_x\rangle_{\omega_2} + |1_y\rangle_{\omega_1}|1_y\rangle_{\omega_2}) d\omega_1 d\omega_2\,, \quad (4)$$

where $\zeta(\omega_1,\omega_2)$ is a weight function peaked at ω_1 and ω_2 with rms widths σ. In order to take into account frequency responses by the polarizers which we also assume to be Gaussian shaped we make a Fourier decomposition of the electric field operators (2) and (3). We finally time–integrate the mean value of the electric field operators and obtain the probability of joint two–photon detection by detectors $D1_{\omega_1}$ and $D2_{\omega_2}$ together with $D1_{\omega_2}$ and $D2_{\omega_1}$:

$$P(\theta_1,\theta_2) = \langle\Psi|\hat{E}_2^\dagger \hat{E}_1^\dagger \hat{E}_1 \hat{E}_2|\Psi\rangle = \eta^2\,\{A^2 + B^2 - 2ABe^{-\sigma^2(\delta\tau)^2/2}\cos[(\omega_1-\omega_2)\delta\tau]\} \quad (5)$$

where $A = \frac{1}{\sqrt{2}}(t_x^2\cos\theta_1\cos\theta_2 + t_y^2\sin\theta_1\sin\theta_2)$, $B = \frac{1}{\sqrt{2}}(r_x^2\cos\theta_1\cos\theta_2 + r_y^2\sin\theta_1\sin\theta_2)$, and η is an overall detection efficiency—constant characteristic of detectors and polarizers—which for the experiments carried out so far was at most 0.8. [13] (Since efficiencies of detectors and prisms for different frequencies do not differ much, we assume that all η's are the same and therefore we put η^2 in Eq.(5) instead of $\eta_1\eta_2$, following Clauser and Shimony. [3]) Assuming positioning of BS so as to have $\delta\tau = 0$, the probability reads

$$P(\theta_1, \theta_2) = \eta^2 s^2 (\cos\theta_1 \cos\theta_2 + r \sin\theta_1 \sin\theta_2)^2 \,, \tag{6}$$

where $s = (t_x^2 - r_x^2)/\sqrt{2}$ and $r = (t_y^2 - r_y^2)/(t_x^2 - r_x^2)$. In the following, we shall limit the values of r to the interval [0,1] because possible negative values of r do not change the obtained results.

On the other hand, the probability of joint detection of both photons on the same sides of the beam splitter is (provided $\sigma \to 0$):

$$P(\theta_1 \times \theta_2) = \eta^2 (t_x r_x \cos\theta_1 \cos\theta_2 + t_y r_y \sin\theta_1 \sin\theta_2)^2 \{1 + \cos[(\omega_1 - \omega_2)\delta\tau]\} \,. \tag{7}$$

We see that for $\cos[(\omega_1 - \omega_2)\delta\tau] = -1$ photons never emerge from the same side of the beam splitter but for such a position of the beam splitter the probability (5) reduces—because of $t_x^2 + r_x^2 = t_y^2 + r_y^2 = 1$—to: $\eta^2 \cos^2(\theta_1 - \theta_2)/2$ what requires 83% detection efficiency. Besides, the assumption $\sigma \to 0$ is not very realistic. It is interesting, however, that the obtained maximal singlet state does not depend on the values of the transmission and reflection coefficients of the beam splitter. In other words, for $\cos[(\omega_1 - \omega_2)\delta\tau] = -1$ the beam splitter is *"non–existing"* while for $\cos[(\omega_1 - \omega_2)\delta\tau] = 1$ it forces photons to behave completely non–classically.

Let us now analyze the following Bell (Clauser–Horne) inequality:

$$f(\theta_1, \theta_2) - f(\theta_1, \theta_2') + f(\theta_1', \theta_2') + f(\theta_1', \theta_2) \le f(\theta_1') + f(\theta_2) \,, \tag{8}$$

where $f(\theta_1, \theta_2)$ given by Eq. (1) approaches $P(\theta_1, \theta_2)$ and $f(\theta_1')$ approaches $P(\theta_1')$ which is given as

$$P(\theta_1') = P(\theta_1', \infty) = \eta\, s^2 (\cos\theta_1'^2 + r^2 \sin\theta_1'^2) \,, \tag{9}$$

where $P(\theta'_1, \infty)$ for $\eta = 1$ describes ideal coincidence detection with "removed" polarizer P2 (= *ordinary* and *extraordinary* beam together). Thus $f(\theta'_1)$ approaches the probability of one photon being detected by D1 and the other entering either D2 or D2$^\perp$ but without necessarily being detected by them due to their inefficiency. However, D1 (as well as D2) also detects counts belonging to photons emerging from the same side of the beam splitter whose counter–counts were not detected by the detectors from the corresponding side again due to their inefficiency. The corresponding counts for both D1 and D2 can be obtained from Eq. (7) as $2(1-\eta)\eta(t_x^2 r_x^2 \cos^2\theta'_1 + t_y^2 r_y^2 \sin^2\theta_2)N$, where N is given by the denominator of Eq. (1). When we therefore subtract that many counts from the total number of singles counts registered by the computer: $n(\mathrm{D1}_{\omega_1} \cup \mathrm{D2}_{\omega_2})/\mathrm{N}$ we get $f(\theta'_1) + f(\theta_2)$ which approximates $P(\theta'_1) + P(\theta_2)$ given by Eq. (9). Thus we obtain that Eq. (8) approaches the following inequality:

$$B \equiv \frac{1}{\eta s^2}[P_{12}(\theta_1,\theta_2) - P_{12}(\theta_1,\theta'_2) + P_{12}(\theta'_1,\theta'_2) + P_{12}(\theta'_1,\theta_2) - P(\theta'_1) - P(\theta_2)] \le 0\,. \qquad (10)$$

We achieve the greatest violation of the Bell inequality when we let $r \to 0$. In order to show this, let us look at Fig. 2 which shows $Max[B](r,\eta)$ surface as obtained by a computer program for appropriate optimal angles. The values above the $B = 0$ plane means violations of the Bell inequality. For $r = 1$ we obtain $Max[B] = 0$ for $\eta = 0.828427$ in accordance with the result of Garg and Mermin. [17] For $r \to 0$ however, we obtain a violation of the Bell inequality already for any efficiency greater then 67%. This is because of the special shape of the $Max[B]$ surface as shown in Fig. 3.

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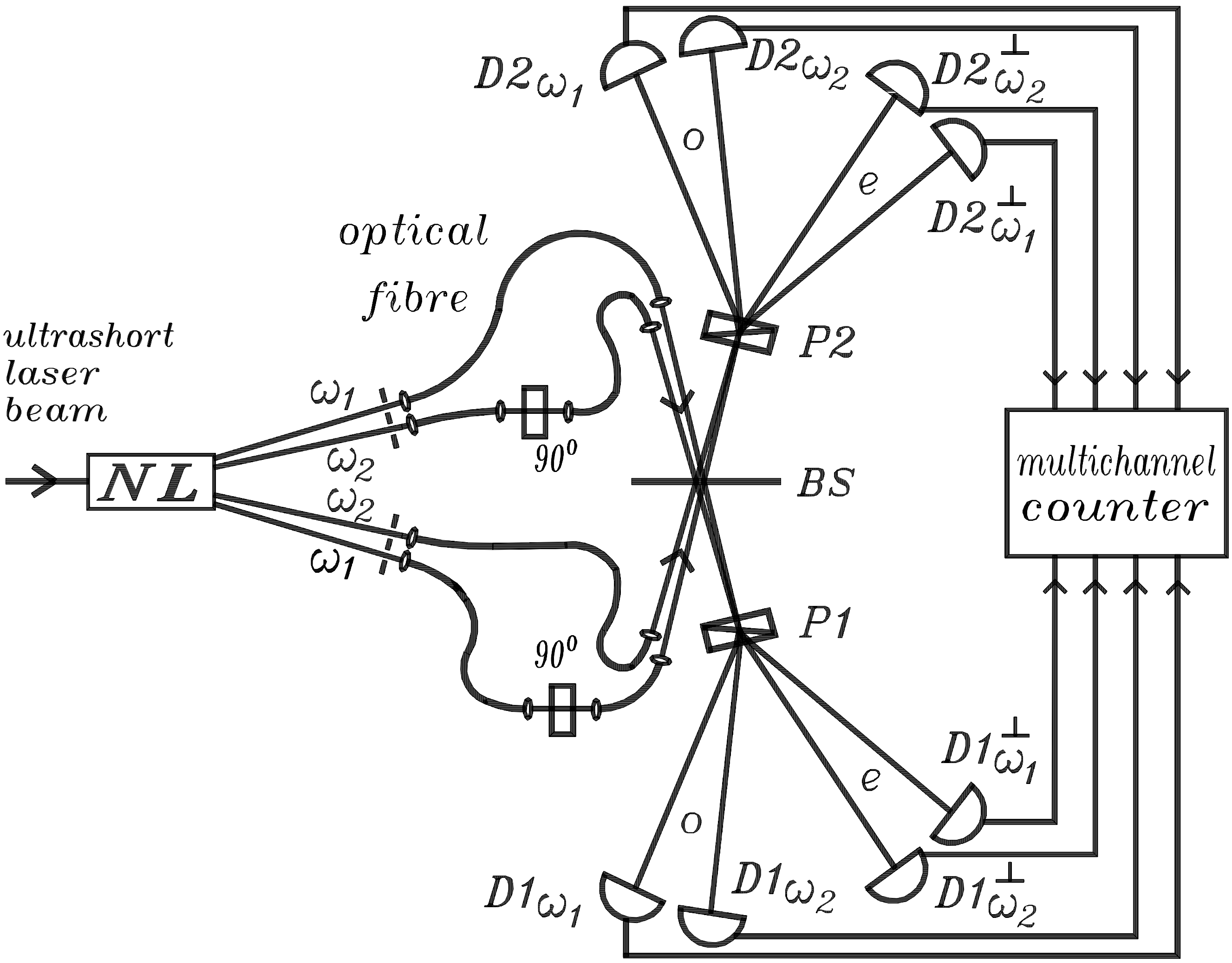
FIGURES

FIG. 1. Outline of the experiment. Single and coincidence counts are collected simultaneously by appropriate computer gates coupled with the laser beam timing. Wollaston prisms P1,P2 are at the same time birefringent polarizers and frequency selectors. The angles between ordinary (as well as extraordinary) beams of different colours (ω_1 and ω_2) are exaggerated so as to avoid graphical presentation of additional prisms and wedge shaped mirrors which would conclusively separate them. The pinholes determining the frequencies (ω_1 and ω_2) coming to the beam splitter BS are positioned as far away from the crystal as possible. The setup is completely symmetrical so that all paths from the middle of the crystal to the detectors have the same *time–of–flight*.

FIG. 2. The surface showing maximal violation of the Bell inequality for the optimal angles of the polarizers. All the values above the B=0 plane violate it.

FIG. 3. The plot shows η's as obtained for $B = 0$ from Eq. (10).



$D2_{\omega_1}$
$D2_{\omega_2}$
$D2^{\perp}_{\omega_2}$
$D2^{\perp}_{\omega_1}$
o
e
optical
fibre
ultrashort
laser
beam
ω_1
ω_2
NL
ω_2
ω_1
90^o
90^o
P2
BS
P1
multichannel
counter
o
e
$D1^{\perp}_{\omega_1}$
$D1_{\omega_1}$
$D1_{\omega_2}$
$D1^{\perp}_{\omega_2}$

Fig. 1

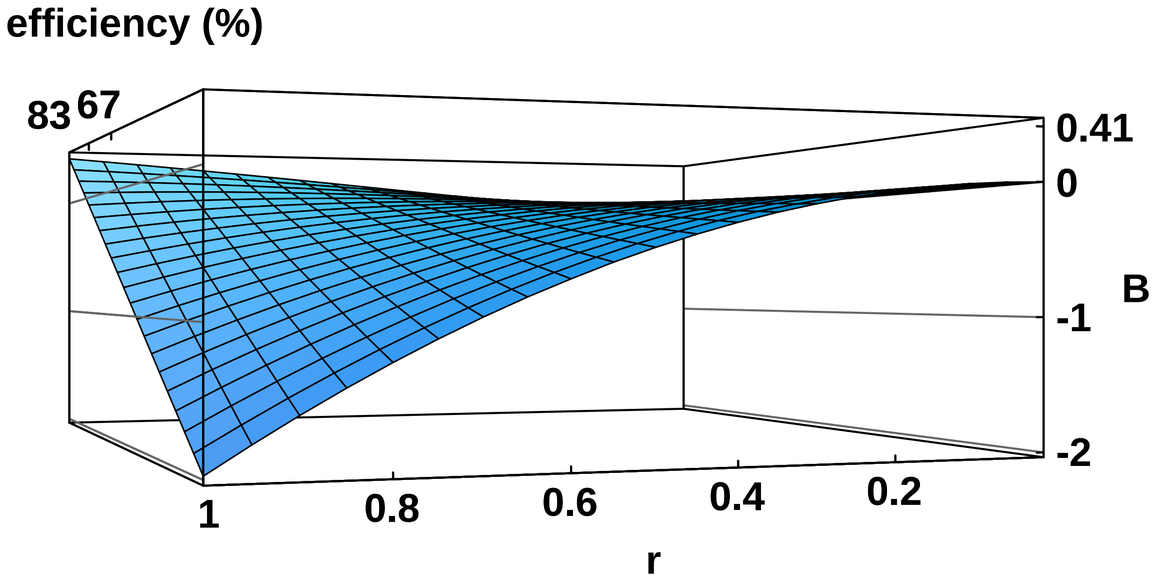


Fig. 2

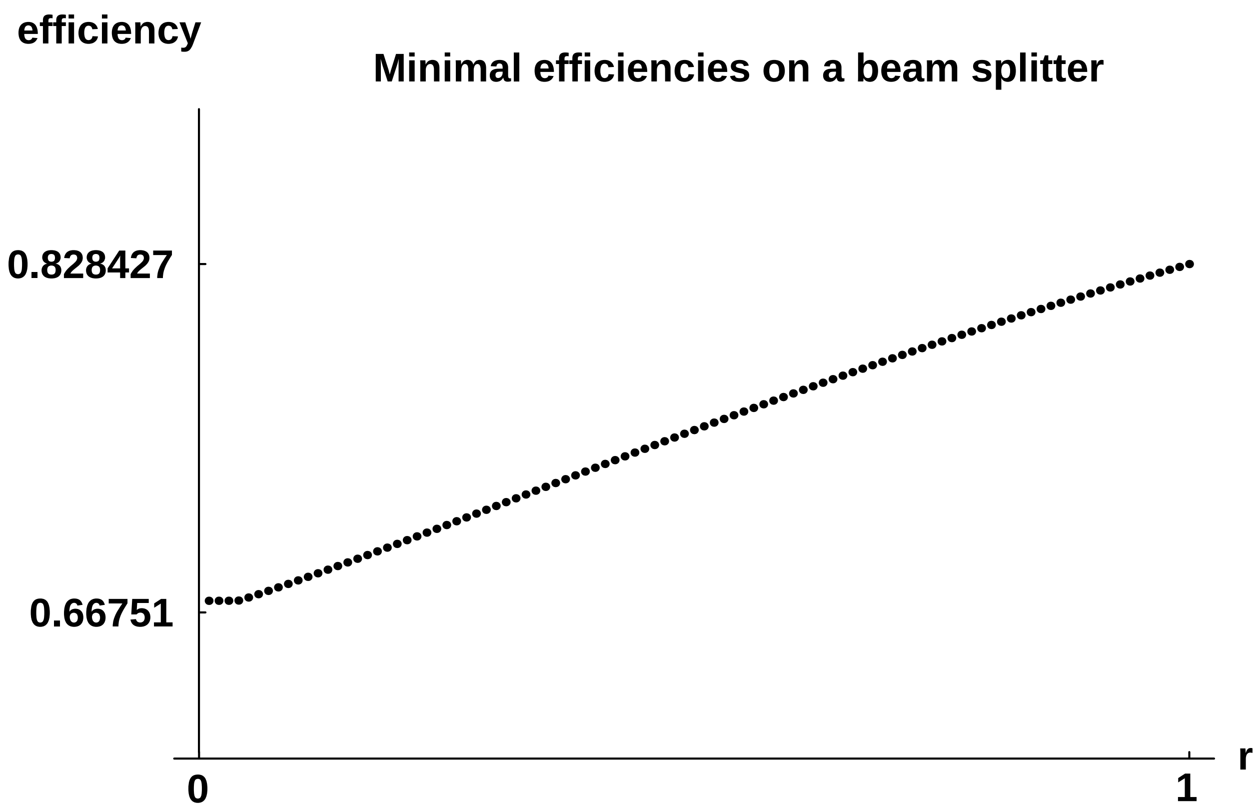

efficiency
Minimal efficiencies on a beam splitter
0.828427
0.66751
r
0
1

Fig. 3