International Journal of Theoretical Physics, **34**, 1653–1665 (1995)

Preselection with Certainty of Photons in a Singlet State from a Set of Independent Photons

Mladen Pavičić [1]

It is shown that one can *preselect with certainty* photons in the singlet state from a set of completely unpolarized and independent photons which did not in any way directly interact with each other — *without in any way affecting them.* The result is based on an experiment which puts together two unpolarized photons from two independent singlet pairs, makes them interfere in the fourth order on a beam splitter so as to preselect the singlet state of the other two photons from the pairs although no polarization measurement has been carried out on the photons coming out from the beam splitter. One can obtain the expectation value for the correlated state of the former two unpolarized photons in the Hilbert space and therefore write down the singlet state for them but one apparently cannot *infer* the state within the Hilbert space. This might sugest that the Hilbert space is not a *maximal* model for quantum measurements.

PACS numbers: 03.65.Bz, 42.50.Wm

---

[1]Department of Mathematics, University of Zagreb, GF, Kačićeva 26, HR–41001 Zagreb, Croatia and Atominstitut der Österreichischen Universitäten, Schüttelstraße 115, A–1020 Wien, Austria; Internets: pavicic@ati.ac.at, mpavicic@faust.irb.hr. Web page: http://m3k.grad.hr/pavicic.

1 INTRODUCTION

Quantum structures and in particular quantum logic are expected to enable us to infer theorems from each other. The theorems are then expected to correspond to physical properties which quantum observables (corresponding to propositions) and states imposed on quantum logic (corresponding to probability measures) should satisfy.

Such an inference, as opposed to classical stuctures, does not rely on an ordering between the subsets of states corresponding to particular observables. For, it can be shown that quantum logic and its models, ortomodular lattices, are ortho–structures in which a unique operation, *bi–implication*, directly corresponds to the *equality* of the subsets of states. (Pavičić, 1993a) Evenmore, it turns out that one can find a representation of this correspondence which in the Hilbert space corresponds to the linear subspaces and their complements. In other words, propositions of quantum logic need not correspond to observables. It is enough that they correspond to states and to their tensor products. (Pavičić, 1993a,b) Of course, the representation might not give us the usual Hilbert space as a *maximal* model for quantum logic in the end but this is exactly the possibility recently considered by *"quantum structuralists"*: It might take us to a wider (Banach?) space which would contain the standard complex Hilbert space as its proper subspace. Many recent papers on effect–algebras, ortho–algebras, and D–posets have such an assumption in the background with the last objective to arrive at a more general theory of quantum measurements than the standard Hilbert space one. However, the whole program depends on whether we can find states which cannot be directly *prepared* by our devices and whether we can find new states which cannot be inferred from the initial states and which do not correspond to any observable in the standard Hilbert space. The purpose of this paper is to show how recent results of us (Pavičić, 1994a,b,c, Pavičić and Summhammer, 1994) may lead us to the existence of such states.

To arrive at the states which are not *"inferrable"* within the Hilbert space (although we obtain their probability in the Hilbert space formalism) we use an experiment which puts together two photons from two independent singlets and make them interfere in the fourth order on a beam splitter so as to allow us to detect polarization (spin) correlations between the other two photons separated in space even when we carry out no polarization measurement on the first two photons. It turns out that one can *preselect with certainty* photons in the singlet state from a set of completely unpolarized and *independent* photons which did not in any way directly interact with each other. Under *"independent photons"* we understand photons which *originate* from completely independent sources.

2 FORMALISM

The formalism we use is the formalism of the second quantization (Paul, 1986; Ou, 1988) but the final result is independent of it. The experiment is based on and uses the results of a newly discovered interference effect of the 4th order on a beam splitter according to which two unpolarized incident photons emerge from a beam splitter correlated in polarization even when they arrive at it unpolarized. (Pavičić, 1994a) It also uses the spin entaglement of two photon pairs on a beam splitter as described in Pavičić and Summhammer (1994) and Pavičić (1994b,c).

Before we present the results from Pavičić (1994a) we shall first introduce some details of a formal description of the 4th order interference on a beam splitter following Campos, Salech, and Teich (1990), Ou (1988), Ou, Hong, and Mandel (1988), and Paul (1986). Here, however, we will not discuss experimental limitations, efficiencies of detectors, etc., and their formal descriptions. For that aspect of our experiment we direct the reader to the afore–cited references of ours.

Let two polarized beams fall on a beam splitter as shown in Fig. 1. Two by two photons (*signal* and *idler* photons) simultatneously emerge from a nonlinear crystal (pumped by a laser beam) in a downconversion process. Signal and idler have random phases relative to each other so that we do not have any interference of the second order. Their frequencies are half the frequency of photons from the pumping laser beam. The state of incoming polarized photons is given by the product of two prepared linear–polarization states:

$$|\Psi\rangle = (\cos\theta_{1'}|1_x\rangle_{1'} \ + \ \sin\theta_{1'}|1_y\rangle_{1'}) \otimes (\cos\theta_{2'}|1_x\rangle_{2'} \ + \ \sin\theta_{2'}|1_y\rangle_{2'}) \tag{1}$$

where $|1_x\rangle$ and $|1_y\rangle$ denote the mutually orthogonal photon states. So, e.g., $|1_x\rangle_{1'}$ means the upper incoming photon polarized in direction x. If the beam spliter were removed it would cause a *"click"* at the detector D1 and no *"click"* at the detector D1$^\perp$ provided the birefringent polarizer P1 is oriented along x. Here D1$^\perp$ means a detector counting photons coming out at the *other exit*, P1$^\perp$. (See Fig. 2) of the birefringent prism P1. Angles $\theta_{1'}$,$\theta_{2'}$ are the angles along which incident photons are polarized with respect to a fixed direction.

The two photons are described by two corresponding electric fields which we obtain in the following way. We describe *"interactions"* of photons with the beamspliter, polarizers, and detectors directly by the appropriate parts of operators in the second quantization formalism as usually employed in quantum optical analysis.

We introduce polarization by means of two orthogonal scalar field components. Thus the scalar component of the stationary electric field operator will read (see Fig. 1)

$$\hat{E}_j(\boldsymbol{r}_j, t) = \hat{a}(\omega_j) e^{i\boldsymbol{k}_j\cdot\boldsymbol{r}_j - i\omega_j t} \tag{2}$$

The anihilation operators describe joint actions of polarizers, beam–splitter, and detectors. The operators act on the states as follows: $\hat{a}_{1x}|1_x\rangle_1 = |0_x\rangle_1, \ \hat{a}^\dagger_{1x}|0_x\rangle_1 = |1_x\rangle_1, \ \hat{a}_{1x}|0_x\rangle_1 = 0$, etc.

The action of the beam–splitter we describe by the following matrix: $\hat{\mathbf{a}}_{out} = \mathbf{c}\,\hat{\mathbf{a}}_{in}$.

The number of output photons must match the number of input ones, i.e., the energy must be conserved. Since the number of photons corresponding to our operators $\hat{a}_i$ is $n_i = \hat{a}^\dagger_i\hat{a}_i$ this demand reads

$$n_{1out} + n_{2out} = \hat{a}^\dagger_{1out}\hat{a}_{1out} + \hat{a}^\dagger_{2out}\hat{a}_{2out} = n_{2in} + n_{1in} = \hat{a}^\dagger_{1in}\hat{a}_{1in} + \hat{a}^\dagger_{2in}\hat{a}_{2in} \tag{3}$$

wherefrom we obtain

$$|c_{11}|^2 + |c_{21}|^2 = 1 \quad \text{and} \quad |c_{11}||c_{12}| = |c_{21}||c_{22}| \tag{4}$$

Next we demand that the input and the output systems be equivalent what boils down to the unitary of the matrix $\mathbf{c}$: $\mathbf{c}^\dagger\mathbf{c} = \mathbf{I}$. This, together with Eqs. (4), gives: $|c_{11}| = |c_{22}|$ and $|c_{12}| = |c_{21}|$.

Introducing transmittance $T = |c_{11}|^2 = |c_{22}|^2$ and reflectance $R = |c_{12}|^2 = |c_{12}|^2$ and denoting $t = |\sqrt{T}|$ and $r = |\sqrt{R}|$ we obtain

$$\mathbf{c} = e^{i\phi_\circ} \begin{pmatrix} te^{i\phi_t} & re^{i(\phi_r+\pi)} \\ re^{-i\phi_r} & te^{-i\phi_t} \end{pmatrix} \tag{5}$$

The freedom which the phases in $\mathbf{c}$ offer is the reason why there are so many varieties of them in the literature depending on aspects authors want to stress. We choose $\phi_\circ = 0$ because the overall phase does not play any role in the interference and $\phi_t = 0$ and $\phi_r = -\pi/2$ to assure the phase shift during the reflection on the beam–splitter. Thus the output operators read

$$\begin{aligned} \hat{a}_{1out} &= t\hat{a}_{1in} + i\,r\hat{a}_{2in} \\ \hat{a}_{2out} &= t\hat{a}_{2in} + i\,r\hat{a}_{1in} \end{aligned} \tag{6}$$

To take the linear polarization along orthogonal directions into account we shall consider two sets of operators, i.e., their matrices

$$\hat{\mathbf{a}}_{x\,out} = \mathbf{c}_x\,\hat{\mathbf{a}}_{x\,in} \quad \text{and} \quad \hat{\mathbf{a}}_{y\,out} = \mathbf{c}_y\,\hat{\mathbf{a}}_{y\,in} \tag{7}$$

So, the action of the polarizers P1,P2 and detectors D1,D2 can be expressed as:

$$\hat{a}_i = \hat{a}_{ix\,out}\cos\theta_i + \hat{a}_{iy\,out}\sin\theta_i. \tag{8}$$

where $i = 1, 2$.

The operators corresponding to the other choices of polarizers and detectors we obtain accordingly. E.g., the action of the polarizer P2$^\perp$ (orthogonal to P2; in the experiment P2 and P2$^\perp$ make a birefringent prism) and the corresponding detector D2$^\perp$ is described by

$$\hat{a}_2 = -\hat{a}_{2x\,out}\sin\theta_2 + \hat{a}_{2y\,out}\cos\theta_2. \tag{9}$$

The electric outgoing field operators describing photons which pass through beam splitter BS and polarizers P1,P2 and are detected by detectors

D1,D2 will thus read

$$\begin{aligned}\hat{E}_1 &= (\hat{a}_{1x}t_x\cos\theta_1 + \hat{a}_{1y}t_y\sin\theta_1)\, e^{i\boldsymbol{k}_1\cdot\boldsymbol{r}_1 - i\omega(t-\tau_1)} \\ &\quad + i\,(\hat{a}_{2x}r_x\cos\theta_1 + \hat{a}_{2y}r_y\sin\theta_1)\, e^{i\tilde{\boldsymbol{k}}_2\cdot\boldsymbol{r}_1 - i\omega(t-\tau_2)} \qquad (10)\end{aligned}$$

$$\begin{aligned}\hat{E}_2 &= (\hat{a}_{2x}t_x\cos\theta_2 + \hat{a}_{2y}t_y\sin\theta_2)\, e^{i\boldsymbol{k}_2\cdot\boldsymbol{r}_2 - i\omega(t-\tau_2)} \\ &\quad + i\,(\hat{a}_{1x}r_x\cos\theta_2 + \hat{a}_{1y}r_y\sin\theta_2)\, e^{i\tilde{\boldsymbol{k}}_1\cdot\boldsymbol{r}_2 - i\omega(t-\tau_1)} \qquad (11)\end{aligned}$$

where τ_j is time delay after which the photon reaches detector D, ω is the frequency of photons, c is the velocity of light. The detectors and crystals are assumed to be positioned symmetrically to the beam splitter so that two time delays suffice.

The joint interaction of both photons with the beam–splitter, polarizers P1,P2, and detectors D1,D2 is given by a projection of our wave function onto the Fock vacuum space by means of $\hat{E}_1$, $\hat{E}_2$ wherefrom we get the following probability of detecting photons by D1,D2:

$$P(\theta_{1'},\theta_{2'},\theta_1,\theta_2) = \langle\Psi|\hat{E}_2^\dagger\hat{E}_1^\dagger\hat{E}_1\hat{E}_2|\Psi\rangle = A^2 + B^2 - 2AB\cos\phi \qquad (12)$$

where

$$\phi = (\tilde{\boldsymbol{k}}_2 - \boldsymbol{k}_1)\cdot\boldsymbol{r}_1 + (\tilde{\boldsymbol{k}}_1 - \boldsymbol{k}_2)\cdot\boldsymbol{r}_2 = 2\pi(z_2 - z_1)/L\,, \qquad (13)$$

where L is the spacing of the intereference fringes, z_1 and z_2 are the cordinates which determine the spacing of the fringes as shown in Fig. 1, and $A = S_{1'1}(t)S_{2'2}(t)$ and $B = S_{1'2}(r)S_{2'1}(r)$ where

$$S_{ij} = s_x\cos\theta_i\cos\theta_j + s_y\sin\theta_i\sin\theta_j \qquad (14)$$

Assuming $t_x = t_y = r_x = r_y = 2^{-1/2}$ and $\cos\phi = 1$ (we can modify ϕ by moving the detectors transversely to the incident beams) the probability reads

$$P(\theta_{1'},\theta_{2'},\theta_1,\theta_2) = (A-B)^2 = \frac{1}{4}\sin^2(\theta_{1'} - \theta_{2'})\sin^2(\theta_1 - \theta_2) \qquad (15)$$

which for unpolarized incident beams becomes

$$P(\infty,\infty,\theta_1,\theta_2) = \frac{1}{2}\sin^2(\theta_1 - \theta_2) \qquad (16)$$

We see that the probability in Eq. (15) unexpectedly factorizes (see Figs. 2 and 3) left–right (corresponding to D1'–D2' $\leftrightarrow$ D1–D2 detections) and not up–down (corresponding to ${}^{\mathrm{BS1}}_{\mathrm{BS2}}\updownarrow$ preparation) as one would be tempted to expect from the up–down initial independence and the product of the upper and lower function in Eq. (1). We also see that by changing the relative angle between the polarization planes of the incoming photons we only change the light intensity of the photons emerging from the beam splitter.

Thus, the photons either leave the beam splitter from its opposite sides correlated according to Eq. (15) or both leave it from the same side according to Eq. (26) of Pavičić (1994a). In case of unpolarized incident beams we obtain the following overall probability for the latter photons

$$P(\infty,\infty,\theta_1\times\theta_2)=\frac{1}{2}[1+\cos^2(\theta_1-\theta_2)] \tag{17}$$

which together with Eq. (16) adds up to one as it should.

We also see that the photon beams leave the beam splitter unpolarized:

$$\begin{aligned}P(\theta_{1'},\theta_{2'},\theta_1,\infty) &= P(\theta_{1'},\theta_{2'},\theta_1,\theta_2)+P(\theta_{1'},\theta_{2'},\theta_1,\frac{\pi}{2}-\theta_2)\\ &= \frac{1}{4}\sin^2(\theta_{1'}-\theta_{2'}) \end{aligned} \tag{18}$$

which for unpolarized incident beams becomes

$$P(\infty,\infty,\theta_1,\infty)=\frac{1}{2} \tag{19}$$

3 PRESELECTION EXPERIMENT

A detailed schematic representation of the experiment is shown in Fig. 2 and a simplified one in Fig. 3. In the presention of of the experiment we shall not discuss the experimental conditions. For this aspect of the experiment we direct the reader to Pavičić (1994b,c) and to Pavičić and Summhammer (1994). In the latter reference we elaborate a wave packet description of a similar experiment.

Two independent beam splitters BS1 and BS2 act as two independent sources of two independent singlet pairs. A laser beam simultaneously pumps up two nonlinear crystals NL1 and NL2 producing in each of them two half–frequency sidebands — signal and idler. Thus, BS1 and BS2 both simultaneously emit two photons (of frequency ω) in the singlet state given by Eq. (15), to the left and to the right. On the left photons we measure polarizations by the polarization filters P1' and P2' and on the right ones by P2 and P1. Before we put beam splitter BS in place we first have to adjust in the setup so as to obtain pure singlet states coming out from BS1 and BS2 and caused by perpendicular beams inciding on these two beam splitters. It follows from Eq. (16) and Fig. 2 that we can do this for $\phi = 0$ by reaching the minimum of coincidences for $\theta_{1'} = \theta_1$ for BS1 and for $\theta_{2'} = \theta_2$ for BS2.[2] Ideally the minimum is zero in which case photons never appear at the opposite sides of the beam splitter so as to both pass parallelly oriented polarizers. Then we put beam splitter BS in place and four photons (of which two pass P1' and P2' on the left and two — after passing BS — pass P2 and/or P1 on the right) form elementary quadruples of counts (coupled firing of D1–D2'$^\perp$) which add up to the probability given by Eq. (27) in the long run. Birefringent polarizers P1,P1',P2,P2' assure registration of *"negative"* detections by the detectors D1$^\perp$–D2'$^\perp$. We impose appropriate time windows on the 4–events coincidence counter. To make sure that there is no phase correlation (and therefore no interference of the 2nd order) between idler photons[3] coming from BS1 and BS2 or between signal photons coming from BS1 and BS2 we introduce[4] a phase modulator (which rotates at small angles at random) in the path between BS2 and BS.

The state of the four photons immediately after leaving BS1 and BS2 is described by the product of the two superpositions corresponding to singlet pairs produced — according to Eq. (15) — on BS1 and BS2, respectively:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|1_x\rangle_{1'}|1_y\rangle_1 \; - \; |1_y\rangle_{1'}|1_x\rangle_1) \otimes \frac{1}{\sqrt{2}}(|1_x\rangle_{2'}|1_y\rangle_2 \; - \; |1_y\rangle_{2'}|1_x\rangle_2) \quad (20)$$

[2]Notice the following difference in notation in equations corresponding to Fig. 1 in Sec. 2 and equtions corresponding to Figs. 2 and 3 in the present section: In Sec.2 the fotons coming out from the beam splitter are denoted 1 and 2, while in the present section the photons coming out from BS1 are 1' and 1, and coming out from BS2 are 2' and 2.

[3]imposed on them by the pumping laser beam in the downconversion processes in the crystals.

[4]following Ou, Gage, Magill, and Mandel (1988)

where $|1_x\rangle$ and $|1_y\rangle$ denote the mutually orthogonal photon states.

The annihilation of photons at detectors D1',D2' after passing the polarizers P1',P2' (oriented at angles $\theta_{1'},\theta_{2'}$) are described by the following electric field operators

$$\hat{E}_{1'} = (\hat{a}_{1'x}\cos\theta_{1'} + \hat{a}_{1'y}\sin\theta_{1'})e^{i\omega(t-\tau_{1'})} \tag{21}$$

$$\hat{E}_{2'} = (\hat{a}_{2'x}\cos\theta_{2'} + \hat{a}_{2'y}\sin\theta_{2'})e^{i\omega(t-\tau_{2'})} \tag{22}$$

Here phases of the photons which accumulate between beam splitters BS1, BS2 and detectors D1',D2' add the factors $e^{i\omega(t-\tau_j)}$, where ω is the frequency of the photons, and τ_j are time delays after which the photons reach detectors D1'.D2'.

The electric outgoing field operators describing photons which pass through beam splitter BS and polarizers P1,P2 and are detected by detectors D1,D2 are given by Eqs. (10) and (11).

The joint interaction of all four photons with the beam splitter, polarizers P1–P2', and detectors D1–D2'$^\perp$ is given by the following projection of our initial state given by Eq. (20) wave function onto the Fock vacuum space:

$$\hat{E}_1\hat{E}_2\hat{E}_{1'}\hat{E}_{2'}|\Psi\rangle = \frac{1}{2}(A\varepsilon_{12} - B\tilde{\varepsilon}_{12})\varepsilon|0\rangle \tag{23}$$

where $\varepsilon_{12} = \exp[i\,(\boldsymbol{k}_1\cdot\boldsymbol{r}_2 + \boldsymbol{k}_2\cdot\boldsymbol{r}_1)]$, $\tilde{\varepsilon}_{12} = \exp\left[i\left(\tilde{\boldsymbol{k}}_1\cdot\boldsymbol{r}_1 + \tilde{\boldsymbol{k}}_2\cdot\boldsymbol{r}_2\right)\right]$, $\varepsilon = \exp\left[-i\omega\,(4t - \tau_1 - \tau_2 - \tau_{1'} - \tau_{2'})\right]$, and $A = Q(t)_{1'1}Q(t)_{2'2}$ and $B = Q(r)_{1'2}Q(r)_{2'1}$, where

$$Q(q)_{ij} = q_x\sin\theta_i\cos\theta_j - q_y\cos\theta_i\sin\theta_j. \tag{24}$$

The corresponding probability of detecting all four photons by detectors D1–D2'$^\perp$ is thus

$$\begin{aligned} P(\theta_{1'},\theta_{2'},\theta_1,\theta_2) &= \langle\Psi|\hat{E}^\dagger_{2'}\hat{E}^\dagger_{1'}\hat{E}^\dagger_2\hat{E}^\dagger_1\hat{E}_1\hat{E}_2\hat{E}_{1'}\hat{E}_{2'}|\Psi\rangle \\ &= \frac{1}{4}(A^2 + B^2 - 2AB\cos\phi) \end{aligned} \tag{25}$$

where

$$\phi = (\tilde{\mathbf{k}}_2 - \mathbf{k}_1)\cdot\mathbf{r}_1 + (\tilde{\mathbf{k}}_1 - \mathbf{k}_2)\cdot\mathbf{r}_2 = 2\pi(z_2 - z_1)/L\,, \tag{26}$$

where L is the spacing of the intereference fringes.

Let us consider 50:50 beam splitter: $t_x = t_y = r_x = r_y = 2^{-1/2}$ for the case $\phi = 0$ for which the above probability reads

$$P(\theta_{1'}, \theta_{2'}, \theta_1, \theta_2) = \frac{1}{4}(A-B)^2 = \frac{1}{16}\sin^2(\theta_1 - \theta_2)\sin^2(\theta_{1'} - \theta_{2'}) \qquad (27)$$

We again see that the probability factorizes left–right and not up–down as one would be tempted to conjecture from the product of the upper and lower function in Eq. (20).

The probability (27) shows that for $\phi = 0$ by removing one of the polarizers we lose any left–right (Bell–like) spin correlation completely:

$$P(\theta_{1'}, \infty, \theta_1, \theta_2) = \frac{1}{16}\sin^2(\theta_1 - \theta_2). \qquad (28)$$

The probability of detecting both photons in one arm, say of D2, one obtains similarly to Eq. (25). If we do not try to separate photons 1 and 2 in the arm according to their possibly different frequency, then we get

$$\begin{aligned} P(\theta_{1'}, \theta_{2'}, 2\times\theta_2) &= \frac{1}{2}\langle\Psi|\hat{E}_{1'}^\dagger \hat{E}_{2'}^\dagger \hat{E}_2^\dagger \hat{E}_2'^\dagger \hat{E}_2' \hat{E}_2 \hat{E}_{2'} \hat{E}_{1'}|\Psi\rangle \\ &= \frac{1}{32}[\sin^2(\theta_{1'} - \theta_2)\sin^2(\theta_{2'} - \theta_2)](1+\cos\eta) \qquad (29) \end{aligned}$$

where $1/2$ matches the possibility of both photons taking the other arm and

$$\eta = (\tilde{\mathbf{k}}_1 - \mathbf{k}_2') \cdot \mathbf{r}_1 + (\tilde{\mathbf{k}}_1' - \mathbf{k}_2) \cdot \mathbf{r}_2\,, \qquad (30)$$

where primes simply refer to the *other* photon possibly of a different frequency. An analogous probability we obtain for D1. Of course one should not try to add up these probabilities and the probability (25) to 1 which we would only get if we registered polarizations of each photon in the arm.

For our experiment we shall primarily use the following probability of detecting all four photons by D1–D2'$^\perp$ in coincidence for 50:50 beam splitter for $\phi = 0$, with equal time delays (that is for a completely symmetrical position of BS), and with the polarizers P1 and P2 removed:

$$P(\theta_{1'}, \theta_{2'}, \infty, \infty) = \langle\Psi|\hat{E}_{1'}^\dagger \hat{E}_{2'}^\dagger \hat{E}_1^\dagger \hat{E}_2^\dagger \hat{E}_2 \hat{E}_1 \hat{E}_{2'} \hat{E}_{1'}|\Psi\rangle = \frac{1}{8}\sin^2(\theta_{1'} - \theta_{2'})\,. \quad (31)$$

and the probability of detecting both photons in one of the arms reads

$$P(\theta_{1'},\theta_{2'},2\times\infty)=\frac{1}{8}[1+\cos^2(\theta_{1'}-\theta_{2'})]. \tag{32}$$

We see that these two probabilitites add up to 1/4. (The other 3/4 correspond to *orthogonal detections* by $\mathrm{D}^{\perp}$ detectors.) The latter probability one obtains so as to add up all the probabilities of detecting polarizations of each photon in one arm, i.e., $P(\theta_{1'},\theta_{2'},\theta_1\times\theta_2)$, $P(\theta_{1'},\theta_{2'},\theta_1\times\theta_2^{\perp})$, etc.

Thus, photons appearing at different sides of the beam splitter behave *quantum like* showing — according to Eq. (31) — 100% *relative modulation*. In other words, by detecting the right photons on different sides of the beam splitter we surprisingly *preselect* the orthogonal individual left photons pairs (25% of all pairs) with probability one, while by detecting the right photons both on one side of the beam splitter we preselect the remaining orthogonal left pairs with probability 0.25 and parallel ones with probability 0.5.

The main point of our experiment is that the correlation between photons 1' and 2', i.e., between photons which never interacted in the past persists even when we do not measure polarization on their companions 1 and 2 at all. Let us therefore concentrate on the experiment without polarizers P1,P2 behind beam splitter BS. To make our point we present the appropriate experimental setup in a simplified and reduced scheme presented in Fig. 3. (Note that D1',D2' are much further away from BS1,BS2, respectively than D1,D2, which is indicated by double wavy lines.) A pair consisting of two photons 1' and 1 appears on BS1 simultaneously with another pair 2'–2 on BS2. Photons are directed towards detectors D1',D2',D1,D2. Of all detections registered by D1,D2 only those pulses which occur within a short enough time (about 100ns) are fed to the *preselection coincidence counter*. That assures that each pair of the pulses really belongs to the two photons which interfered on BS so as to appear at the opposite sides of the beam splitter. The coincidence pulses then open a computer gate for a selection of counts from the *checking coincidence counter* after a time delay calculated from the time–of–flight difference.

We should stress here that there exists a definite probability, given by Eq. (29) that both photons from each of the pairs originated at BS1 and BS2 leave BS1 or BS2 from the same side and enter BS together. Then we might have two or more photons in both of the detectors D1,D2 which they cannot discern from each other. We solved this *"problem"* in Pavičić (1994b) by using photons of different frequencies.[5] However, the *"problem"* is not a real problem at all because we can always discard *checking coincidence* counts which show only one or none pulse and include such a discarding into the preselection sheme. Thus, we are always able to tell four from only three or two counts in the quadruple recording sheme and to keep only "valid" quadruples here. It is important to keep in mind here that we preselect polarization correlation between photons 1' and 2' and not an entaglement between them in the configuration space. So, discarding "nonvalid" quadruples (with void recordings of detectors D1'–D1'$^{\perp}$ and/or D2'–D2'$^{\perp}$) does not influence the statistics[6] which is obtained so that non–discarded checking coincidence data divided by the corresponding preselection coincidence data give the probability given by Eq. (31) multiplied by 4. Multiplication by 4 compensates for the photons which appeared at the same side of BS and therefore were not recorded by the preselection coincidence counter.

4 CONCLUSION

We have shown that one is able to *preselect with certainty* a subset of photon pairs in the singlet state out of a set of completely random, unpolarized, and independent photons without directly interacting with them.[7] Such a preselection can later on be confirmed by the polarization measurements carried out on photons from the subset. We arrive at such a result by means of the probability given by Eq. (31) (multiplied by by 4)[8] and by means of the following equation [obtained from Eq. (28) after multiplication by 4]

$$P(\theta_{1'}, \infty, \infty, \infty) = \frac{1}{2} \tag{33}$$

[5]The 4th order interference with photons of different colors have been elaborated both theoretically and experimentally by Ou, Gage, Magill, and Mandel (1988) and Larchuk, Campos, Rarity, Tapster, Jakeman, Saleh, and Teich (1993).

[6]Discarded events do not belong to "our" set of events

[7]This also shows that quantum nonlocality is basically a property of selection.

[8]See the last sentence of Sec.3.

which expresses the fact that the photons from the subset are unpolarized. In plain words, the probability of both photons being detected by D1' and D2' (see Fig. 3) being $\frac{1}{2}cos^2(\theta_{1'} - \theta_{2'})$ and the photons being unpolarized *means* that they are in the *singlet state*. However, we apparently cannot *infer* such a singlet state as a pure state from the initial state [Eq.(20)] because the final state which gives the probability [Eq.(27)] is a four photon state which without involving mean values cannot tell us whether one of its substates is pure or not. This might suggest that the Hilbert space is not a *maximal* model for quantum measurements.

ACKNOWLEDGMENTS

The author is grateful to his host Dr. Johann Summhammer for his hospitality at the Atominstitut der Österreichischen Universitäten, Wien, Austria. He also acknowledges supports of the Technical University of Vienna and the Ministry of Science of Croatia.

REFERENCES

Campos, R. A., Saleh, B. E. A., and Teich, M. A. (1990). *Phys. Rev. A* **42**, 4127.

Larchuk, T. S., Campos, R. A., Rarity, J. G., Tapster, P.R., Jakeman, E., Saleh, B. E. A., and Teich, M. C. (1993) *Phys. Rev. Lett.* **70**, 1603

Ou, Z. Y. (1988). *Phys. Rev. A* **37**, 1607.

Ou, X. Y., Gage, E. C., Magill, B. E., and Mandel, L. (1988). *Opt. Commun.* **69**, 1.

Ou, X. Y., Hong, C. K., and Mandel, L. (1988). *Opt. Commun.* **67**, 159.

Paul, H. (1986). *Rev. Mod. Phys.* **57**, 209.

Pavičić, M. (1993a). *International Journal of Theoretical Physics*, **32**, 1481.

Pavičić, M. (1993b). *Phys. Lett. A* **174**, 353.

Pavičić, M. (1993c). *Interference of four correlated beams from nonlocal sources*, (presented at the Inst. f. Theor. Phys., Techn. Univ. Berlin, on July 16, 1993).

Pavičić, M. (1994a). *Phys. Rev. A* **50**, 3486–3491.

Pavičić, M. (1994b). *Preselection of spin correlated photons*, (submitted).

Pavičić, M. (1994c). *Spin correlated interferometry on beam splitter: preselection of spin correlated photons*, (submitted).

Pavičić, M. and Summhammer, J. (1994). *Phys. Rev. Lett.* (to appear).

FIGURES

Fig. 1. Beam splitter.

Fig. 2. Outline of the experiment. Distances between BS1 (BS2) and D1' (D2') are in the experiment assumed to be much larger then distances between BS1 (BS2) and D1 (D2) as indicated by the wavy lines.

Fig. 3. Reduced scheme of the experiment.

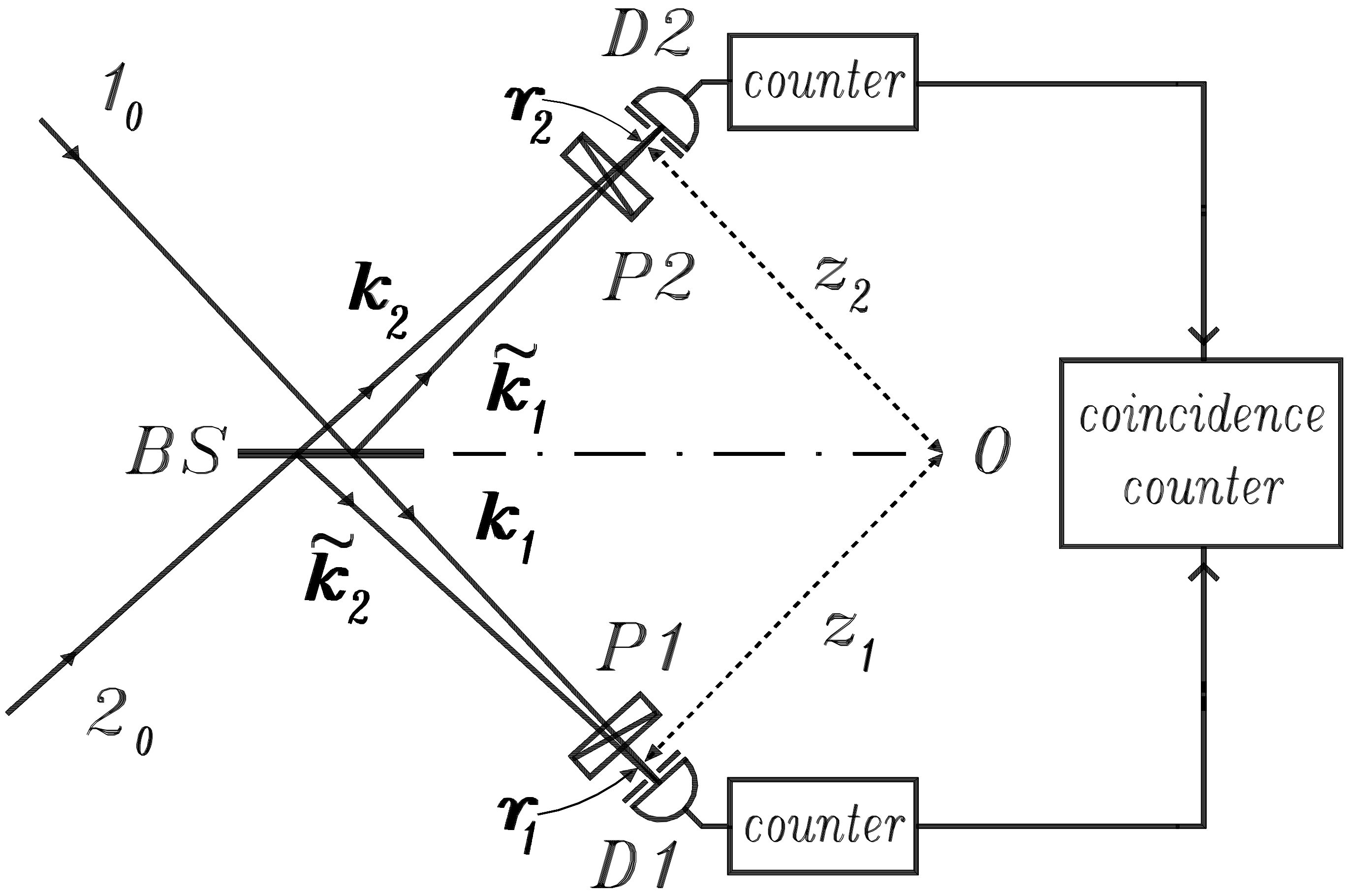
D2
counter
1_0
r_2
P2
z_2
k_2
$\tilde{k}_1$
coincidence
counter
BS
O
k_1
$\tilde{k}_2$
P1
z_1
2_0
r_1
counter
D1

Fig. 1

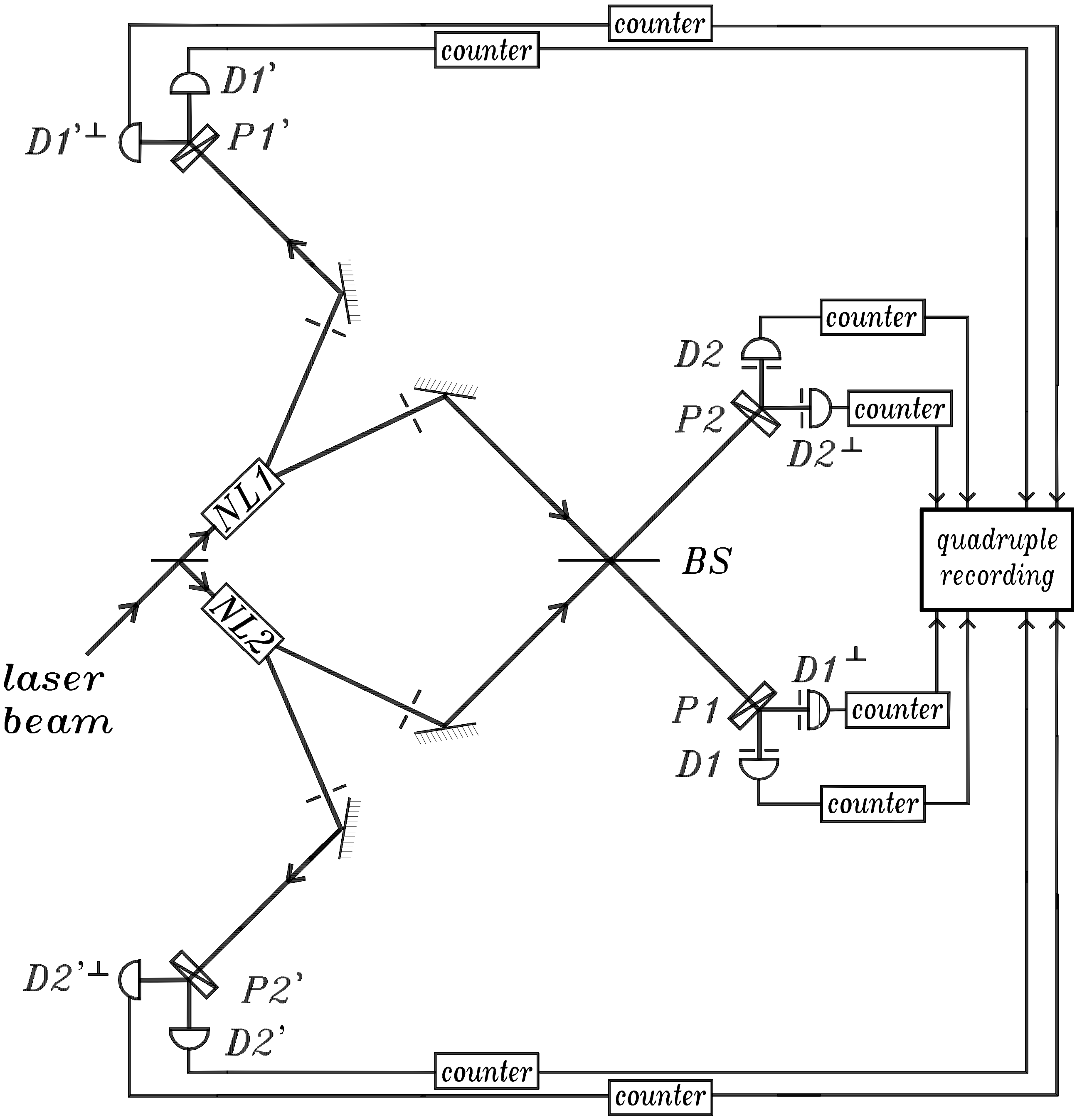
counter
counter
D1'
D1'⊥
P1'
counter
D2
P2
counter
D2⊥
NL1
BS
quadruple recording
NL2
laser beam
D1⊥
P1
counter
D1
counter
D2'⊥
P2'
D2'
counter
counter

Fig. 2

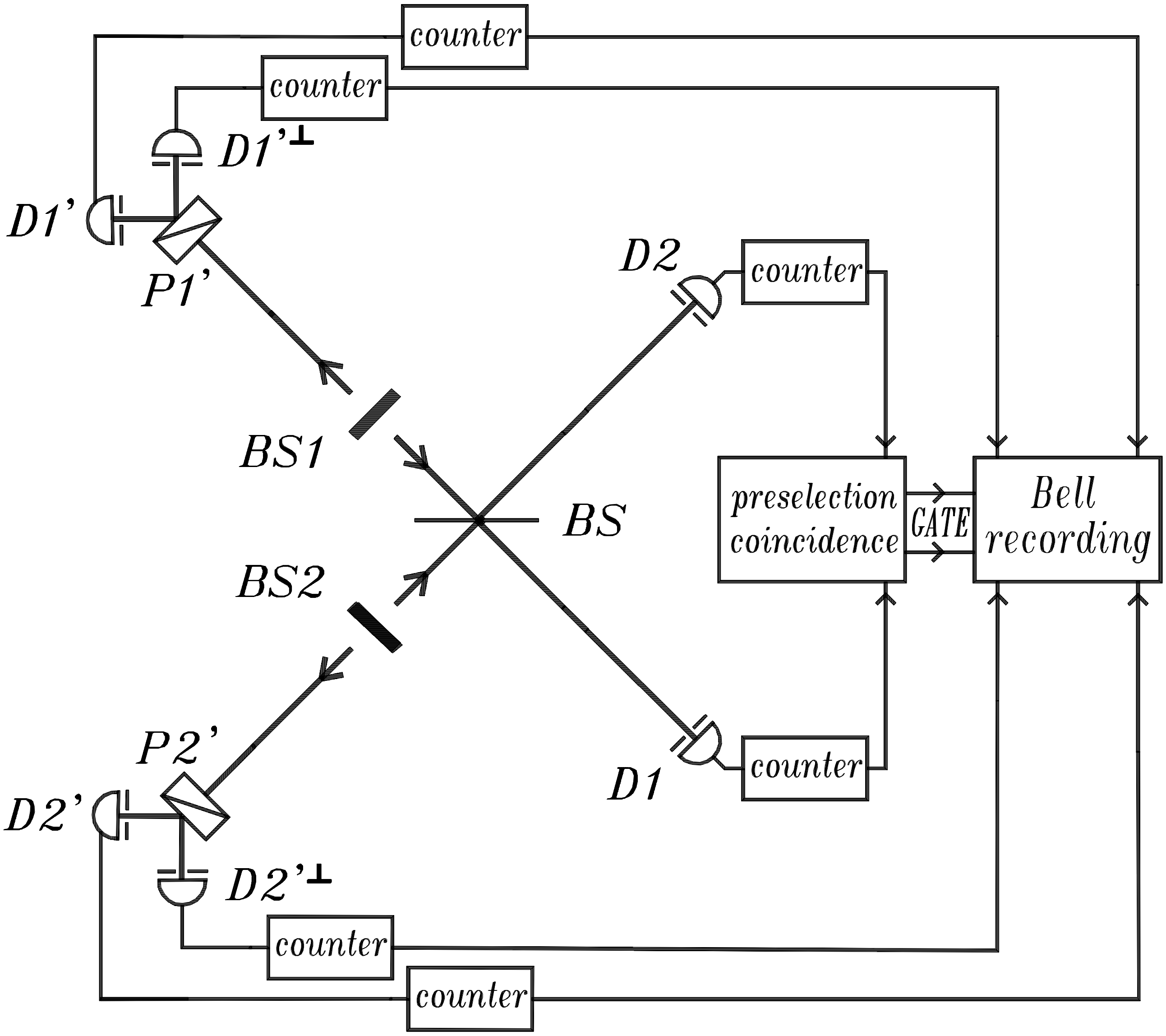
counter
counter
D1'⊥
D1'
P1'
D2
counter
BS1
preselection coincidence
GATE
Bell recording
BS
BS2
P2'
D1
counter
D2'
D2'⊥
counter
counter

Fig. 3