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Spin Correlated Interferometry on Beam Splitters: Preselection of Spin Correlated Photons

Mladen Pavičić

Institut für Theoretische Physik, TU Berlin, Hardenbergstraße 36, D–10623 Berlin 12, Germany
Atominstitut der Österreichischen Universitäten, Schüttelstraße 115, A–1020 Wien, Austria
and Department of Mathematics, University of Zagreb, GF, Kačićeva 26, HR–41001 Zagreb, Croatia[*]

Abstract

A nonclassical feature of the fourth–order interference at a beam splitter, that genuine photon spin singlets are emitted in predetermined directions even when incident photons are unpolarized, has been used in a proposal for an experiment that imposes quantum spin correlation on truly independent photons. In the experiment two photons from two such singlets interfere at a beam splitter, and as a result the other two photons—which nowhere interacted and whose paths nowhere crossed—exhibit a 100% correlation in polarization, even when no polarization has been measured in the first two photons. The propsed experiment permits closure of the remaining loopholes in the Bell theorem proof, reveals the quantum nonlocality as a property of selection, and pioneers an experimental procedure for exact preparation of unequal superposition.

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1. INTRODUCTION

The fourth order interference of photons has been given a growing attention in the literature in the last few years mostly because it provided several rather unexpected results which differ from the classical intensity interference counterparts.[1–23] Let us mention the down–converted induced coherence,[18] non–dependence of the interference on the relative intensity of the incoming beams,[12] a disproval[1,15] of Dirac's dictum: *Interference between two different photons never occurs*,[24] interference of photons of different colors,[5] entaglement of photons which did not in any way directly interact whith each other in the configuration space[20] and in the spin space,[21,23] as well as particularly successful testing of both local[7,9,13] and nonlocal[17,18] hidden variable theories.

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[*] Permanent address. E-mail: mpavicic@faust.irb.hr; Web-page: http://m3k.grad.hr/pavicic

In this paper we close the *no enhancement* and *low efficiency* loopholes in the Bell theorem proof, show that quantum nonlocality is essentially a property of selection, and establish a procedure for recording unequal superpositions without loss of detection counts. In accomplishing these objectives we rely on the spin features of the interferencethe fourth order on a beam splitter which we previously used for an entaglement of two photon pairs coming out from two cascade sources.[21,23] In the interference both, polarized and unpolarized incident photons emerge from two different sides of the beam splitter unpolarized but correlated. (Section 2) This enables us to devise an experiment in which two photons from two such singlet states interfere in the fourth order on a third beam splitter and as a result two other companion photons from each pair turn out entangled and correlated in polarization even when we do not measure polarization on the first two at all. In Section 3 we elaborate the theory of such an entaglement and in Section 4 we present the experiment in a realistic approach discussing the spatial visibility of the correlations.

Correlated photons come out from cascading atoms in all directions allowed by such a three–body process and by registering only those pairs which reach detectors we actually select a subset of all correlated photons for which one can raise doubts as to whether it properly represents the whole set. An affirmative assumption, known as the *no enhancement* assumption, has been been widely adopted since Clauser and Horne[25] first made it. Recently however, Santos[26] surfaced the problem and calculated that no experiment carried out on the photons born in a cascade process can confirm the assumption. As opposed to this situation photons coming from a beam splitter build spin (polarization) correlated pairs *only* in particular precisely determined directions but, on the other hand, it was believed that such set–ups force the experimentalists to discard more than 50% of the data because detectors cannot tell one photon from two and one has to rely only on coincidence counts. We, however, show that one can devise an experiment in which no data need to be discarded thus avoiding Santos' objection. We do this in Section 4 by providing a device for preselecting spin directed correlated photons among those photons which have not in any way directly interacted with each other. The experiment can close all the existing loopholes in disproofs of local hidden variable theories, including the *low effiency* one, thanks to preselection of photons and might provide a scheme for disproving nonlocal theories as well. In closing the *low effiency* lopphole we find out how to measure unequal superpositions exactly as presented at the end of Section 4.

2. SPIN INTERFEREOMETRY ON A BEAM SPLITTER

Our experimental set–up rests on the fact that under particular conditions the interference of the fourth order makes unpolarized and independent incident photons correlated in polarization (spin) and turns polarized incident photons into unpolarized ones. We recognized this property only recently[22] because although the interference of the fourth order in the configuration space has been elaborated in detail in the literature[1–5,7–18] it lacked a detailed elaboration and apparently a proper understanding in the spin space. One of the rare partial elaborations was provided by Ou, Hong, and Mandel for a special case of orthogonally polarized photons.[27] They clearly recognized that orthogonally polarized photons incoming to a symmetrically positioned beam splitter produce a *singlet–like* state at a beam splitter[2,7,8,27] and that parallelly polarized photons incoming to a symmetrically positioned beam splitter never appear on its opposite sides[28] but it does not seem to have been recognized that the polarization of incoming photons does not have any effect on the correlation in polarization of the outgoing photons and that it only affects the intensity of the photons emerging from the opposite sides of the beam splitter. In the following we carry out the spin elaboration of the fourth order interference on a beam splitter using some results obtained by Pavičić.[22]

Let two photons interfere on a beam splitter as shown in Fig. 1. First, we describe the interference of polarized and later on of unpolarized photons. The state of incoming polarized photons is given by the product of two prepared linear–polarization states:

$$|\Psi\rangle = (\cos\theta_{1_0}|1_x\rangle_{1_0} \;+\; \sin\theta_{1_0}|1_y\rangle_{1_0}) \otimes (\cos\theta_{2_0}|1_x\rangle_{2_0} \;+\; \sin\theta_{2_0}|1_y\rangle_{2_0}) \;, \tag{1}$$

where $|1_x\rangle$ and $|1_y\rangle$ denote the mutually orthogonal photon states. So, e.g., $|1_x\rangle_{1_0}$ means the upper incoming photon polarized in direction x. If the beam spliter were removed it would cause a *"click"* at the detector D1 and no *"click"* at the detector D1$^\perp$ provided the birefringent polarizer P1 is oriented along x. Here D1$^\perp$ means a detector counting photons coming out at the *other exit* of the birefringent prism P1. Angles θ_{1_0},θ_{2_0} are the angles along which incident photons are polarized with respect to a fixed direction.

We do not consider any interference of the second order because the signal and idler photons emerging from the nonlinear crystals which we use in our experiment in Sec. 3 have random phases relative to each other. Thus we are left with the interference of the fourth order, i.e., with two interacting photons described by two corresponding electric fields. To describe the appropriate interaction of photons with the beamspliter, polarizers, and detectors we make use of the second quantization formalism employed, e.g., by Paul,[1] Mandel, Ou, Hong, Zou, and Wang,[4,14,15] and Campos, Saleh, and Teich.[16]

We introduce polarization by means of the stationary electric field operator whose orthogonal components read (see Fig. 1)

$$\hat{E}_j(\mathbf{r}_j,t) \,=\, \hat{a}_j(\omega_j)e^{i\mathbf{k}_j\cdot\mathbf{r}_j - i\omega_j t}\,. \tag{2}$$

The anihilation operators describe joint actions of polarizers, beam splitter, and detectors. The operators act on the states as follows: $\hat{a}_{1x}|1_x\rangle_1 \,=\, |0_x\rangle_1$, $\hat{a}^\dagger_{1x}|0_x\rangle_1 \,=\, |1_x\rangle_1$, $\hat{a}_{1x}|0_x\rangle_1 \,=\, 0$, etc. Thus, the action of the polarizers P1,P2 and detectors D1,D2 can be expressed as:

$$\hat{a}_i \,=\, \hat{a}_{ix\,out}\cos\theta_i + \hat{a}_{iy\,out}\sin\theta_i\,, \tag{3}$$

where $i = 1, 2$.

The operators corresponding to the other choices of detectors we obtain accordingly. E.g., the action of the polarizer P2 and the corresponding detector D2$^\perp$ (as shown in Fig. 2) is described by

$$\hat{a}_2 \,=\, -\hat{a}_{2x\,out}\sin\theta_2 + \hat{a}_{2y\,out}\cos\theta_2\,. \tag{4}$$

The outgoing electric–field operators describing photons which pass through beam splitter BS and through polarizers P1 and P2 and are detected by detectors D1 and D2 will thus read

$$\hat{E}_1 = \left(\hat{a}_{1x}t_x\cos\theta_1 + \hat{a}_{1y}t_y\sin\theta_1\right)e^{i\mathbf{k}_1\cdot\mathbf{r}_1 - i\omega_1 t_1} + i\left(\hat{a}_{2x}r_x\cos\theta_1 + \hat{a}_{2y}r_y\sin\theta_1\right)e^{i\tilde{\mathbf{k}}_2\cdot\mathbf{r}_1 - i\omega_2 t_1}\,, \tag{5}$$

$$\hat{E}_2 = (\hat{a}_{2x} t_x \cos\theta_2 + \hat{a}_{2y} t_y \sin\theta_2)\, e^{i\mathbf{k}_2\cdot\mathbf{r}_2 - i\omega_2 t_2} + i\,(\hat{a}_{1x} r_x \cos\theta_2 + \hat{a}_{1y} r_y \sin\theta_2)\, e^{i\tilde{\mathbf{k}}_1\cdot\mathbf{r}_2 - i\omega_1 t_2}\,, \qquad (6)$$

where i assures the phase shift during the reflection on the beam splitter, t_j is the time of detection of a photon by detector D_j, ω_j is the frequency of photon j, c is the velocity of light. Here the crystal as a supposed source of (idler and signal down–converted) photons is assumed to be positioned symmetrically to the beam splitter (with respect to the photon paths from the center of the crystal to the beam splitter). This is just the opposite to the elaboration carried in Pavičić[22] where detectors were assumed to be positioned symmetrically to the beam splitter while time delays for the sources were introduced in order to describe photons born in atomic cascade processes used in Pavičić and Summhammer.[23]

The joint interaction of both photons with the beam splitter, polarizers P1,P2, and detectors D1,D2 is given by a projection of our wave function onto the Fock vacuum space by means of $\hat{E}_1$,$\hat{E}_2$ wherefrom we get the following probability of detecting photons by D1,D2:[22]

$$P(\theta_{1_0}, \theta_{2_0}, \theta_1, \theta_2) = \langle\Psi|\hat{E}_2^\dagger \hat{E}_1^\dagger \hat{E}_1 \hat{E}_2|\Psi\rangle = A^2 + B^2 - 2AB\cos\phi\,, \qquad (7)$$

where $|\Psi\rangle$ is given by Eq. (1) and where

$$\phi = (\tilde{\boldsymbol{k}}_2 - \boldsymbol{k}_1)\cdot\boldsymbol{r}_1 + (\tilde{\boldsymbol{k}}_1 - \boldsymbol{k}_2)\cdot\boldsymbol{r}_2 + (\omega_1 - \omega_2)(t_1 - t_2)\,, \qquad (8)$$

$A = S_{1'1}(t)S_{2'2}(t)$ and $B = S_{1'2}(r)S_{2'1}(r)$, where

$$S_{ij} = s_x \cos\theta_i \cos\theta_j + s_y \sin\theta_i \sin\theta_j\,. \qquad (9)$$

Assuming $\omega_1 = \omega_2$ we obtain (see Fig. 1) $\phi = 2\pi(z_2 - z_1)/L$, where L is the spacing of the intereference fringes.[2]

For $t_x = t_y = r_x = r_y = 2^{-1/2}$ and $\cos\phi = 1$ (we can modify ϕ by moving the detectors transversely to the incident beams) the probability reads

$$P(\theta_{1_0}, \theta_{2_0}, \theta_1, \theta_2) = (A - B)^2 = \frac{1}{4}\sin^2(\theta_{1_0} - \theta_{2_0})\sin^2(\theta_1 - \theta_2)\,, \qquad (10)$$

which for removed polarizers makes

$$P(\theta_{1_0}, \theta_{2_0}, \infty, \infty) = \frac{1}{2}\sin^2(\theta_{1_0} - \theta_{2_0})\,. \qquad (11)$$

We see that the probability in Eq. (10) factorizes (see Fig. 1) left–right (corresponding to 1_0–2_0–preparation $\leftrightarrow$ D1–D2–detections) and not up–down (corresponding to ${}^{1_0}_{2_0}\updownarrow$ preparation) in spite of the up–down initial independence described by the product of the upper and lower function in Eq. (1). We also see that by changing the relative angle between the polarization planes of the incoming photons we only change the light intensity of the photons emerging from the beam splitter at particular sides. Thus the photons either emerge on two different sides of the beam splitter correlated according to Eq. (10) or both emerge on one side according (when we do not measure their outgoing polarization) to the following overall probability

$$P(\theta_{1_0}, \theta_{2_0}, \infty \times \infty) = \frac{1}{2}[1 + \cos^2(\theta_{1_0} - \theta_{2_0})]\,, \qquad (12)$$

which together with Eq. (11) adds up to one.

We also see that the photon beams leave the beam splitter unpolarized:

$$P(\theta_{1_0},\theta_{2_0},\theta_1,\infty)=\frac{1}{4}\sin^2(\theta_{1_0}-\theta_{2_0})\,. \tag{13}$$

If both incoming photons come in unpolarized — coming, e.g., from two simultaneously cascading independent atoms or better from two other beam splitters what as a possibility directly follows from just obtained Eq. (13) — they appear[22] correlated whenever they appear at the opposite sides of the beam splitter:

$$P(\infty,\infty,\theta_1,\theta_2)=\frac{1}{2}\sin^2(\theta_1-\theta_2) \tag{14}$$

and partially correlated whenever they both emerge from one side of the beam splitter:

$$P(\infty,\infty,\theta_1,\theta_2)=\frac{1}{2}[1+\cos^2(\theta_1-\theta_2)]\,. \tag{15}$$

The latter probability can be checked experimentally with the help of an additional beam splitter in each arm following Rarity and Tapster[29] or by means of photons of different colors which one can distinguish using frequency filters (prisms).[5,30,31]

In case of nondegenerate idler and signal down–converted photons (by means of asymmetrically positioned pinholes), i.e. in case of photons of different colors we should, according to Eq. (8), obtain a space–time combination of space–like intensity interference and time–like frequency–difference beating. The latter effect, however, cannot be measured together with observing the intensity interference fringes because the fast photon beating would wipe out the spatial fringes. For observation of the beating itself one uses the optical path–lenght difference method by which the coincidences are recorded.[5,30] So, in our notation we simply drop the dot products in Eq. (8) and then the method consists in moving the beam splitter up or down in order to obtain the optical path–lenght difference $\delta=c|t_1-t_2|$ and thus have $|\phi|=|\omega_1-\omega_2|\delta/c$. In this way one can register beating corresponding to 30 fs by means of detectors and counters whose resolving time is 10 ns.[5] Our main coincidence probability for particular polarization measurements given by Eq. (7) remains the same for the beating between photons of different frequencies as it was for the degenerate idler and signal photons. The fact that we can trace the path of each photon is here not contradictory because, first, we deal not with the beam intensity but with the intensity correlation, and, secondly, as we already stressed, polarization *preparation* of photons is *erased* by the beam splitter anyhow.

The most important consequence of the obtained equations for our experiment is that the photons appear entangled in a singlet state whenever they appear on different sides of the beam splitter provided the condition $\phi=0$ is satisfied no matter whether incident photons were polarized or not. For, Eqs. (10) and (14) tells us that the probability of *such* photons passing parallel polarizers is equal to zero.

3. THEORY OF THE ENTAGLEMENT IN THE EXPERIMENT

A schematic representation of the experiment is shown in Fig. 2. Two independent beam splitters BS1 and BS2 act as two independent sources of two independent singlet pairs which is enabled by Eq. (10) as elaborated in the previous section. Two photons from each pair interfere on the beam splitter BS and as a result the other two photons, under particular conditions elaborated below, appear to be in the singlet state although the latter photons are completely independent and nowhere interacted.

An ultrashort[32] laser beam (a subpicosecond one) of frequency ω_0 simultaneously (split by a beam splitter) pumps up two nonlinear crystals NL1 and NL2 producing in each of them pairs of signal and idler photons (simulataneously and with equal probability) of frequencies ω_1 and ω_2, respectively, which satisfy the following energy and momentum conservation conditions: $\omega_0 = \omega_1 + \omega_2$ and $\boldsymbol{k}_0 = \boldsymbol{k}_1 + \boldsymbol{k}_2$.[33] By means of the appropriately symmetrically positioned pinholes we select half–frequency sidebands so as to have $\omega_2 = \omega_1$. The idler and signal photon pairs coming out from the crystals do not have definite phases[28,34] with respect to each other and consequently one can have a second order interference neither on BS1 nor on BS2. In order to prevent any coherence which might be induced by the split pumping beam between the idler (or signal) photon from the first crystal and the idler (or signal) photon from the second crystal we introduce a phase modulator (which rotates to and fro at random and destroy the second order phase coherence) following Ou, Gage, Magill, and Mandel.[6] (We do take a correction term corresponding to the modulator into account when estimating the visibility below, but do not show it in the equations for the sake of their simplicity.)

Thus, two *sources* BS1 and BS2 both simultaneously emit two photons in the singlet states given by Eq. (10) to the left and to the right. But before we put beam splitters BS1 and BS2 in place we first have to adjust detectors the beam splitter and D1–D2$^\perp$ so as to obtain $\phi = 0$. After that we take out BS and put in BS1 and BS2 to adjust them and detectors D1'–D2'$^\perp$ (while leaving detectors D1–D2$^\perp$ fixed) so as to obtain pure singlet states coming out from BS1 and BS2. It follows from Eq. (10) and Fig. 2 that we can do this for $\phi = 0$ by reaching the minimum of coincidences (ideally the minimum should be zero) for $\theta_{1'} = \theta_1$ for BS1 and for $\theta_{2'} = \theta_2$ for BS2. It is interesting that this step of tuning BS1,BS2 and D1'–D2'$^\perp$ is not crucial, because the four photon entanglement is not dependent on the positions of D1'–D2'$^\perp$ detectors in directions perpendicular to the photon paths, i.e., according to Eq. (21) there are no interference fringes for photons 1' and 2' — only for photons 1 and 2. Then we put beam splitter BS in place and four photons form elementary quadruples of counts which add up to the below calculated probabilities in the long run. The quadruple recording is obtained by the following *preselection* procedure: whenever exactly two of the *preselection detectors* D1–D2$^\perp$ fire in coincidence (see Fig. 3) a gate for counters D1'–D2'$^\perp$ opens. In case only one or none of the so preselected D1'–D2'$^\perp$ detectors fires we discard the records (because they correspond to four or three photons detected by D1–D2$^\perp$, respectively). In case exactly two of four D1'–D2'$^\perp$ detectors fire, the corresponding counts contribute to our statistics. The possibility of two photons going into one arm of the beam splitter as well as the possibility that a detector fails to react because of its inefficiency we discuss in Sec. 4.

The state of the four photons immediately after leaving BS1 and BS2 from their oppposite sides is described by the product of the two superpositions corresponding to singlet pairs produced — according to Eq. (10) — on BS1 and BS2, respectively:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|1_x\rangle_{1'}|1_y\rangle_1 \; - \; |1_y\rangle_{1'}|1_x\rangle_1) \otimes \frac{1}{\sqrt{2}}(|1_x\rangle_{2'}|1_y\rangle_2 \; - \; |1_y\rangle_{2'}|1_x\rangle_2)\,, \tag{16}$$

where $|1_x\rangle$ and $|1_y\rangle$ denote the mutually orthogonal photon states.

The annihilation of photons at detectors D1',D2' after passing the polarizers P1',P2' (oriented at angles $\theta_{1'},\theta_{2'}$) are described by the following electric field operators

$$\hat{E}_{1'} = (\hat{a}_{1'x}\cos\theta_{1'} + \hat{a}_{1'y}\sin\theta_{1'})e^{-i\omega'_1 t_{1'}}\,, \tag{17}$$

$$\hat{E}_{2'} = (\hat{a}_{2'x}\cos\theta_{2'} + \hat{a}_{2'y}\sin\theta_{2'})e^{-i\omega'_2 t_{2'}}\,. \tag{18}$$

Here, phases of the photons which accumulate between beam splitters BS1,BS2 and detectors D1',D2' add the factors $e^{-i\omega_j t_j}$, where t_j is the time of detection of a photon by detector D$_j$' and ω_j is the frequency of the photon. [Until Eq. (21) we shall consider the frequencies of photons different for the sake of generality.]

The electric outgoing field operators describing photons which pass through beam splitter BS, polarizers P1,P2 and detectors D1,D2 are given by Eqs. (5) and (6).

The joint interaction of all four photons with the beam splitter, polarizers P1–P2', and detectors D1–D2'$^\perp$ is given by the following projection of our initial state given by Eq. (16) wave function onto the Fock vacuum space:

$$\hat{E}_{1'}\hat{E}_{2'}\hat{E}_1\hat{E}_2|\Psi\rangle = \frac{1}{2}(A\varepsilon_{12} - B\tilde{\varepsilon}_{12})\varepsilon|0\rangle\,, \tag{19}$$

where $|\Psi\rangle$ is given by Eq. (16), where $\varepsilon_{12} = \exp[i\,(\boldsymbol{k}_1\cdot\boldsymbol{r}_1 + \boldsymbol{k}_2\cdot\boldsymbol{r}_2 - \omega_1 t_1 - \omega_2 t_2)]$, $\tilde{\varepsilon}_{12} = \exp\left[i\left(\tilde{\boldsymbol{k}}_1\cdot\boldsymbol{r}_2 + \tilde{\boldsymbol{k}}_2\cdot\boldsymbol{r}_1 - \omega_1 t_2 - \omega_2 t_1\right)\right]$, $\varepsilon = \exp\left[-i\,(\omega'_1 t_{1'} + \omega'_2 t_{2'})\right]$, and $A = Q(t)_{1'1}Q(t)_{2'2}$ and $B = Q(r)_{1'2}Q(r)_{2'1}$, where

$$Q(q)_{ij} = q_x\sin\theta_i\cos\theta_j - q_y\cos\theta_i\sin\theta_j\,. \tag{20}$$

The corresponding probability of detecting all four photons by detectors D1–D2'$^\perp$ is thus

$$P(\theta_{1'},\theta_{2'},\theta_1,\theta_2) = \langle\Psi|\hat{E}^\dagger_{2'}\hat{E}^\dagger_{1'}\hat{E}^\dagger_2\hat{E}^\dagger_1\hat{E}_1\hat{E}_2\hat{E}_{1'}\hat{E}_{2'}|\Psi\rangle = \frac{1}{4}(A^2 + B^2 - 2AB\cos\phi)\,, \tag{21}$$

where

$$\phi = (\tilde{\mathbf{k}}_2 - \mathbf{k}_1)\cdot\mathbf{r}_1 + (\tilde{\mathbf{k}}_1 - \mathbf{k}_2)\cdot\mathbf{r}_2 + (\omega_1 - \omega_2)(t_1 - t_2)\,. \tag{22}$$

For $\omega_1 = \omega_2 = \omega'_1 = \omega'_2$ we obtain (see Fig. 1 which applies on BS from Fig. 2 as well) $\phi = 2\pi(z_2 - z_1)/L$, where L is the spacing of the intereference fringes. ϕ can be changed by moving the detectors transversely to the incident beams.

To make Eq. (21) more transparent, without loss of generality, we here consider 50:50 beam splitter: $t_x = t_y = r_x = r_y = 2^{-1/2}$. In Sec. 4 we also consider a polarized beam splitter.

For $\phi = 0$ the above probability reads

$$P(\theta_{1'},\theta_{2'},\theta_1,\theta_2) = \frac{1}{4}(A - B)^2 = \frac{1}{16}\sin^2(\theta_{1'} - \theta_{2'})\sin^2(\theta_1 - \theta_2)\,. \tag{23}$$

We again see that the probability factorizes left–right (corresponding to D1'–D2' $\leftrightarrow$ D1–D2 detections — see Fig. 2) and not up–down (corresponding to $\substack{\text{BS1}\\\text{BS2}}\updownarrow$ preparation) as one would be tempted to conjecture from the product of the upper and lower function in Eq. (16). For removed polarizers P1,P2 Eq. (23) gives:

$$P(\theta_{1'},\theta_{2'},\infty,\infty)=\frac{1}{8}\sin^2(\theta_{1'}-\theta_{2'})\,. \tag{24}$$

The overall probability of detecting both photons in one arm of BS is given by:

$$P(\theta_{1'},\theta_{2'},\theta_1\times\theta_2)=\frac{1}{16}[\cos(\theta_{1'}-\theta_1)\cos(\theta_{2'}-\theta_2)+\cos(\theta_{1'}-\theta_2)\cos(\theta_{2'}-\theta_1)]^2. \tag{25}$$

which for removed polarizers reads:

$$P(\theta_{1'},\theta_{2'},\infty\times\infty)=\frac{1}{8}[1+\cos^2(\theta_{1'}-\theta_{2'})]\,. \tag{26}$$

The latter probability one obtains so as to add up all the probabilities of detecting polarizations of each photon in one arm, i.e., $P(\theta_{1'},\theta_{2'},\theta_1\times\theta_2)$ [given by Eq. (25)], $P(\theta_{1'},\theta_{2'},\theta_1\times\theta_2^{\perp})$, etc. We see that the probabilities (24) and (26) add up to one.

The probability (23) shows that for $\phi=0$ by removing one of the polarizers we lose any left–right (Bell–like) spin correlation completely: $P(\theta_{1'},\infty,\theta_1,\theta_2)=\frac{1}{16}\sin^2(\theta_1-\theta_2)$. On the other hand, for $\phi\neq 0$ we obtain a partial left–right correlation even when two polarizers, one on each side, are removed.

4. THE EXPERIMENT AND THE BELL ISSUE

The main point of our experiment is that the correlation between photons 1' and 2', i.e., between photons which never interacted in the past, persists even when we do not measure polarization on their companions 1 and 2 at all as follows from Eq. (24). Therefore we shall concentrate on the experiment without polarizers P1,P2 behind beam splitter BS. To make our point we present the appropriate experimental set–up in a simplified and reduced scheme presented in Fig. 3. The set–up deals with four photons of the same frequency and relies on (computer) time windows for coincidence detections which compensate for the long responding time of the detectors. Afterwards we shall consider the experiment in a more realistic approach making use of polarizers P1,P2 as shown in Fig. 2.

In the idealized approach from Sec. 3. the probability of detecting all four photons by D1–D2'$^{\perp}$ in coincidence for 50:50 beam splitter for $\phi=0$ and with equal time delays (that is for a completely symmetrical position of BS) is given by Eq. (24) and the probability of detecting both photons in one of the arms by Eq. (26). We see that these two probabilitites add up to 1/4. (The other 3/4 correspond to *orthogonal detections* by D$^{\perp}$ detectors.) The former probability given by Eq. (24) and describing coincidence detections by D1' and D2' corresponds — when multiplied by 4 — to the following singlet state:

$$|\Psi_s\rangle=\frac{1}{\sqrt{2}}(|1_x\rangle_1|1_y\rangle_2-|1_y\rangle_1\,|1_x\rangle_2)\,. \tag{27}$$

Multiplication by 4 is for photons which emerge from the same side of BS and which therefore do not belong to our statistics. Analogously, the probability of coincidental detection by D1' and D2'$^{\perp}$ (which we will make use of later on), given by Sec. 4)

$$P(\theta_{1'}, \theta_{2'}^{\perp}, \infty, \infty) = \frac{1}{8}\cos^2(\theta_{1'} - \theta_{2'})\,. \tag{28}$$

corresponds to the following triplet–like state:

$$|\Psi_t\rangle = \frac{1}{\sqrt{2}}(|1_x\rangle_1|1_y\rangle_2 + |1_y\rangle_1\,|1_x\rangle_2)\,. \tag{29}$$

Thus, photons 1',2' belonging to quadruples containing photons 1,2, which appear at different sides of the beam splitter behave *quantum–like* showing — according to Eq. (24) — 100% *relative modulation.*[7] In other words, by detecting the right photons on different sides of the beam splitter we *preselect* the orthogonal individual left photons pairs (25% of all pairs) with probability one, while by detecting the right photons both on one side of the beam splitter we (*would* have — if it had been experimentally possible) preselect the parallel pairs (75% of all pairs) with probability 1/3. When we compare this result with the classical formulation of Pavičić[22] carried out by Paul and Wegmann[35] we see that the former case (photons emerging from different sides of the beam splitter) is "completely non–classical." This means that it is the nonclassical feature of the intensity correlations that enables our experiment.

Let us now dwell on the details of the experiment without polarizers P1,P2 behind beam splitter BS as shown in Fig. 3. A pair consisting of two photons 1' and 1 appears from BS1 simultaneously with another pair 2'–2 on BS2. Photons 1', 2', and 1 *and* 2 are directed towards detectors D1' *or* D1'$^{\perp}$, D2' *or* D2'$^{\perp}$, and D1 *and* D2, respectively. Of all detections registered by D1,D2 only those counts which occur within a short enough time windows (about 10ns) are fed to the *preselection coincidence counter*. Thanks to the ultrashort pumping beam (ω_0), which ensure an average appearance of down–converted pairs of photons ($\omega = \omega_0/2$ — coming out from the crystals and passing through symmetrically positioned pinholes) every 50 ns, we are able to effectively control coincidences each of which occurs (as a property of downconversion) well within our time windows. In this way we overcome the problem of having the detector reaction time longer than the fourth order correlation time and the coherence time. So, each pair of the pulses belongs to the two photons which interfered on BS so as to appear at the opposite sides of the beam splitter. (Realistically, as we will see below, it boils down to about 85%; A possibility of having detected 3 or 4 photons due to a possibility of both photons emerging from one side of BS1 or BS2 we resolve below.) Each D1–D2 time window is coupled (as calculated from the time–of–flight difference) with a computer gate for counts from detectors D1',D2',D1'$^{\perp}$,D2'$^{\perp}$. If D1,D2 counters do not register coincidence counts but a only a single count, then the "gated" D1',D2',D1'$^{\perp}$,D2'$^{\perp}$ recordings are discarded. If they do, we get potential data for our statistics what we call *Bell recording* in Fig. 3. Since we use birefringent polarizers we have to have a coincidence firing of exactly two of the counters D1',D2',D1'$^{\perp}$,D2'$^{\perp}$ in order to obtain definite data for the statistics. Firing of one or none of the counters as well as of three or all four discard the corresponding data because they do not belong to our set of quadruple events. $P(\theta_{1'}, \theta_{2'}, \infty\times\infty)$ of Eq. (24) is then given by the following ratio between the numbers of coincidence counts:

$$f(\theta_{1'}, \theta_{2'}) = \frac{n(\mathrm{D1}' \cap \mathrm{D2}')}{\mathrm{n}[(\mathrm{D1}' \cup \mathrm{D1}'^{\perp}) \cap (\mathrm{D2}' \cup \mathrm{D2}'^{\perp})]}\,. \tag{30}$$

divided by 4. Division by 4 compensates for the photons which emerged from the same side of BS and were therefore discarded from the statistics as not belonging to the considered set of events. Of course, we produce an error here because counters can remain inactive because of their inefficiency but we can always make use of Mach–Zehnder interferometers instead of BS1 and BS2 to avoid this problem. Their adventages would be, first, that we can adjust them so that photons almost always emerge from the oposite sides of their second beam splitters and almost never from the same sides and, secondly, that a detector resolution time which is much longer then the coherence time is not any more a problem (in contradistinction to a single beam splitter) — it is even required.[14,16] We did not use the interferometers here so as not to overcomplicate our presentation, but we will comment on them in some detail later on. Alternatively we can use photons of different frequencies for each pair and relying on their beating instead on the spatial fringes as explained at the end of Sec. 2.

The assumed 100% visibility above is of course an oversimplification since the measurement of probability (21) cannot be measured at a point (see Fig. 1) but only over a detector width Δz. Therefore, in order to obtain a more realistic probability following Ghosh and Mandel[3] we integrate Eq. (21) over z_1 and z_2 over Δz to obtain

$$\begin{aligned}\mathcal{P}(\theta_{1'},\theta_{2'},\theta_1,\theta_2) =& \frac{1}{4}\int_{z_1-\Delta z/2}^{z_1+\Delta z/2}\int_{z_2-\Delta z/2}^{z_2+\Delta z/2}\Big[A^2+B^2-2AB\cos[2\pi(z_2-z_1)/L]\Big]dz_1dz_2\\ =&\frac{1}{4}(A^2+B^2-v2AB\cos\phi)\,,\end{aligned}\tag{31}$$

where $v=[\sin(\pi\Delta z/L)/(\pi\Delta z/L)]^2$ is the *visibility* of the coincidence counting. Visibility of 95% has been estimated as achiveable in principle,[12] 80% and 87% has been reached recently.[36,37]

Thus, Eq. (24) corrected for a realistic visibility reads:

$$P(\theta_{1'},\theta_{2'},\infty,\infty)=\frac{1}{8}[1-v\cos^2(\theta_{1'}-\theta_{2'})]\,.\tag{32}$$

To see that our results really tighten all the remaining loopholes in disproving local hidden varable theories, let us in the end discuss the corresponding Bell's inequality:

$$S\equiv P(\theta_{1'},\theta_{2'})-P(\theta_{1'},\theta'_{2'})+P(\theta'_{1'},\theta'_{2'})+P(\theta'_{1'},\theta_{2'})-P(\theta'_{1'},\infty)-P(\infty,\theta_{2'})\le 0\,,\tag{33}$$

where $P(\theta_{1'},\theta_{2'})=4P(\theta_{1'},\theta_{2'},\infty,\infty)$, etc. The singlet states of photons 1' and 2' and the corresponding probabilities $\frac{1}{2}\sin^2(\theta_{1'}-\theta_{2'})$ correspond to the D1'–D2' coincidence counts preselected by D1–D2 coincidence counts. Since, ideally, no one of so preselected photons escapes detection we have thus satisfied Santos' demand.[26] To be more specific, $P(\theta_{1'},\theta_{2'})$ is not obtained as a coincidence counting rate like in the previous experiments[2,7,8] but as the ratio (frequency) $f(\theta_{1'},\theta_{2'})$ given by Eq. (30) where the total number of counts in the denominator can actually be recorded.

Ideally, for a violation of Bell's inequality, and hence for a possible exclusion of hidden variable theories, v must be[8] larger than $2^{-1/2}$. If we also take into account the overall efficiency of detectors η defined by $P(\theta_{1'},\theta_{2'})=\eta f(\theta_{1'},\theta_{2'})$ for the case of equal superposition given by Eq. (16) the inequality (33) can be violated only if[38,39]

$$\eta(1+v\sqrt{2})>2\,.\tag{34}$$

So, for the visibility $v = 1$ we must have $\eta > 83\%$. For the recently achieved visibilities $v = 0.8$[37] and $v = 0.87$[36] according to Eq. (34) this means $\eta > 0.94$ and $\eta > 0.9$ which is already anounced as achievable.[40,41] So, the experiment in the presented set–up is just about to be feasible. However, using our most recent result we can adjust it so as to be comfortably over this verge and conclusively feasible with the present technology. Let us elaborate this in some detail.

As "forerunners" of our singlet states *selected* among photons whose paths nowhere crossed in the space, several simpler set–ups involving only two fotons interfering on beam splitters were reported. In particular, Mach–Zehnder interferometer was recognized as a possible source of 100% correlated photons (i.e., without both photons emerging from the same sides of the second beam splitter).[14,16] Only, until our result[22] it was not recognized that these photons appear correlated in polarization and automatically satisfy Santos' demand up to the efficiency of detectors. But, after Kwiat, Eberhard, Steinberg, and Chiao[41] in the meantime carried out an explicit calculation for the single Mach–Zehnder interferometer they immediatelly addressed detector efficiency limitations and focussed a recent result by Eberhard[42] as a possible remedy. (It should be stressed here that in the light of dector efficiencies Hardy's[43] proposal cannot be considered as an answer to Santos' objection because today's visibility in his proposal is 30%.) Eberhard has shown that if one used unequal superpositions

$$|\Psi_r\rangle = \frac{1}{\sqrt{1+r^2}}(|1_x\rangle_1|1_y\rangle_2 + r|1_y\rangle_1\,|1_x\rangle_2) \tag{35}$$

instead of equal ones (given by r=1), then one would be able to lower the required efficiency of detectors down to 67%. The problem was how to achieve this. Eberhard himself connected the effect with the background noise and the drawback of this definition was, first, that one can hardly specify *the background* and, secondly, that one loses counts. We have however found the following way how to use Eberhard's result without any losses and without invoking any background noise.

From Eqs. (21) and (4) it follows that the probability of having coincidence counts by detectors D1' and D2'$^{\perp}$ after a selection by (see Fig. 2) detectors D1 and D2 with the orientation of polarizers $\theta_2 = 0$ and $\theta_1 = \pi/2$ and with $t_y = r_y = 2^{-1/2}$ is given by

$$P(\theta_{1'}, \theta_{2'}^{\perp}) = \frac{1}{1+r^2}(\cos\theta_{1'}\cos\theta_{2'} + r\sin\theta_{1'}\sin\theta_{2'})^2\,, \tag{36}$$

where $r = \frac{r_x}{t_x}$ and where we also take counts registered by D1'$^{\perp}$ and D2' into account in order to obtain the proper probability. Since it can easily be shown that the detected photons are in the state described by Eq. (35) we have thus recognized Eberhard's term r as the ratio between the reflection and transmission coefficient of the polarized beam splitter. So, for $r = 0.31$, i.e., for $T_x = t_x^2 = 0.91$, an efficiency greater than 70% suffices for a loophole free Bell's experiment depending on the visibility on the beam splitter. On the other hand, Eq. (36) establishes an experimental procedure for measuring unequal superposition without loss of detection counts since the probability $P(\theta_{1'}, \theta_{2'}^{\perp})$ can be obtained as the frequency

$$f(\theta_{1'}, \theta_{2'}^{\perp}) = \frac{n(\mathrm{D1'} \cap \mathrm{D2'}^{\perp})}{\mathrm{n}[(\mathrm{D1'} \cup \mathrm{D1'}^{\perp}) \cap (\mathrm{D2'} \cup \mathrm{D2'}^{\perp})]}\,. \tag{37}$$

where both $n(\mathrm{D1'}\cap\mathrm{D2'}^{\perp})$ and $n[(\mathrm{D1'}\cup\mathrm{D1'}^{\perp})\cap(\mathrm{D2'}\cup\mathrm{D2'}^{\perp})]$ can be recorded with equal accuracy.

This hopefully[44] closes all the remaining loophoes in the Bell's proof, and constitutes a most discriminating test of Bell's inequality.

5. CONCLUSION

The experiment we proposed is a realization of a polarization correlation between two independent and unpolarized photons. The experiment is based on a newly discovered nonclassical effect in the interference of the fourth order on a beam splitter according to which two unpolarized incident photons emerge from a beam splitter correlated in polarization as follows from Eqs. (7) and (14). The essential new element of the experiment is that it puts together two photons from two singlets formed on two beam splitters, makes them interfere on a third beam splitter, and as a result we find polarization correlations between the other two photons which nowhere interacted and whose paths nowhere crossed even when no polarization measurement have been carried out on the former two photons as follows from Eqs. (21) and (23). As for the latter two photons which nowhere interacted, one of their subsets turns out to contain only photons in the singlet state and since we are able to extract these photons with probability one one can consider them *preselected* by their pair–companion photons which interfered on the beamsplitter. By using birefringent prisms we can in principle detect all the photons from the subset and obtain the probability (24) as a proper frequency (a ratio of counts) given by Eq. (30). In this way we close the *enhancement loop* of the Bell theorem proof. On the other hand, the experiment shows that it is not a direct interaction between photons or their common origin what entagles them in a polarization singlet state but particular correlations which one can preselect without resorting to polarization measurement at all. We conclude that nonlocality is essentially a property of selection. This might exclude all nonlocal hidden–variable theories which rely on some kind of a physical entaglement via a common origin.

The realistic estimation of the experiment carried out in Sec. 4 for the equal superposition given by Eq. (27) shows that such a set–up is just about to be feasible within the so–called (see Sec. 4) *83% limit* thus narrowing the second, *efficiency* loophole in the Bell theorem proof. It should be stressed here that a very helpful feature of the considered effect is that the entaglement (quadruple firing of *preselection detectors* D1–D2 behind the beam splitter *and* two of D1'–D2$^{\perp}$ catching the other two "free" photons) is independent of positions of D1'–D2$^{\perp}$ and also of the moment of their firing as follows from Eqs. (21) and (31). In other words the visibility of the whole entaglement and the visibility of the two photon coincidence on BS practically do not differ.

To narrow down the efficiency loophole completely we resort to the polarization measurement and unequal superposition [given by Eq. (35)] whereby we can make the experiment comfortably within the so–called *67% limit* by recognizing Eberhard's *r–term* not as a measure of a background noise but as the ratio of the reflection to the transmission coefficient in one of the measured polarization directions (on a polarized BS). At the same time this approach establishes to our knowledge the first experimental procedure for *exact* measurement of unequal superposition without loss of detection counts. In other words, when we, with the help of partially polarized beam splitter BS and detectors D1–D2$^{\perp}$, preselect photons in the subset of photons in the unequal triplet–like state given by Eq. (35) we do not lose counts because the detectors by means of the birefringent polarizers P1' and P2' register all counts so that we can form a proper frequency, given by Eq. (37), in order to verify the corresponding probability, given by Eq. (36). This closes the *efficiency* loop in the Bell theorem proof.

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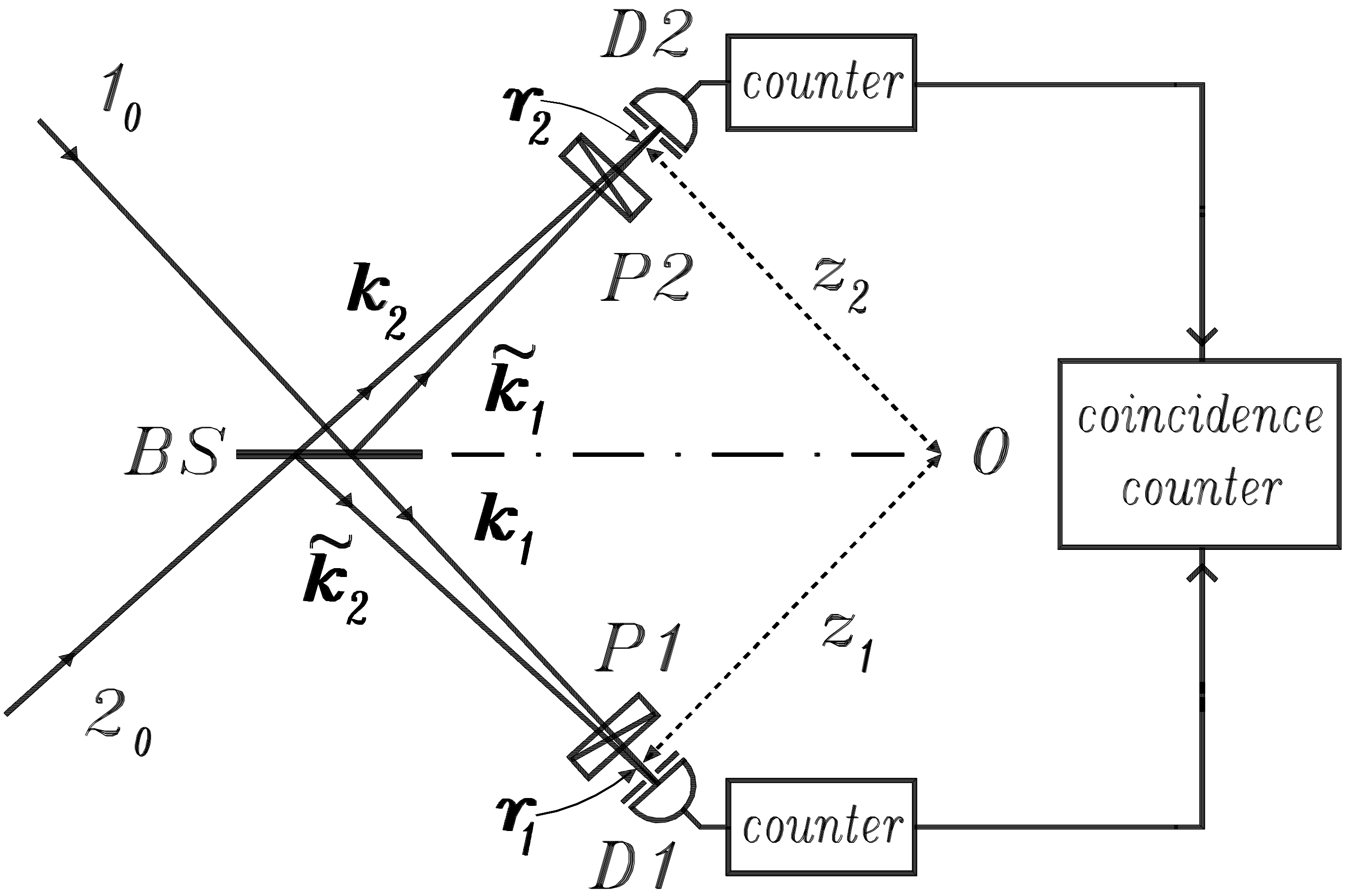
FIGURES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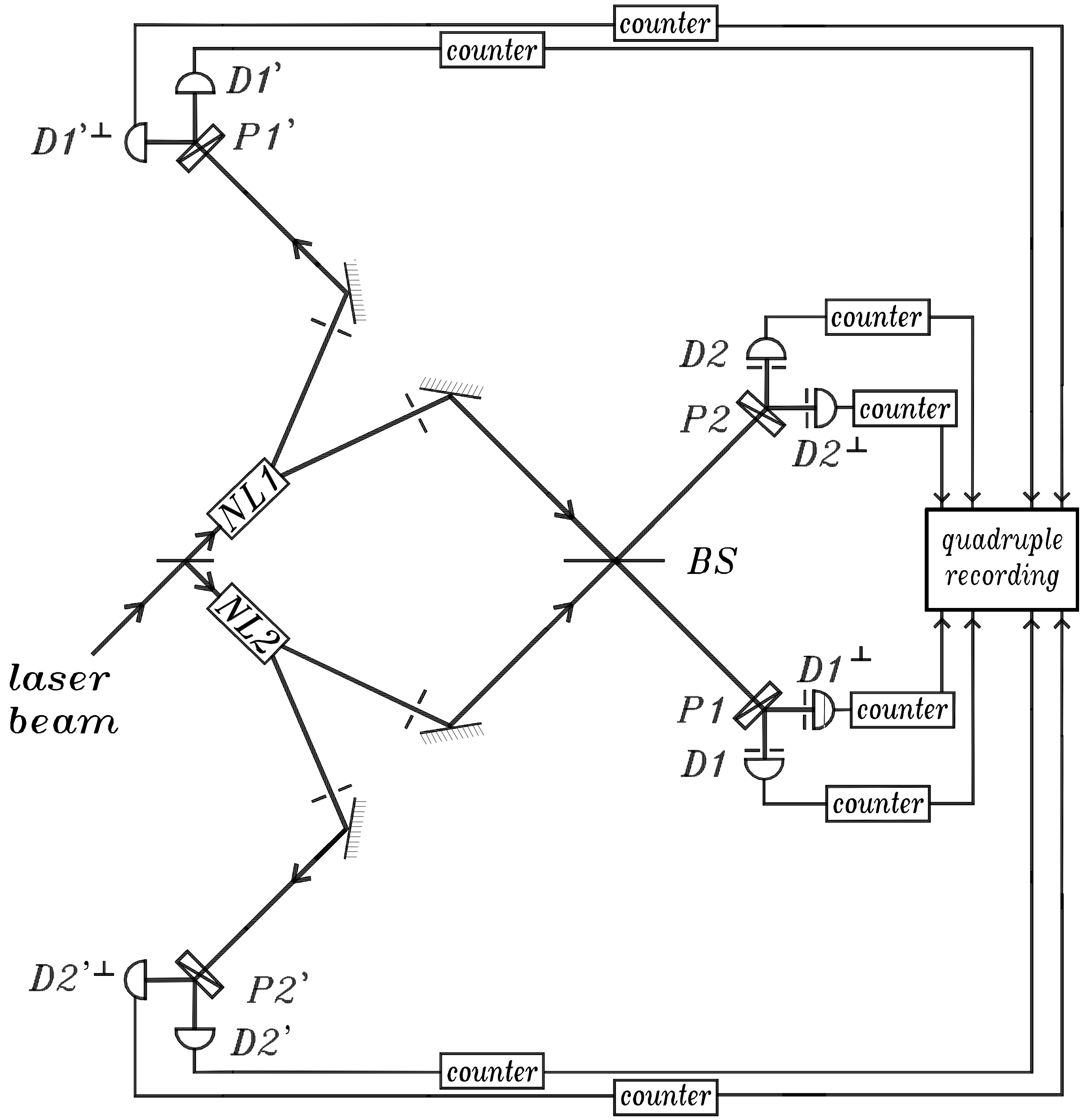
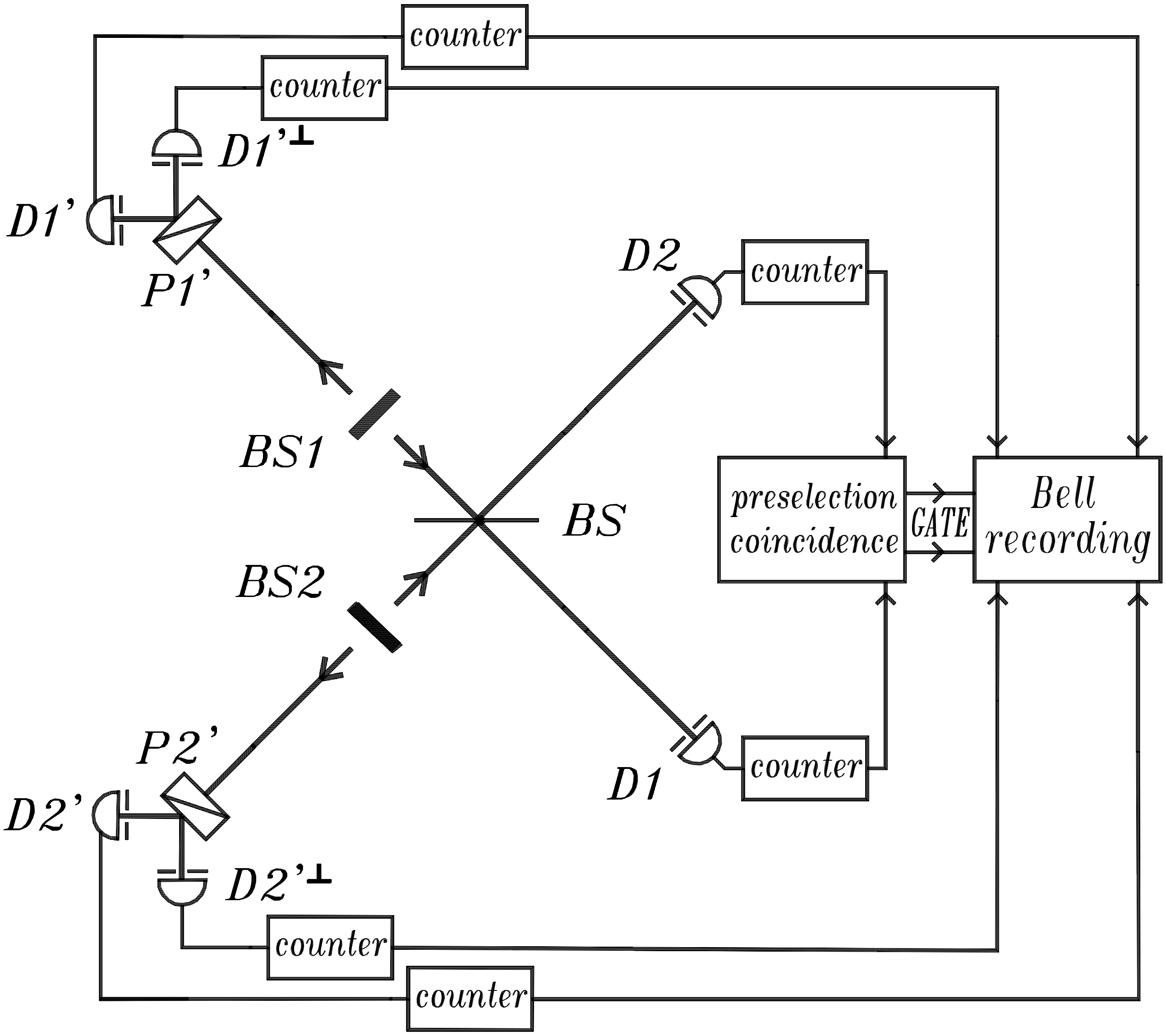
counter
1_0
r_2
P2
k_2
z_2
$\tilde{k}_1$
BS
O
coincidence counter
k_1
$\tilde{k}_2$
P1
z_1
2_0
r_1
counter
D1

Fig. 1



counter
counter
D1'
D1'⊥
P1'
counter
D2
counter
P2
D2⊥
NL1
BS
quadruple recording
NL2
laser beam
D1⊥
P1
counter
D1
counter
D2'⊥
P2'
D2'
counter
counter

Fig. 2



counter
counter
D1'⊥
D1'
P1'
D2
counter
BS1
BS
preselection coincidence
GATE
Bell recording
BS2
P2'
D1
counter
D2'
D2'⊥
counter
counter

Fig. 3