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Deterministic mediated superdense coding with linear optics

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ABSTRACT

We present a scheme of deterministic mediated superdense coding of entangled photon states employing only linear-optics elements. Ideally, we are able to deterministically transfer four messages by manipulating just one of the photons. Two degrees of freedom, polarization and spatial, are used. A new kind of source of heralded down-converted photon pairs conditioned on detection of another pair with an efficiency of 92% is proposed. Realistic probabilistic experimental verification of the scheme with such a source of preselected pairs is feasible with today's technology. We obtain the channel capacity of 1.78 bits for a full-fledged implementation.

1. Introduction

Superdense coding (SC) [1] (sending up to two bits of information, i.e., four messages, by manipulating just one of two entangled subsystems of a quantum system) is considered to be a protocol that can give quantum computation yet another edge over a classical one.

So far the attempts to implement photon SC concentrated on the Bell states. The idea was to send four messages via four Bell states [see Eq. (1)] and herewith achieve a $\log_2 4 = 2$ bit transfer. To this aim, a recognition of all four Bell states was required. However, Vaidman's [2] and Lütkenhaus' [3] groups proved the following no-go result: Deterministic discrimination of all four Bell states with linear optics elements and only one degree of freedom (DOF) (e.g., polarization) is not possible. One can deterministically discriminate only three Bell states and they enable the so-called *dense coding* (channel capacity $\log_2 3 = 1.585$ bits) [4]. Fortunately, the no-go proof does allow a deterministic discrimination with two DOFs in a hyperentanglement setup. Such hyperentanglement experiments have been put forward and carried out [5–8].

Hyperentanglement of photon polarization and its orbital angular momentum recently served Barreiro, Wei, and Kwiat to beat the channel capacity of the dense coding [4] by a tight margin $1.63 > 1.585$ [8] in a postselection experiment. The result has been recognised as "breaking the communication barrier" and such a SC by means of a chosen primary DOF supported by another DOF has been referred as a *mediated SC* [9].

Another kind of hyperentanglement of photon polarization mediated by a time-spatial DOF has been proposed by Kwiat and Weinfurter [10] and carried out by Schuck, Huber, Kurtsiefer, and Weinfurter [5]. They make use of the spatial DOF in order to achieve a time delay.

The main feature of mediated SCs is that photons states are defined by one main DOF (e.g., polarization) and one ancillary DOF (e.g., a time-spatial, spatial, or photon angular momentum). The latter one enables a discrimination of the states of the former one. They require a sophisticated level of controlling qubit states, but at the same time in the existing designs we actually loose more information than in the dense coding. For instance, in the aforementioned hyperentanglement "each hyperentangled state is a unique superposition of four of the sixteen possible combinations of two-photon spin–orbit Bell states" [8].

On the other hand, it was shown that "more entanglement" does not necessarily imply "more computational power" [11] and therefore we considered it viable examining whether SC with mediated photons might be "less entangled." We make use of the so-called *mixed basis* states, two of which are mediated by a spatial DOF, to implement an ideally deterministic 2 bit transfer.

We proposed another mixed basis SC protocol previously in Ref. [12] but that one could not transfer more than 1.43 bits. The present protocol enables Alice to transfer $\log_2 4 = 2$ bits of information, via sending 4 messages to Bob, by manipulating only one photon—called a "travel" photon—from a pair of entangled photons in a Bell state $|\Psi^-\rangle$ generated by Bob. Bob keeps the other photon—called a "home" photon—delayed in a fibre spool. Alice encodes 2 of 4 messages by manipulating the travel photon so as

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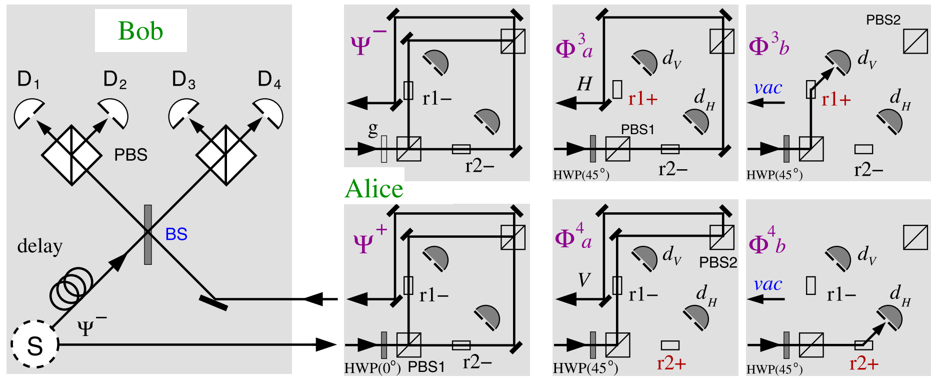

Fig. 1. Schematic of the protocol; Alice sends messages $\Psi^{\mp}, \Phi^{3,4}$; S is a source of photons in state $|\Psi^-\rangle$—see Subsec. 3.1; r1, r2 are routers (see text) which either let the photons through (*off* mode, r1−, r2−) or deflect them (*on* mode, r1+, r2+) into detectors d_V, d_H, respectively; D_{1-4} are photon number dissolving detectors; BS is a standard beam splitter; PBSs are polarizing beam splitters; g is a glass plate which preserves polarization and makes Ψ^--path length identical to the others; Alice sends Ψ^- by turning the routers off and Ψ^+ by keeping them off and sliding in HWP(0°); she sends Φ^3 (Φ^4) by sliding in HWP(45°) and turning r1 (r2) on and r2 (r1) off, indicated by r1+ (r2+) and r2− (r1−), respectively; photons randomly "choose" to exit PBS1 either in the H or V state—indicated as Φ^3a vs. Φ^3b and Φ^4a vs. Φ^4b options; in Φ^3b and Φ^4b vacuum (*vac*) is sent to Bob; d_V (d_H) is triggered [Φ^3b (Φ^4b)] or not [Φ^3a (Φ^4a)]; Bob receives Φ^3-*message* (Φ^4-*message*) as $|H\rangle_1|H\rangle_2$ ($|V\rangle_1|V\rangle_2$)—Φ^3a (Φ^4a), or as $|V\rangle_1|vac\rangle_2$ ($|H\rangle_1|vac\rangle_2$)—Φ^3b (Φ^4b).

to generate $|\Psi^{\mp}\rangle$ states and sends the travel photons to Bob who combines them with his home photons at a beam splitter (BS) and measures them. To send the other 2 messages Alice first generates a $|\Phi^-\rangle$ Bell state and then collapses it to 2 computational states mediated by a spatial DOF: two photon paths; one leads to Bob's BS and he measures the travel and home photons; the other leads to Alice's detector and Bob combines his home photon with the vacuum state at his BS.

As in the aforementioned experiments [8,5], we consider the SC protocol primarily as a computational resource. Thus, we only elaborate on the information transferred from Alice to Bob without Eve (eavesdropping) being involved although we briefly discuss a possible cryptographic implementation in Sec. 4.

The spatial DOF, Bob makes use of, when measuring the photons encoded by Alice, does not contain any information about the polarization states Alice imposes on photons taking different paths and therefore there is no classical information transfer involved in Alice's encoding. The classical information carried by photon spatial DOF is tantamount to the mediation of messages via these modes as in [5].

For our protocol to be feasible, a source of entangled photon pairs on demand or a very efficient source of heralded pairs are required because, for an equal efficiency of measuring both vs. only one of two photons, Bob cannot rely on a postselection as in a cryptography application where only detection of both photons are kept and those of single ones are discarded. None of the so far experimentally implemented candidates for such a source, even the most developed quantum dots, is sufficiently reliable and efficient. Therefore in this paper we come forward with a proposal for a very efficient source of heralded preselected entangled photon pair in a Bell state conditioned on a detection of another pair. The source can be implemented with today's technology so as to have a realistic efficiency of 92%.

An experiment in a postselection mode, similar to postselection experiments carried out in [5,6,8], can be carried out with today's technology as proposed in detail in Sec. 2. Actually, also a full-fledged experiment of the proposal can be carried out with today's technology with even higher efficiency, however, with high end versions of all components.

The paper is organised as follows. In Sec. 2 we give physical and technical details of our protocol and all the definitions of states, messages, and optical elements used in the paper. In Sec. 3 we describe the new source of preselected entangled photon pairs (Subsec. 3.1) and propose a postselection proof-of-principle experiment (Subsec. 3.3). At the end of the section we compare channel capacity of our proposal with previous experimentally obtained ones. In Sec. 4 we summarise and discuss the obtained results. At the end of the section we discuss (in)applicability of our SC protocol to quantum cryptography.

2. Protocol

The superdense coding (SC) is an encoding of four messages into the states of entangled pairs of qubits by means of an interaction with one of the qubits only.

We make use of the following three Bell states

$$|\Psi^{\mp}\rangle = \frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_2 \mp |V\rangle_1|H\rangle_2),$$
$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2 - |V\rangle_1|V\rangle_2), \quad (1)$$

and the following two states from the computational basis

$$|H\rangle_1|H\rangle_2, \qquad |V\rangle_1|V\rangle_2. \quad (2)$$

The Bell states $|\Psi^{\mp}\rangle$ given by Eq. (1) together with the states given by Eq. (2) form a basis called *mixed state basis* or simply *mixed basis*.

Bob prepares $|\Psi^-\rangle$ photon pairs ideally by using a source of entangled photon pairs on demand but realistically by making use of the source we propose in Subsec. 3.1 which can be realised with today's technology so as to have the efficiency of preselecting pairs of 92%. Bob then sends one photon from each pair to Alice who manipulates it so as to send four different messages to Bob. We call her photon a *travel* photon. The other (Bob's) photon from a pair we call a *home* photon. Alice ideally deterministically encodes the following four messages and sends them to Bob:

$$\Psi^+\text{-}message, \quad \Psi^-\text{-}message, \quad \Phi^3\text{-}message, \quad \Phi^4\text{-}message, \quad (3)$$

as shown in Fig. 1. We will discuss non-ideal realistic implementation of the protocol and take losses into account in Sec. 3. We also discuss a particular aspect of a realistic implementation at the end of this section.

To send a Ψ^--*message* Alice keeps both routers (r1, r2) off, meaning that they let photons through without affecting their states, indicated as r1− and r2− in Fig. 1. The routers make use of electro-optical modulators based on rubidium titanite phosphate [13]. When they are turned on, they can deflect incoming photons independently of their polarization unlike the standard optical switches like, e.g., Pockels cells, based on polarization selection.

The home and travel photons are directed to Bob's BS and $|\Psi^-\rangle$ exits from it again as $|\Psi^-\rangle$, triggering either D_1 and D_3 or D_2 and D_4.

To send a Ψ^+-*message* Alice substitutes a halfwave plate whose optical axis and the direction of the horizontal polarization make an angle of 0° (HWP(0°)) for the glass plate g in the path of her photons in front PBS1, while keeping both routers off. HWP(0°) changes the sign of the vertical polarization of her travel photon and turns Ψ^- state into Ψ^+ state, i.e., into a Ψ^+-*message*. The photons bunch on either side of Bob's BS and are detected either by D_1 and D_2 or by D_3 and D_4.

To send a Φ^3-*message* [Fig. 1($\Phi^3 a, b$)] Alice puts HWP(45°) in front of PBS1 and turns r1 on and r2 off. HWP(45°) turns $|\Psi^-\rangle$ into $|\Phi^-\rangle$. At PBS1 $|\Phi^-\rangle$ collapses either into $|H\rangle_1|H\rangle_2$ (with the probability of 50%) or into $|V\rangle_1|V\rangle_2$ (also with the probability of 50%). If the travel photon passed through PBS1 (it and the home photon are then in state $|H\rangle_1|H\rangle_2$), Bob would receive both photons bunched at either side of his BS (according to the Hong–Ou–Mandel effect [14]) and would verify the reception of a Φ^3-*message*. Alice knows she sent and Bob received it because her d_V detector remained silent. This is shown in Fig. 1($\Phi^3 a$). On the other hand, if Alice's travel photon gets reflected from PBS1 (it and Bob's home photon are in state $|V\rangle_1|V\rangle_2$), Bob would receive only one photon which is in state $|V\rangle_1$ and would know that Alice has sent a Φ^3-*message*. Alice also knows that she sent and Bob received it because her travel photon in state $|V\rangle_2$ triggers her detector d_V. This is shown in Fig. 1($\Phi^3 b$).

To send a Φ^4-*message* [Fig. 1($\Phi^4 a, b$)] Alice keeps HWP(45°) and turns r1 off and r2 on. If the travel photon reflected from PBS1 Bob would receive both photons bunched at either side of his BS. Alice knows she sent it because her d_H remained silent [Fig. 1($\Phi^4 a$)]. If a travel photon passed through Alice's PBS1, Bob would receive $|H\rangle_1$. Alice's travel photon in state $|H\rangle_2$ triggers d_H [Fig. 1($\Phi^4 b$)].

To sum up, Alice sends Φ^3-*message* by turning r1 on and r2 off and Φ^4-*message* by turning r2 on and r1 off. Alice cannot control whether her Φ^3-*message* (Φ^4-*message*) will reach Bob as a $|H\rangle_1|H\rangle_2$ ($|V\rangle_2|V\rangle_2$) or as a $|V\rangle_1$ ($|H\rangle_1$), i.e., whether the travel photon will be mediated to Bob via one or none paths (spatial DOF). These alternative paths are created by the "choice" of the photons at PBS1 completely at random, but ideally, Alice nevertheless has a full deterministic control over her sending and Bob over his receiving of each message.

In order to discriminate between $|H\rangle_1|H\rangle_2$ ($|V\rangle_2|V\rangle_2$) and $|H\rangle_1$ ($|V\rangle_1$) behind Bob's BS, we make use of superconducting transition edge sensor (TES) photon number resolving detectors which detect two photons in one step. The highest efficiency of such detectors is currently over 98% [15–17]. The dark count probability of TES detectors is practically zero.

The coincidence clicks shown in Table 1 correspond to an ideal deterministic discrimination of all four messages with photon number resolving detectors.

Photon number resolving detectors are nonlinear devices and, strictly speaking, state analysis would not be considered linear in their presence. However, TES detectors are inherently photon number resolving [16], i.e., the same detectors are used as single and as multiple photon detectors, and on the other hand, one can always use concatenated beam splitters and linear single photon detectors, instead (however at the cost of exponentially increased number of detectors). Also the crystal sources are always nonlinear. Thus, it has been accepted to call a setup linear when linear optical elements are used together with photon number resolving detectors—see, e.g., [5].

Let us summarise ideal deterministic sending and receiving of messages:

Table 1
Ideal discrimination of all four Alice's messages with photon number resolving detectors; D^2 in the first two rows of Φ^3-*message* and Φ^4-*message* indicate a detection of 2 photons by the same number resolving detector; next two rows of Φ^3-*message* and Φ^4-*message* indicate Bob's detection of home photons by his detectors $D_{2,3}$ and $D_{1,4}$ and Alice's detection of travel photons by her detectors d_V and d_H—see Fig. 1.

	"clicks" at		
Ψ^--*message*	D_1 & D_3	OR	D_2 & D_4
Ψ^+-*message*	D_1 & D_2	OR	D_3 & D_4
Φ^3-*message*	D_1^2	OR	D_4^2
	D_2 & d_V	OR	D_3 & d_V
Φ^4-*message*	D_2^2	OR	D_3^2
	D_1 & d_H	OR	D_4 & d_H

$$
\begin{array}{lcl}
\text{Alice} & & \text{Bob}\\
\Psi^-\textit{-message}: & |\Psi^-\rangle & \rightarrow\ \Psi^-\textit{-message}\\
\Psi^+\textit{-message}: & |\Psi^-\rangle \ \rightarrow\ |\Psi^+\rangle & \rightarrow\ \Psi^+\textit{-message}\\
\Phi^3\textit{-message}: & |\Psi^-\rangle\rightarrow|\Phi^-\rangle\rightarrow\left\{\begin{array}{c}|H\rangle_1|H\rangle_2\\ (|V\rangle_2\rightarrow d_V)\quad |V\rangle_1\end{array}\right\} & \rightarrow\ \Phi^3\textit{-message}\\
\Phi^4\textit{-message}: & |\Psi^-\rangle\rightarrow|\Phi^-\rangle\rightarrow\left\{\begin{array}{c}|V\rangle_1|V\rangle_2\\ (|H\rangle_2\rightarrow d_H)\quad |H\rangle_1\end{array}\right\} & \rightarrow\ \Phi^4\textit{-message}
\end{array}
$$

In a realistic implementation with losses the symmetry of Bob's reception of Φ^3 and Φ^4 *messages* via $|H\rangle_1|H\rangle_2$ and $|V\rangle_1|V\rangle_2$, in contrast to his reception of these messages via $|V\rangle_1|vac\rangle_2$ and $|H\rangle_1|vac\rangle_2$, is broken. In particular, when one of the photons from $\Psi^{\mp}$, $\Phi^3 a$ ($|H\rangle_1|H\rangle_2$), or $\Phi^4 a$ ($|V\rangle_1|V\rangle_2$) is lost in transmission, a detection of the other photon is indistinguishable from a detection of $\Phi^3 b$ ($|V\rangle_1|vac\rangle_2$) or $\Phi^4 b$ ($|H\rangle_1|vac\rangle_2$). We shall elaborate on this aspect of losses and take it into account while calculating the efficiency of the protocol in Subsec. 3.2.

3. Source, efficiency, and postselection proof-of-principle experiment

A realistic implementation of our setup as well as its postselection proof-of-principle experiment are feasible with the current technology. For the former experiment we need TES photon number resolving detectors and for the latter at least three very low dark count rate ones.

Another crucial point of these two experiments—and of any their future implementation—is the choice of a source which would ideally be a source of entangled photon pairs *on demand* or realistically a high efficiency source of *preselected* entangled photon pairs.

3.1. Source of preselected entangled photon pairs

Before we dwell on the design of our source we have to review some previous sources and theoretical results behind them.

Recently generated entangled photon pairs in quantum dots [18,19] have been announced to be on-demand but so far they are on demand only with respect to its high purity (very small probability that multiple pairs will be generated instead of single pairs). The problem with them is their photon collection efficiency; the probability of collecting photon pairs (and therefore also sending them in a chosen direction) is currently under 1%: "The external efficiency, that is, the collection efficiency of the set-up, was estimated [to be] ∼0.4%" [19].

Similar collection efficiency problem exists with cascade emissions from atoms [20] and down-converted photon sources [21, 22] because in neither of the processes the generation location and direction of the photons can be well determined, in principle. However, apart from the low collection efficiency, by using only two pairs coming from two sources, we can preselect a photon pair conditioned on detection of another pair. For instance, for photon pairs coming from atoms in a spontaneous cascade emission this amounts to a preselection of an entangled pair conditioned on detection of another pair [20]. The spontaneous emission occurs only once and with them we have a genuine preselection: by detecting two photons at a beam splitter we entangle the other two photons into the state $|\Psi^-\rangle$ "although their trajectories never mix or cross" [20]. With photon pairs down-converted in two nonlinear crystals, on the other hand, the probabilities of their conversion in each of the crystals or in just one of them are about the same and the latter ones will ruin the preselection if we stick to the standard way of combining photons from two different crystals at a BS in an attempt to entangle the other two photons.

Having such standard way of combining photons at a BS in mind, Śliwa and Banaszek wrote: "generation of a double pair in one crystal and none in the second crystal [23] [is] a fundamental obstacle in the conditional preparation of maximal entanglement from four down-converted photons: it has been shown [by Kok and Braunstein [24]] that a maximally entangled state cannot be generated [conditioned] on detection of two auxiliary photons. This rules out the possibility [making use] of overall four photons, to produce maximally entangled pairs by means of conditional detection" [25].

Kok and Braunstein obtained their result under the following assumption: "every detector needs to detect at most one photon" [24] and if we found a way to preselect a pair of entangled photons conditioned on detecting more than one photon by some detectors we would be able to go around Śliwa and Banaszek's conclusion. This is exactly what we did in the design of our source below.

However, before we dwell on it, let us first deal with the problem of low collection efficiency. To overcome it, we turn to experiments with heralded generations of entangled signal and idler colinear photons in a pair of opposite polarization down-converted in a type II poled potassium titanyl phosphate (PPKTP) crystal placed in a cavity carried out by Benson's groups [26–29]. Signal and idler are filtered from the central peak of a comb-like distribution of photon frequencies in a cavity [30] so as to have the same frequency what makes them indistinguishable up to their polarization. A very important feature of so filtered signal and idler is that the visibility of their coincidence rate is over 96.5%, without background subtraction [28,31]. Closely related is the efficiency of generating an entangled pair in contrast to generating just one of the photons (i.e., loosing one of them) which can be extrapolated from the recently obtained experimental results [31] for which data show that with the current setup an efficiency of 80% can be achieved.

Now, in Fig. 2 we present the design of our source in which we make use of a Mach–Zehnder interferometer (MZI) to single out photons entangled in a Bell state while discriminating between them and the photons coming from just one crystal—the latter photons will bunch at either exit of the MZI obeying its basic symmetry: what comes in from one of its sides, will exit from the other. As for the former photons, the design enables them to interfere after taking either of two indistinguishable paths, like in Franson's interferometer [32–34]—only here we do not have long vs. short paths but equally balanced *i* vs. *ii* paths as well as correlated *iii* vs. *iv* paths.

The intensity of the pump beam is lowered down so as to down-convert predominantly two pairs of entangled photons, each in state $|HV\rangle$ within a chosen time window. Each pair can be down-converted in each crystal or both of them in one of the crystals, with the same probability.

Photons from a cavity can take four different paths after being split at the first polarizing beam splitter (PBS). When we look at options to detect exactly two photons by our detectors behind MZI, we see that this is realised in exactly two ways.

The first way is when the signal (H) coming from the top cavity takes path *i* and idler (V) from the bottom one takes path *ii*. The

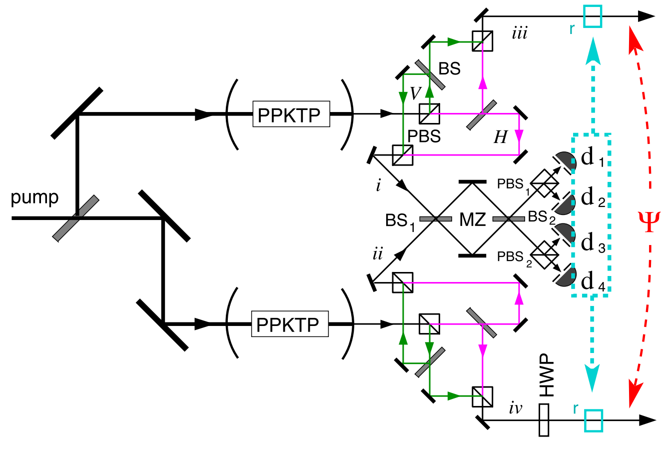

Fig. 2. Proposal of a heralded generation of an entangled photon pair in the $|\Psi^-\rangle$ state conditioned on detecting another pair of photons in the $|\Psi^+\rangle$ state. All paths are equal in length to enable interference (not shown). Routers (r) which block exit photons that are not in state $|\Psi^-\rangle$ are triggered by detectors d_i, $i = 1, \dots, 4$.

top idler (V) is then going to *iii* and the bottom signal (H) to *iv*. This combination is indistinguishable (with respect to BS_1) from the top idler (V) going to *i* and bottom signal (H) to *ii* while top signal (H) is going to *iii* and bottom idler (V) to *iv*. Because of the aforementioned indistinguishability, the input state to BS_1 is

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_2 + |V\rangle_1|H\rangle_2). \tag{4}$$

At BS_1 it transforms to [35, Eqs. (1.160,1), pp. 72,73], [36, Eq. (4.28), p. 68]

$$|\Psi^+\rangle \quad \xrightarrow{BS_1} \quad \frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_1 - |H\rangle_2|V\rangle_2) = |\Psi^-\rangle_{12}, \tag{5}$$

and at BS_2 it gives the following output

$$|\Psi^-\rangle_{12} \quad \xrightarrow{BS_2} \quad \frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_2 + |V\rangle_1|H\rangle_2) = |\Psi^+\rangle. \tag{6}$$

The photons will be detected at opposite sides of BS_2 and the detectors will trigger two r's so as to let the *iii*, *iv* photons through.

The companion photons go to *iii* and *iv* in two aforementioned indistinguishable combinations and form the following state

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_2 + |V\rangle_1|H\rangle_2), \tag{7}$$

which HWP transforms into the final output state:

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_2 - |V\rangle_1|H\rangle_2). \tag{8}$$

When routers (r) do let the photons through in state $|\Psi^-\rangle$ they must relay photons so as not to affect their polarization. Therefore we make use of routers with electro-optical modulators based on rubidium titanite phosphate so as to preserve the entangled state [13].

The only other way of having only two photons with different polarization going to *i* and *ii* is when either both the top idler and signal are going to *i* (while the bottom signal and idler are going to *iv*) or (indistinguishably) the bottom idler and signal are going to *ii* (while the top signal and idler are going to *iii*) in which case we have

$$|\Psi^+\rangle_{12} = \frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_1 + |H\rangle_2|V\rangle_2) \tag{9}$$

as an input to BS_1. MZI does not change the state, i.e., the photons will be detected bunched at one side of BS_2 and the detectors will trigger two routers (r) so as not to let the *iii*, *iv* photons through.

These two ways of having only two photons with different polarization going to *i* and *ii* constitute the most important part of our design since they amount to a discrimination between pairs of photons coming from two crystals given by Eq. (7) and those coming from either one or the other given by Eq. (9). This is what boosts the overall efficiency of the combined source so as to become considerably higher than the efficiency of each of its sub-sources as we show below.

Also the other combinations of 0, 1, 3, 4 photons that arrive at MZI are eliminated by the number resolving detectors d_1–d_4 and r's. In particular, when both pairs (or even three) are generated in only one of the crystals, then all photons entering in whatever state from one side of BS_1 will exit from the opposite side of BS_2 and trigger either d_1 and/or d_2 or d_3 and/or d_4 so as to activate r's and block the exits of *iii* and *iv* photons. For instance $|H\rangle_1|V\rangle_1$ will become $\frac{1}{2}(|H\rangle_1|V\rangle_1 + |H\rangle_2|V\rangle_2 - |H\rangle_1|V\rangle_2 - |V\rangle_1|H\rangle_2)$ after BS_1 and $|H\rangle_2|V\rangle_2$ after BS_2.

Let us now calculate the efficiency of our source. We have stated above that the efficiency of obtaining a pair (in contrast to obtaining just one of the photons or none) experimentally achieved with a PPKTP crystal in a cavity and with a frequency filter can be as high as 80%. For a crude combination of two such sources the overall efficiency would be $0.8^2 = 0.64$, but by means of triggering d_1–d_4 we actually rise the overall efficiency of the combined source so as to become considerably higher than the efficiency of each of the sub-sources taken independently. $1 - 0.64 = 0.36$, i.e., 36% of losses equal the probability of obtaining all the unwanted photons that are *not* those 4 photons which belong to 2 different pairs that were down-converted in 2 different crystals. The possibilities of having 1 or 3 pairs down-converted are also included in this probability, but with taking into account reduced chances of occurring within Poissonian distribution. We neglect possible 4 or more down-converted pairs.

The only cases for which r's let the exiting *iii*, *iv* photons through are those for which 2 photons in state $|\Psi^+\rangle$ enter MZI and are measured by d_{1-4} which trigger r's. The cases that do not belong to such 2 photon measurements are: 0 and 1 photons from 1, 2, or 3 pairs; any number of photons coming from the pairs down-converted in the same crystal; 3 to 6 photons coming to d_{1-4}. Cases with 2 photons in state $|\Psi^+\rangle$ arriving at d_{1-4} from different cavities and triggering a release of unwanted *iii*, *iv* photons are: 2 photons from 2 pairs will send a vacuum state out with the probability of 12.5%; 2 from 3 pairs—vacuum—12.5%; 3 from 2 pairs—will send single photons—6.2%; 3 from 3—single photons—6.2%; 4 from 3—2 photons in state which is not $|\Psi^-\rangle$—18.7%; 5 from 3—3 photons—25%; 6 from 3—4 photons—3.1%. We also have 4 photons from 3 pairs which will send out 2 photons in state $|\Psi^-\rangle$ with the probability of 25% and which we have to add to 64%. A more detailed description and calculations will be given elsewhere [37].

Taking into account that the probabilities with which 1 or 3 pairs are down-converted in a Poissonian distribution (in contrast to just 2 of them) is ca. 4%, we get, after taking out all the cases eliminated by detectors d_1–d_4, that two entangled photons will be generated in state $|\Psi^-\rangle$ with the probability of 98%. When we include the efficiency of TES detectors of 98% we obtain that the overall efficiency of our source is $0.98 \times 0.98^3 \approx 0.922$, i.e., ca. 92%, since on average (less than) ca. 3 detectors are required to fire simultaneously.

3.2. *Efficiency of the setup*

To obtain the overall efficiency of our SC setup, the efficiency of the source we calculated in the previous section should be combined with the efficiency of detectors and optical elements, i.e., the losses in them. Variable losses are those in fibres which rise exponentially with their length, so, we will calculate them after we estimate the other fixed losses to determine the distances at which the protocol can be efficiently implemented as a computational resource.

The BS losses can be lower than 1%. Misalignment losses are up to 2%. Routers losses are below 1%. TES detector efficiencies are 0.98 and their dark count probability is zero. This amounts to an efficiency of $0.99^2 \times 0.98^2 \approx 0.941$ for elements and detectors which we multiply by the efficiency of the source to obtain the overall efficiency $0.941 \times 0.92 \approx 0.87$.

Let us now calculate the lower bound for our protocol so as to have the channel capacity just over 1.63 bits achieved in [8] (for dense coding with 3 messages it is 1.585 bits), i.e., the maximal fibre losses that would allow it. The transmission of a fibre at a distance L, i.e., probability p that Bob would detect a photon at the end of a fibre he makes use of, is $p = 10^{-\alpha L/10}$ [38], where α is the attenuation of the fibre; $\alpha = 0.16$ dB/km in commercially available ultralow-loss fibres [39].

When calculating the losses we take into account that Bob's detection of $\Phi^3 b$ and $\Phi^4 b$ messages, i.e., of single V and H photons, respectively, are less efficient than his two-photon detection of $\Psi^\mp$, $\Phi^{3,4}a$ messages since he cannot distinguish them from the latter messages that lost one photon and therefore also have one-photon detection. For instance, Bob cannot distinguish V photon of $\Phi^3 b$ from V photons of $\Psi^\mp$ whose H photon was absorbed in the fibre or from $\Phi^4 a$ ($|V\rangle_1|V\rangle_2$) whose other V photon was absorbed. Let us calculate the probabilities for the latter events. Bob will detect both photons with the probability of p^2 and none with the probability of $(1-p)^2$. That means that the probability of detecting one photon of $\Psi^\mp$, $\Phi^{3,4}a$ messages is $1 - p^2 - (1-p)^2 = 2p(1-p)$, i.e., $p(1-p)$ for each of H, V polarizations. As for Bob's detection of single home photons from $\Phi^{3,4}b$ messages we have to take into account that Alice must have registered the travel photons in her $d_{V,H}$ detectors. Since $d_{V,H}$ are at half the distance ($L/2$) the probability of the travel photon reaching it is $\sqrt{p}$, as follows from the above given expression for p. Hence, the probability of Bob registering a $\Phi^{3,4}b$ message is $p\sqrt{p} = p^{3/2}$.

So, the probability that Bob will register a single V or H photons per sent message is $(0.25 + 0.25 + 0.125)p(1-p) + 0.125p^{3/2} = 0.125p(5 - 5p + \sqrt{p})$, the probability that they stem from $\Phi^3 b$ is $P_b(p) = \sqrt{p}/(5 - 5p + \sqrt{p})$, the probability that they stem from $\Psi^\mp$–$\Phi^4 a$ is $P_a(p) = 5(1-p)/(5 - 5p + \sqrt{p})$, and their ratio is $R_{ab}(p) = 5(1-p)/\sqrt{p}$. Four messages which Bob should receive when there are no losses, are therefore, in the presence of losses, reduced to: $N(p) = 3p^2 + \sqrt{p}/(5 - 5p + \sqrt{p})$. Number of messages that correspond to 1.63 bits obtained in [8] is $2^{1.63} \approx 3.095$. With our efficiency of 0.87 that would correspond to transferred 3.56 messages without losses. From $N(p) = 3.56$ we get $p \approx 0.956$ as a lower bound for p. It yields the maximal distance of 1.22 km for the aforementioned low-loss fibres. This suffices for a verification of our protocol as a computational resource under today's realistic losses. For a 100 m fibre in a laboratory, we get 3.45 messages with losses included and that corresponds to the channel capacity of 1.78 bits. The corresponding attenuated probability $p \approx 0.996$ yields $P_a \approx 0.018$ and $P_b \approx 0.982$ i.e., when measuring single-photon-messages, Bob will detect a wrong message with the probability of ca. 1.8%. With a near-future source of multiplexed heralded (almost on demand) entangled photons and ultralow-loss fibres with $\alpha = 0.1$ dB/km [39] we should have >4 km as the upper distance bound and the channel capacity of >1.98 bits at 100 m which might be the length suitable for an incorporation in a would-be quantum circuit.

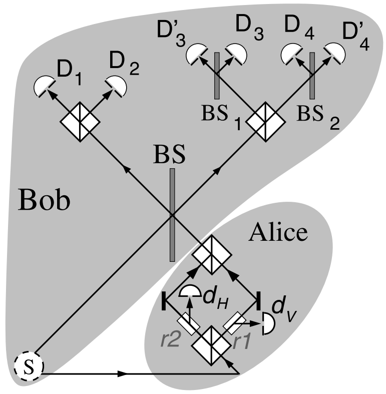

Fig. 3. Proposal of a proof-of-principle postselection experiment. Paths that lead to BS are balanced.

3.3. Postselection experiment

We make use of the source described in Subsec. 3.1. As for the other components needed for the postselection experiment we will keep to the standard off-the-shelf elements, except for at least three of the detectors that should have a rather low dark count rate. The superconducting TES detectors have practically no dark counts at all, but InGaAs avalanche photodiodes operating at $-60\,^\circ$C with ca. 0.1% dark count probability [40] would serve our purpose as D_4 and D'_4 as shown in Fig. 3. A review of 16 detectors with respect to their efficiencies, dark count rates, etc., is given in [41]. For our postselection experiment it is actually better to choose detectors with lower efficiency because the dark count probability always rises with the efficiency. We shall assume to have detectors with an efficiency of 50% and the dark count probability of 1%. Alice needs to have reliable confirmations on sent messages with only home photons arriving at D_4 or D'_4 so that her d_H should also be an InGaAs one.

We need not have a full fledged setup but only its essential part as for example in the SC experiment carried out by Weinfurter's group [5]. This means that we should be able to carry out calibrated comparison of data collected for all four messages. For that Bob needs only his two InGaAs detectors D_4 and D'_4 and the four other ones (D_1–D'_3) can be the standard off-the-shelf detectors operating at the room temperature, because he can always calibrate the obtained data with respect to the former two detectors. To simplify the calculation we shall, however, assume that all six Bob's detectors D_1–D'_4 are of the same kind.

When two photons with parallel polarization arrive at BS_1 and BS_2 shown in Fig. 3 they have 50% probability of splitting at them and we should calibrate all other measurements with respect to the measurements by detectors after such a possible splitting as follows. Bob detects both photons of $\Psi^{\mp}$ messages. Ψ^{-}-*message* triggers either [D_1 and (either D_3 or D'_3)] or [D_2 and (either D_4 or D'_4)] (see Table 1) with the detection probability of 100% (ideally). Similarly, Ψ^{+}-*message* triggers either [D_1 and D_2] or [(either D_3 or D'_3) and (either D_4 or D'_4)] also with the detection probability of 100% (ideally).

As for $\Phi^{3,4}$-*messages* with both home and travel photons at BS, they will, according to the Hong–Ou–Mandel effect [14], either go to the left or to the right from BS in Fig. 3. Those going to the left (50% of them) will end up either in D_1 (both photons) or D_2 (both photons). Since they will give Bob a single "click" in either D_1 or D_2, Bob will discard these recordings. The other half of $\Phi^{3,4}$-*messages* with both home and travel photons will go to the right and half of them will bunch together and the other half will split at either BS_1 (Φ^4) or BS_2 (Φ^3). Here the Hong–Ou–Mandel effect does not apply because both photons come from the same side. Thus D_3-and-D'_3 simultaneous clicks as well as D_4-and-D'_4 simultaneous clicks will altogether collect 25% of all $\Phi^{3,4}$ messages of home and travel photons and the statistics of postselected measurements of all four kinds of messages should be then calibrated because when a correlated detection is carried out and two detectors are used and the efficiency of each of them is 0.5, then their joined detection is carried out with the efficiency of $0.5^2 = 0.25$. However, when single detections are carried out for the other half $\Phi^{3,4}$-*messages* with only home photons which did not split at either BS_1 or BS_2 (altogether 75% of $\Phi^{3,4}$-*messages*) then they are carried out with the efficiency of 0.5, which means that Bob will have to divide the number of these detections by 2 what corresponds to the efficiency of $0.5^2 = 0.25$. for correlated detections of two detectors for all the other measurements. In the end, he multiplies all the obtained $\Phi^{3,4}$ data by two to compensate for the discarded D_1, D_2 data.

The next step is to see which losses and efficiencies can affect our final result. The efficiency of the source is 0.92 and the efficiencies of the optical elements we obtain as follows. BS losses can be lower than 1% but only the differences between the right side with additional two BS and the left side without them are relevant for our postselection experiment. Misalignments and path differences might cause up to 3% of losses. Router losses can be as low as 1%. Then we take the dark counts into account by multiplying the 50% of split $\Phi^{3,4}$-*messages* by 0.99, corresponding to the detector efficiency ($1 - 0.01 = 0.99$, where 0.01 is the dark count probability). We should add ca. 4% for miscalibration. So, the element losses, detector dark count "gains," and miscalibration amount (by multiplication) to an efficiency of ca. 0.91, what is lower than for a full fledged experiment obtained in Subsec. 3.2 but this is due to the miscalibration and dark count errors. Altogether, an overall efficiency of $0.92 \times 0.91 \approx 0.84$ covers losses in the source, in the optical elements, by dark counts, and miscalibration.

To obtain the number of successfully transferred messages we assume to have 10 m low-loss fibre and following the procedure from Subsec. 3.2 we get 3.36 messages. From this we obtain that the mutual information between Alice and Bob, i.e., the channel capacity is $\log_2 3.36 \approx 1.75$ bits.

This can be compared with the best postselection channel capacities achieved so far: $1.63 > \log_2 3 \approx 1.585$ [8] and 1.18 [5].

4. Results and discussion

We have shown that Alice can carry out a full mediated deterministic superdense coding (SC) by manipulating only one photon from a pair of entangled photons to generate four messages which Bob can unambiguously discriminate by beam splitters (BS) and two polarizing beam splitters (PBS) as shown in Fig. 1.

For this to work, ideally a would-be source of such pairs on demand would be required but for an immediate implementation with today's technology, in Subsec. 3.1, we propose a new source of heralded pairs in a Bell state conditioned on detection of another pair of ancillary photons (see Fig. 2) whose realistic efficiency can be as high as 92%.

Alice makes use of two Bell states $|\Psi^{\mp}\rangle$ to send $\Psi^{\mp}$-*messages* and one $|\Phi^{-}\rangle$ to send $\Phi^{3,4}$-*messages* [see Eqs. (1) and (3)]. To send the latter messages she first collapses $|\Phi^{-}\rangle$ into two states from the computational basis, $|H\rangle_1|H\rangle_2$ and $|V\rangle_1|V\rangle_2$, at her PBS1 with the 50:50 probability, i.e., completely randomly. That means that Alice is not able to obtain $|H\rangle_1|H\rangle_2$ or $|V\rangle_1|V\rangle_2$ at will (such a possibility would be tantamount to her sending superluminal messages to Bob) but a clever design with optical routers and her own detectors d_V and d_H shown in Fig. 1 enables her to nevertheless ideally deterministically send Φ^3-*message* (Φ^4-*message*) mediated by the spatial degrees of freedom (DOF)—two paths: one leading her travel photon $|H\rangle_2$ ($|V\rangle_2$) to Bob's BS and the other leading her travel photon $|V\rangle_2$ ($|H\rangle_2$) to her detector d_V (d_H). So, Bob receives Φ^3-*message* (Φ^4-*message*) either in a $|H\rangle_1|H\rangle_2$ ($|V\rangle_1|V\rangle_2$) state or in a single $|V\rangle_1$ ($|H\rangle_1$) home photon state. Alice sends Φ^3-*message* (Φ^4-*message*) by switching her r1 (r2) on and her r2 (r1) off. Alice knows that she sent Φ^3-*message* (Φ^4-*message*), and via which path, according to whether her d_v (d_H) triggered or remained silent. This is why we had to design a high efficiency (realistically 92%) source of photon pairs preselected in a Bell state in Subsec. 3.1. We also had to estimate and calculate Bob's ambiguities in reading $\Phi^{3,4}$-*messages* mediated by single home photons in presence of losses in fibres in Subsec. 3.2. This gave us the efficiency of SC and the channel capacity as a function of the fibre length.

It is important to recognise here that a bare *classical information* on whether Bob received two photons or one photon, i.e., on his reading of the spatial DOF, does not enable Bob to read off the Φ-*message* Alice's has sent him. He has to carry out a polarization measurement in the polarization DOF to find that out. In other words, the classical information refers only to the mediation of the messages, not to the messages themselves—similarly to the experiment carried out by the Weinfurter's group [10] where Bob also cannot discriminate between the states only by measuring their time delay (time DOF).

In the latter experiment, Alice only controls HWPs to generate the Bell states wile in our setup, Alice controls both DOFs but in neither experiment the supporting DOF carries any information on supported polarization DOF taken separately. The latter DOF is only mediated by the former one.

In the Barreiro–Wei–Kwiat protocol (mentioned in Sec. 1), photons are in superposed states of H and V polarization and paraxial spatial modes carrying $+\hbar$ and $-\hbar$ units of orbital angular momentum (OAM). This enables one to encode and decode all four polarization Bell states. Here, also by reading off *only* OAM states one cannot read off any polarization encoded states with any probability higher than the one of casting dice. Again, the polarization DOF is only mediated by the angular momentum DOF.

We do not consider any cryptography application of our protocol since mediated SC protocols are primarily of importance for a possible engineering of quantum gates within quantum computation circuits and for this application, the comparatively short maximal applicable fibre lengths we obtained in Subsec. 3.2 are acceptable.

At the end of Sec. 3 we compare our postselection channel capacity (for the fibre attenuation and length specified there) with the best other mediated SC ones achieved so far: 1.63 bits [8] and 1.18 bits [5]. For a realistic and feasible postselection experiment presented in Subsec. 3.3, the overall efficiency would be ca. 84%. That would enable our Alice to unambiguously transfer ca. 3.36 messages by acting on her photon only, i.e., to reach the channel capacity of ca. 1.75 bits. This capacity convincingly beat all the previously achieved ones. However, the most important result is that with a full-fledged realistic implementation, feasible with today's technology one can achieve the channel capacity of 1.78 bits, as shown in Subsec. 3.2, with *preselected* pairs of entangled photons.

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