Physical Review A **50**, 3486–3491 (1994)

SPIN CORRELATED INTERFEROMETRY FOR POLARIZED AND UNPOLARIZED PHOTONS ON A BEAM SPLITTER

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(May 20, 1994)

Abstract

Spin interferometry of the 4th order for independent polarized as well as *unpolarized* photons arriving simultaneously at a beam splitter and exhibiting spin correlation while leaving it, is formulated and discussed in the quantum approach. Beam splitter is recognized as a source of genuine singlet photon states. Also, typical nonclassical beating between photons taking part in the interference of the 4th order is given a polarization dependent explanation.

Ms number AP5065. PACS numbers: 03.65.Bz, 42.50.Wm

I. INTRODUCTION

Quite a number of papers were recently engaged in the nonclassical 4th order interference of independent sources [1–18]. It proved to be a powerful tool for checking on possible new quantum principles and features as well as on the nonstandard interpretations of quantum phenomena. E.g., Dirac's principle *each photon interferes only with itself* seems to be valid only for the standard interference of the 2nd order while for the nonclassical interference of the 4th order it should read: *each pair of photons interferes only with itself* [3,6]. As for the nonstandard interpretations, the nonclassical interference of the 4th order, in particular with the down–converted beams, was recently used for disproving both local and nonlocal hidden–variable theories. Ou and Mandel [1,7,9] have elaborated and carried out a new type of the Bell–like experiments against local hidden–variable theories which was then extended by Yurke and Stoler [12] to three independent sources, by Żukowski, Zeillinger, Horne, and Ekert [13] to independent correlated pairs in the configuration space, and by Pavičić and Summhammer [14,15] to independent correlated pairs in the spin space. On the other hand Wang, Zou, and Mandel [17] carried out an experiment to test de Broglie–Bohm pilot (guiding, "ghost") waves (without the latter being physically blocked) according to a set–up proposed by the Selleri–Croca school and obtained a negative result.

The afore mentioned extention in the spin space brought us to a new phenomenon — spin correlated interferometry — which asks for an independent elaboration. So, in this paper we elaborate the *spin correlated interferometry* of the 4th order for two polarized as well as *unpolarized* photons arriving simultaneously at a beam splitter and in a forthcoming paper [16] the spin correlated interferometry for the independent pairs of spin (polarization) correlated photons. We call the phenomena *spin correlated interferometry* because it turns out that two photons which simulataneously leave a beam splitter, always leave it correlated in spin no matter how they were prepared, *i.e.*, no matter whether they were previously polarized or not. The interferometry is based on an experiment we put forward in Refs. [14,15] which is a realization of the 4th order interference of randomly prepared independent photons correlated in polarization and coming from independent sources.

Of the two experiments we consider in this paper, the first one puts together two polarized photons, makes them interact on a beam splitter, and allows us inferring the dependence of the typical nonclassical 4th order *beating* on the mutual polarization of the incoming photons when no polarization is measured as well as inferring modulated polarization (spin) correlations when it is measured. The second experiment puts together two unpolarized photons coming out from two simultaneous but independent cascade processes of two simulataneously excited independent atoms, makes them interact on a beam splitter, and then allows us inferring polarization (spin) correlations by simultaneous measurement of the polarizations of the photons. Thus we recognize beam splitter as a device for detection and preparation of spin correlation between incoming and/or outgoing photons as well as a source of genuinely unpolarized photons. This in turn allows us to substitute two additional beam splitters for the above cascade sources.

On the other hand, the present elaboration of the 4th order interference on a beam splitter in the spin space attempts to fill a gap in the literature. For, while the interference of the 4th order in the configuration space has been elaborated in detail in the literature [1,3,4,10,11], the interference lacks a detailed elaboration and apparently a proper understanding in the spin space. One of the rare partial elaborations was provided by Ou, Hong, and Mandel for a special case of orthogonally polarized photons [19]. They clearly recognized that orthogonally polarized photons incoming to a symmetrically positioned beam splitter produce a *singlet–like* state at a beam splitter [1,7,9,19] and that parallelly polarized photons incoming to a symmetrically positioned beam splitter never appear on its opposite sides [20] but it does not seem to have been recognized that the polarization of incoming photons actually does not have any effect on the correlation in polarization of the outgoing photons and that it only affects the intensity of the outgoing photons. That was also apparently the reason for not realizing that a beam splitter produces not a *singlet–like* state but a genuine singlet state and not only for polarized but for unpolarized incident photons as well.

To be able to follow the main features of the experiments presented in Sec. II we develop the basic formalism in Sec. III and we shall calculate the setups in the plane–wave approach in Sec. IV. In Sec. V we discuss the obtained interference patterns.

II. EXPERIMENTS

The essential part of both experiments — for polarized as well as for unpolarized photons — is presented in Fig. 1 which is, in effect, a slightly modified figure from Ref. [21]. The only difference for the two cases below are the sources.

A. Polarized photons

Two incoming independent photons in Fig. 1 are emerging as signal and idler photons from a nonlinear crystal as in the experiment of Ou and Mandel [7] with the only difference that the polarization rotator can be turned in any appropriate direction without affecting the output correlation. Signal and idler photons of frequences ω_1 and ω_2 are produced in the process of parametric down conversion of a laser beam (of the frequency $\omega_0 = \omega_1 + \omega_2$) that interacts with the nonlinear crystal (*e.g.*, $LiIO_3$).

The two so obtained independent photons are then directed to a beam splitter from opposite sides. Photons coming out from the beam splitter pass polarizers P1 and/or P2 and fall on detectors D1 and/or D2. In an actual setup a birefringent prisms should be used for polarizers (allowing detection of polarization P and the perpendicular polarization $P^\perp$) so as to enable *zero detections* by appropriate $D1^\perp$,$D2^\perp$ detectors (not shown in Fig. 1). Pulse pairs arriving within an appropriate time interval (typically 5 ns or shorter) are taken as coincidence counts. The obtained coincidence counts are ascribed to the probability of detecting two photons for possible settings of incoming and outgoing (polarizers P1 and P2) polarizations.

Such a setup can however be objected that it fails to adequately record photons when they both go to one arm and when their triggering of the detectors should be disabled. To match this possibility we can use frequency filters (prisms) that would separate the photons emerging from the beam splitter according to their frequency and would direct them to two birefringent polarizers P_{ω_1} and P_{ω_2} and through them to four detectors D_{ω_1},$\mathrm{D}^{\perp}_{\omega_1}$,$\mathrm{D}_{\omega_2}$,$\mathrm{D}^{\perp}_{\omega_2}$ in each arm. Simultaneous firing (i.e., within the shortest time feasible) of at least two detectors in one arm then discards the corresponding recording in both arms from the set of *coincidence counts*. In such a way we are able to *preselect* a genuine singlet state of the photon pair emerging from different sides of the beam splitter (see Sec. IV A 1). The setup also enables an experimental verification of the behavior of photons when they both emerge from the same side of a beam splitter presented in Sec. IV A 2.

B. Unpolarized photons

Signal and idler down–converted photons emerging from independent nonlinear crystals are parallelly polarized ($\pm 2^\circ$ relative to the uv pump laser beam [7]). So, such sources cannot be used to obtain unpolarized incoming photons but we have at least two available sources. One is a cascade process, e.g., $(J = 0) \rightarrow (J = 1) \rightarrow (J = 0)$. It can be triggered by a simultaneous pumping of a split laser beam. Due to the random phases of photons emitted from two distinct atoms we shall have no interference of the 2nd order at all. In previous experiments where sources of unpolarized photons were needed, the sources were poorly localized because a better localization was not necessary. E.g., in Aspect's experiments [22] the source atoms where located in a $60 \times 60\ \mu$m region, i.e. within the laser beam waist diameter of the focused pumping laser beam. Recently, however, trapping of single atoms (as opposed to 3×10^{10} atoms/cm^3 in Aspect's experiment) down to $1 \times 1\ \mu$m has been achieved [23].

The other possible sources are two other beam splitters. For, as will see below, beam splitters emit completely unpolarized photons in particular directions. Of course, the beam splitters would be much easier to handle than cascade processes but we introduced the cascade sources first because of the conceptual clarity they give to the proposal of the experiment.

The rest of the experiment is the same as above for polarized photons.

III. FORMALISM

The state of polarized photons immediately after leaving the sources is described by the product of two prepared linear–polarization states:

$$|\Psi\rangle = (\cos\theta_{1'}|1_x\rangle_1 \;+\; \sin\theta_{1'}|1_y\rangle_1)$$
$$\otimes\,(\cos\theta_{2'}|1_x\rangle_2 \;+\; \sin\theta_{2'}|1_y\rangle_2)\ , \tag{1}$$

where $|1_x\rangle$ and $|1_y\rangle$ denote the mutually orthogonal photon states. So, e.g., $|1_x\rangle_1$ means the state of a photon leaving the upper source polarized in direction x. If the beam spliter were removed it would cause a *"click"* at the detector D1 and no *"click"* at the detector D1$^\perp$ provided the birefringent polarizer P1 is oriented along x. Here D1$^\perp$ means a detector counting photons coming out at the *other exit* P$^\perp$ (perpendicular polarization; not shown in the Fig. 1) of the birefringent prism P1. Angles $\theta_{1'}$,$\theta_{2'}$ are the angles along which incident photons are polarized with respect to a fixed direction.

For unpolarized photons the density matrix is proportional to the unit matrix and this means that we only need products $|1_x\rangle_1\,|1_x\rangle_2$, $|1_x\rangle_1\,|1_y\rangle_2$, $|1_y\rangle_1\,|1_y\rangle_2$, and $|1_y\rangle_1\,|1_y\rangle_2$ to form partial probabilities which then sum up to the total correlation probability as shown in Sec. IV.

To describe the interaction of photons with the beam spliter, polarizers and detectors we use the quantized electric field operators often employed in quantum optical analysis, e.g., by Paul [3], Mandel's group [5,6,8], and Campos et al. [10]. Because we use independent sources, resulting random constant phases will give no interference of the 2nd order so that we dispense with them. As for polarization we introduce it by means of two orthogonal scalar field components. Thus the scalar components of the stationary electric field operators read:

$$\hat{E}_j(\mathbf{r}_j,t) = \frac{1}{\sqrt{\mathcal{V}}}\sum_{\{\omega_j\}} l(\omega_j)\hat{a}(\omega_j)\xi(\omega_j)e^{i\mathbf{k}_j\cdot\mathbf{r}_j - i\omega_j t}\,, \tag{2}$$

where $l(\omega) = i\sqrt{\frac{\hbar\omega}{2\varepsilon_\circ}}$, $\mathbf{k}$ is the wave vector ($k = \omega/c$), $j = 1, 2$ refer to a particular photon in question, $\mathcal{V}$ is the quantization volume, $\{\omega\}$ is the frequency set with a bandwidth $\Delta\omega$, $\hat{a}(\omega)$ is the annihilation (lowering) operator at the angular frequency ω, and $\xi(\omega)$ is the frequency density of the chosen form for the wave packet. In a subsequent paper [16] we use the Gaussian wave packets and therefore we have

$$\xi(\omega) = [2\pi(\Delta\omega)]^{-1/4} \exp\left[-\left(\frac{\omega - \omega_\circ}{2\Delta\omega}\right)^2\right]. \tag{3}$$

In this paper we consider only monochromatic waves, i.e., $\Delta\omega = 0$ and $\xi(\omega) = 1$. So, we deal here with plane waves represented by the following field operators:

$$\hat{E}_j(\mathbf{r}_j, t) = \hat{a}(\omega_j)e^{i\mathbf{k}_j\cdot\mathbf{r}_j - i\omega_j t}. \tag{4}$$

Of course, we tacitly assume that photons must arrive at the beam splitter practically simultaneously, i.e. with appropriate short delays. In the plane wave approach we cannot derive the conditions under which events gain a particular visibility but that does not affect the reasoning here, since only the overall visibility is affected by greater delays. In Ref. [16] we carry out the appropriate calculations in detail using Gaussian wave packets and we show that the experiment is feasible.

The annihilation operators describe joint actions of polarizers, beam splitter, and detectors. The operators act on the states as follows: $\hat{a}_{1x}|1_x\rangle_1 = |0_x\rangle_1$, $\hat{a}^\dagger_{1x}|1_x\rangle_1 = |2_x\rangle_1$, $\hat{a}_{1x}|0_x\rangle_1 = 0$, etc.

The action of the beam splitter we describe by the input annihilation operators $\hat{a}_{1in}$ and $\hat{a}_{2in}$ operators and the following output ones:

$$\begin{aligned} \hat{a}_{1out} &= t\hat{a}_{1in} + i\,r\hat{a}_{2in}\,, \\ \hat{a}_{2out} &= i\,r\hat{a}_{1in} + t\hat{a}_{2in}\,, \end{aligned} \tag{5}$$

where $t = |\sqrt{T}|$ and $r = |\sqrt{R}|$, where T and R denote transmittance and reflectance, respectively.

To take the linear polarization along orthogonal directions into account we shall consider two sets of operators, i.e., their matrices

$$\hat{\mathbf{a}}_{x\,out} = \mathbf{c}\,\hat{\mathbf{a}}_{x\,in} \quad \text{and} \quad \hat{\mathbf{a}}_{y\,out} = \mathbf{c}\,\hat{\mathbf{a}}_{y\,in}\,. \tag{6}$$

So, the action of the polarizers P1,P2 and detectors D1,D2 can be expressed as:

$$\hat{a}_i = \hat{a}_{ix\,out}\cos\theta_i + \hat{a}_{iy\,out}\sin\theta_i\,, \tag{7}$$

where $i = 1, 2$.

Projections corresponding to the other choices of polarizers and detectors we obtain by using appropriate transformations instead of the ones given by Eqs. (9) and (10). E.g., we obtain the action of the polarizer P2$^\perp$ (orthogonal to P2; in the experiment P2 and P2$^\perp$ make a birefringent prism) and the corresponding detector D2$^\perp$ if we substitute

$$\hat{a}_2 = -\hat{a}_{2x\,out}\sin\theta_2 + \hat{a}_{2y\,out}\cos\theta_2 \tag{8}$$

for Eq. (7).

Hence the appropriate outgoing electric field operators read

$$\begin{aligned}\hat{E}_1 =& \left(\hat{a}_{1x}t_x\cos\theta_1 + \hat{a}_{1y}t_y\sin\theta_1\right)e^{i\mathbf{k}_1\cdot\mathbf{r}_1 - i\omega_1(t-\tau_1)}\\ &+i\left(\hat{a}_{2x}r_x\cos\theta_1 + \hat{a}_{2y}r_y\sin\theta_1\right)e^{i\tilde{\mathbf{k}}_2\cdot\mathbf{r}_1 - i\omega_2(t-\tau_2)}\,,\end{aligned} \tag{9}$$

$$\begin{aligned}\hat{E}_2 =& \left(\hat{a}_{2x}t_x\cos\theta_2 + \hat{a}_{2y}t_y\sin\theta_2\right)e^{i\mathbf{k}_2\cdot\mathbf{r}_2 - i\omega_2(t-\tau_2)}\\ &+i\left(\hat{a}_{1x}r_x\cos\theta_2 + \hat{a}_{1y}r_y\sin\theta_2\right)e^{i\tilde{\mathbf{k}}_1\cdot\mathbf{r}_2 - i\omega_1(t-\tau_1)}\,,\end{aligned} \tag{10}$$

where τ_j is time delay after which the photon reaches detector D, ω_j is the frequency of photon j, and c is the velocity of light. The detectors and the crystal are assumed to be positioned symmetrically with regard to the beam splitter so that two time delays suffice.

IV. DETECTION PROBABILITIES

A. Polarized photons

1. Each photon in one arm

The joint interaction of both photons with the beam splitter, polarizers P1,P2, and detectors D1,D2 is given by the following projection of our wave function onto the Fock vacuum space

$$
\begin{aligned}
\hat{E}_1\hat{E}_2|\Psi\rangle = \Big[&(t_x^2\varepsilon_{12} - r_x^2\tilde{\varepsilon}_{12})\cos\theta_{1'}\cos\theta_{2'}\cos\theta_1\cos\theta_2 \\
&+(t_x t_y\varepsilon_{12}\sin\theta_1\cos\theta_2 - r_x r_y\tilde{\varepsilon}_{12}\cos\theta_1\sin\theta_2)\sin\theta_{1'}\cos\theta_{2'} \\
&+(t_x t_y\varepsilon_{12}\cos\theta_1\sin\theta_2 - r_x r_y\tilde{\varepsilon}_{12}\sin\theta_1\cos\theta_2)\cos\theta_{1'}\sin\theta_{2'} \\
&+(t_y^2\varepsilon_{12} - r_y^2\tilde{\varepsilon}_{12})\sin\theta_{1'}\sin\theta_{2'}\sin\theta_1\sin\theta_2\Big]\varepsilon|0\rangle\,, \qquad (11)
\end{aligned}
$$

where $\varepsilon = \exp\Big\{-i\left[\omega_1(t-\tau_1)+\omega_2(t-\tau_2)\right]\Big\}$, $\varepsilon_{12} = \exp[i(\mathbf{k}_1\cdot\mathbf{r}_1+\mathbf{k}_2\cdot\mathbf{r}_2)]$, and $\tilde{\varepsilon}_{12} = \exp\left[i\left(\tilde{\mathbf{k}}_1\cdot\mathbf{r}_2+\tilde{\mathbf{k}}_2\cdot\mathbf{r}_1\right)\right]$.

The corresponding probability of detecting the photons by detectors D1,D2 is thus

$$
P(\theta_{1'},\theta_{2'},\theta_1,\theta_2) = \langle\hat{E}_2^\dagger\hat{E}_1^\dagger\hat{E}_1\hat{E}_2\rangle = A^2+B^2-2AB\cos\phi\,, \qquad (12)
$$

where

$$
\begin{aligned}
A = &\, t_x^2\cos\theta_{1'}\cos\theta_{2'}\cos\theta_1\cos\theta_2 + t_y^2\sin\theta_{1'}\sin\theta_{2'}\sin\theta_1\sin\theta_2 \\
&+t_x t_y\left(\cos\theta_{1'}\sin\theta_{2'}\cos\theta_1\sin\theta_2+\sin\theta_{1'}\cos\theta_{2'}\sin\theta_1\cos\theta_2\right)\,, \qquad (13)
\end{aligned}
$$

$$
\begin{aligned}
B = &\, r_x^2\cos\theta_{1'}\cos\theta_{2'}\cos\theta_1\cos\theta_2 + r_y^2\sin\theta_{1'}\sin\theta_{2'}\sin\theta_1\sin\theta_2 \\
&+r_x r_y\left(\cos\theta_{1'}\sin\theta_{2'}\sin\theta_1\cos\theta_2+\sin\theta_{1'}\cos\theta_{2'}\cos\theta_1\sin\theta_2\right)\,, \qquad (14)
\end{aligned}
$$

$$
\phi = (\tilde{\mathbf{k}}_2-\mathbf{k}_1)\cdot\mathbf{r}_1+(\tilde{\mathbf{k}}_1-\mathbf{k}_2)\cdot\mathbf{r}_2 = 2\pi(z_2-z_1)/L\,, \qquad (15)
$$

where L is the spacing of the intereference fringes [1]. ϕ can be changed by moving the detectors transversely to the incident beams.

To make the formula more transparent, without loss of generality, in the following we shall consider 50:50 beam splitter: $t_x = t_y = r_x = r_y = 2^{-1/2}$ and three characteristic locations of the detectors so as to have $\cos\phi = -1, 0, 1$

Let us first consider the case $\phi = 0$ for which the above probability reads

$$
\begin{aligned}
P(\theta_{1'},\theta_{2'},\theta_1,\theta_2) &= (A-B)^2 \\
&= \frac{1}{4}\sin^2(\theta_{1'}-\theta_{2'})\sin^2(\theta_1-\theta_2)\,. \qquad (16)
\end{aligned}
$$

We see that the probability unexpectedly factorizes left–right and not up–down as one would be tempted to conjecture from the initial up–down independence expressed by the product of the "upper" and "lower" function in Eq. (1).

On the other hand, the incoming polarizations influence the coincidence counting even when we remove the polarizers P1 and P2. Then, provided the right photons arrive at the beam splitter within a sufficiently short time and are separately detected by D1 and D2, we obtain

$$P(\theta_{1'},\theta_{2'},\infty,\infty) = \frac{1}{2}\sin^2(\theta_{1'}-\theta_{2'})\,. \tag{17}$$

This equation clarifies the minimum of the coincidence rates obtained for the $z_1 = z_2$ positions of detectors in Refs. [24–27]. We just have to recall again that signal and idler down–converted photons emerging from independent nonlinear crystals used in these experiments are parallelly polarized [7]. Conversely, by inserting $\theta_{2'} = \theta_{1'} + \pi/2$ in Eq. (16) we obtain exactly what — for $\phi = 0$ — Ou, Hong, and Mandel obtained in Ref. [9] and what Ou and Mandel should have obtained also in Refs. [1] and [7]. [28]

For $\phi = \pi$ our probability reads

$$\begin{aligned} P(\theta_{1'},\theta_{2'},\theta_1,\theta_2) &= (A+B)^2 \qquad (18)\\ &= \frac{1}{4}[\cos(\theta_{1'}-\theta_2)\cos(\theta_{2'}-\theta_1)+\cos(\theta_{1'}-\theta_1)\cos(\theta_{2'}-\theta_2)]^2, \end{aligned}$$

while for $\phi = \pi/2$ it becomes

$$\begin{aligned} P(\theta_{1'},\theta_{2'},\theta_1,\theta_2) = \frac{1}{4}[&\cos^2(\theta_{1'}-\theta_2)\cos^2(\theta_{2'}-\theta_1)\\ &+\cos^2(\theta_{1'}-\theta_1)\cos^2(\theta_{2'}-\theta_2)]\,. \end{aligned} \tag{19}$$

The probability shows that, for $\phi = \pi$, by removing the polarizers we lose the spin correlation completely and the coincidence counting remains unchanged no matter how we turn the polarization planes of the incoming photons. This is just opposite to $\phi = 0$ above where because of Eq. (17) we could not have a coincidence for parallel incident polarizations. The latter means that we obtain the typical non–classical 100% (ideally) coincidence rate [24–26] as opposed to the classical treatment (maximum 50%), i.e., that both photons go into only one of the arms. Let us therefore have a closer look at the case of two photons in a particular arm.

2. Both photons in one arm

In order to treat both photons going into one arm properly (i.e., so as to make all the probabilities add up to one) we have to switch to the experimental setup described in the last paragraph of Sec. II A and employ four detectors in each arm: $D2_{\omega_1}$–$D2^{\perp}_{\omega_2}$ and $D1_{\omega_1}$–$D1^{\perp}_{\omega_2}$ in the upper and lower arm, respectively. Let us do that for the upper arm. Instead of $\hat{E}_1$ from Eq. (9) we must use

$$\begin{aligned}\hat{E}'_2 = &\left(\hat{a}_{2x}t_x\cos\theta_1 + \hat{a}_{2y}t_y\sin\theta_1\right)e^{i\mathbf{k}'_2\cdot\mathbf{r}'_2 - i\omega_2(t-\tau_2)}\\ &+i\left(\hat{a}_{1x}r_x\cos\theta_1 + \hat{a}_{1y}r_y\sin\theta_1\right)e^{i\tilde{\mathbf{k}}'_1\cdot\mathbf{r}'_2 - i\omega_1(t-\tau_1)}\end{aligned} \tag{20}$$

so as to obtain the following analogue of Eq. (11)

$$\begin{aligned}\hat{E}'_2\hat{E}_2|\Psi\rangle = \Big[&t_xr_x(\eta_{2'}+\eta_2)\cos\theta_{1'}\cos\theta_{2'}\cos\theta_1\cos\theta_2\\ &+t_xr_y(\eta_{2'}\cos\theta_1\sin\theta_2 + \eta_2\sin\theta_1\cos\theta_2)\sin\theta_{1'}\cos\theta_{2'}\\ &+t_yr_x(\eta_{2'}\sin\theta_1\cos\theta_2 + \eta_2\cos\theta_1\sin\theta_2)\cos\theta_{1'}\sin\theta_{2'}\\ &+t_yr_y(\eta_{2'}+\eta_2)\sin\theta_{1'}\sin\theta_{2'}\sin\theta_1\sin\theta_2\Big]\varepsilon|0\rangle\,,\end{aligned} \tag{21}$$

where $\varepsilon = \exp\Big[-i\left[\omega_1(t-\tau_1)+\omega_2(t-\tau_2)\right]\Big]$, $\eta_{2'} = \exp[i\left(\mathbf{k}_1\cdot\mathbf{r}_2 + \mathbf{k}'_2\cdot\mathbf{r}'_2\right)]$, and $\eta_2 = \exp\left[i\left(\tilde{\mathbf{k}}_2\cdot\mathbf{r}_2 + \tilde{\mathbf{k}}'_1\cdot\mathbf{r}'_2\right)\right]$.

The corresponding probability of detecting the photons by detectors $D2_{\omega_1}$,$D2_{\omega_2}$ is thus

$$P(\theta_{1'},\theta_{2'},\theta_1\times\theta_2) = \frac{1}{2}\langle\hat{E}^{\dagger}_2\hat{E}'^{\dagger}_2\hat{E}'_2\hat{E}_2\rangle = \frac{1}{2}(C^2+D^2+2CD\cos\psi)\,, \tag{22}$$

where 1/2 matches the possibility of both photons taking the other arm and

$$\begin{aligned}C = &\,t_xr_x\cos\theta_{1'}\cos\theta_{2'}\cos\theta_1\cos\theta_2 + t_yr_y\sin\theta_{1'}\sin\theta_{2'}\sin\theta_1\sin\theta_2\\ &+t_xr_y\sin\theta_{1'}\cos\theta_{2'}\sin\theta_1\cos\theta_2 + t_yr_x\cos\theta_{1'}\sin\theta_{2'}\cos\theta_1\sin\theta_2\,,\end{aligned} \tag{23}$$

$$D = t_x r_x \cos\theta_{1'}\cos\theta_{2'}\cos\theta_1\cos\theta_2 + t_y r_y \sin\theta_{1'}\sin\theta_{2'}\sin\theta_1\sin\theta_2$$
$$+t_x r_y \sin\theta_{1'}\cos\theta_{2'}\cos\theta_1\sin\theta_2 + t_y r_x \cos\theta_{1'}\sin\theta_{2'}\sin\theta_1\cos\theta_2\,, \tag{24}$$

$$\psi = (\tilde{\mathbf{k}}_1 - \mathbf{k}_2)\cdot\mathbf{r}_2 + (\mathbf{k}'_2 - \tilde{\mathbf{k}}'_1)\cdot\mathbf{r}'_2 = 2\pi(Z_2 - Z'_2)/L\,, \tag{25}$$

where primes refer to the *other* photon of a different frequency and where the geometry of the detectors is of course not any more the one shown in Fig. 1 but is, e.g., following the Fig. 1 of Ref. [29]. An analogous probability we obtain for the lower arm.

For 50:50 beam splitter and $\psi = 0$ the probability reads

$$P(\theta_{1'},\theta_{2'},\theta_1\times\theta_2) \tag{26}$$
$$= \frac{1}{8}[\cos(\theta_{1'}-\theta_2)\cos(\theta_{2'}-\theta_1)+\cos(\theta_{1'}-\theta_1)\cos(\theta_{2'}-\theta_2)]^2.$$

To obtain the corresponding probability with the polarizers removed we have to add up probabilities for all four possible outcomes from the birefringent P2 which we obtain by using Eqs. (7), (8), and two other ones what for both arms amounts to

$$P(\theta_{1'},\theta_{2'},\infty\times\infty) = \frac{1}{2}[1+\cos^2(\theta_{1'}-\theta_{2'})]. \tag{27}$$

We see that this equation and Eq. (17) add up to one.

Another possible way of detecting both photons in one arm, although far less reliable, is by means of *non–coincidental* recording of only one of detectors D1,D2 assuming the recording be triggered by two simultaneously arriving photons. In this case we keep to the setup described in the second paragraph of Sec. II A and employ no additional detectors. Then the probability of detecting both photons in the arm of, e.g., D2 we obtain similarly to Eq. (12)

$$P(\theta_{1'},\theta_{2'},2\times\theta_2) = \frac{1}{2}\langle\hat{E}_2^{\dagger 2}\hat{E}_2^2\rangle$$
$$= \frac{1}{8}[\cos^2(\theta_{1'}-\theta_2)\cos^2(\theta_{2'}-\theta_2)](1+\cos\psi)\,, \tag{28}$$

where ψ is automatically zero because of the coincidental spatial recording of both photons. Of course, we cannot add up this probability for the removed polarizers and the probability (17) to 1 because the the corresponding counts are from two different spaces of events.

B. Unpolarized photons

To obtain the general probability for unpolarized light, $\hat{E}_1, \hat{E}_2$ given by Eqs. (9) and (10) should be applied to $|1_x\rangle_1|1_x\rangle_2$, $|1_x\rangle_1|1_y\rangle_2$, $|1_y\rangle_1|1_x\rangle_2$, and $|1_y\rangle_1|1_y\rangle_2$ so as to give four probabilities which then sum up to the following correlation probability:

$$
\begin{aligned}
&P(\infty,\infty,\theta_1,\theta_2)\\
&=\frac{1}{4}(t_x^2\cos^2\theta_1+t_y^2\sin^2\theta_1)(t_x^2\cos^2\theta_2+t_y^2\sin^2\theta_2)\\
&+\frac{1}{4}(r_x^2\cos^2\theta_1+r_y^2\sin^2\theta_1)(r_x^2\cos^2\theta_2+r_y^2\sin^2\theta_2)\\
&-\frac{1}{2}(t_x^2r_x^2\cos\theta_1\cos\theta_2+t_y^2r_y^2\sin\theta_1\sin\theta_2)^2\cos\phi\,. \qquad (29)
\end{aligned}
$$

For 50:50 beam splitter this probability reads

$$P(\infty,\infty,\theta_1,\theta_2)=\frac{1}{8}[1-\cos\phi\cos^2(\theta_2-\theta_1)]\,. \qquad (30)$$

Comparing this result with the classical formula obtained by Paul [30] for two amplitude–stabilized beams of equal intensity which, apart from a normalization factor, reads

$$P_{\rm cl}(\theta_1,\theta_2)=3+2(1-\cos\phi)\cos^2(\theta_2-\theta_1)\,, \qquad (31)$$

we see that the quantum mechanical *visibility* reaches its maximum for $\phi=0$ while the corresponding classical one cannot be equal to zero at all.

In the end, for unpolarized photons and for $\phi=0$ we obtain:

$$P(\infty,\infty,\theta_1,\theta_2)=\frac{1}{8}\sin^2(\theta_2-\theta_1)\,. \qquad (32)$$

So, photons that arrive at the beam splitter unpolarized emerge from it perpendicularly polarized whenever they appear at the opposite sides of the beam splitter. The overall probability of their appearance on one side of the beam splitter is

$$P(\infty,\infty,\theta_1\times\theta_2)=\frac{1}{8}[1+\cos^2(\theta_1-\theta_2)]. \qquad (33)$$

V. CONCLUSION

We have therefore shown that the 4th order interference interaction between beam splitter and two incoming photons imposes polarization correlation on the emerging photons no matter whether they arrive at the beam splitter polarized or unpolarized. In particular we have shown [Eqs. (30) and (32)] that for an appropriate position of the beam splitter incoming unpolarized photons always emerge perpendicularly polarized in particular directions. More specifically, they appear prepared in a genuine singlet state and enable conceiving a novel experiment in which we can preselect spin correlated photons from completely unpolarized independent photons which nowhere interacted without in any way affecting them [31].

When polarized photons arrive at a beam splitter and the 4th order interference takes place, one can use the modulation of the polarizations in order to determine the coincidence counting even when no outgoing polarization is being measured. In particular we have shown [Eq. (17)] that in predetermined directions incoming parallelly polarized photons never emerge on two different sides of the beam splitter. It is also interesting that the probability of detecting both photons together on one side of the beam splitter (by one detector) is structurally different from the the probability of finding them on both sides. The former depends on the direction of leaving the beam splitter and allows a transmission of the left–right information of the Bell type [Eq. (28)].

ACKNOWLEDGMENTS

I am grateful to Dr. Summhammer for particularly fruitful and rich collaboration and for his hospitality at the Atominstitut der Österreichischen Universitäten, Vienna, to Prof. K.–E. Hellwig for valuable discussions and the hospitality at the Institut für Theoretische Physik, TU Berlin and to Prof. H. Paul, Humboldt–Universität zu Berlin for valuable discussions and correspondence and a pronounced active interest for the idea developed in the papers [14–16].

I also acknowledge supports of the Alexander von Humboldt Foundation, the Technical University of Vienna, and the Ministry of Science of Croatia.

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[28] Eqs. (56), (57), etc. of Ref. [1] and Eqs. (2), (5a), etc. of Ref. [7] should be corrected so as to change signs in order to assure the preservation of boson commutation relations for the annihilation operators at the beam splitter. I would like to mention here that Prof. H. Paul noticed this mistake as well (in a private communication).

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1993). Actually, Prof. Paul's paper triggered the way in which I presented the matter in the present paper and he himself was brought to consider the classical approach through the papers [14] and [15].

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FIGURES

FIG. 1. Outline of the experiment.

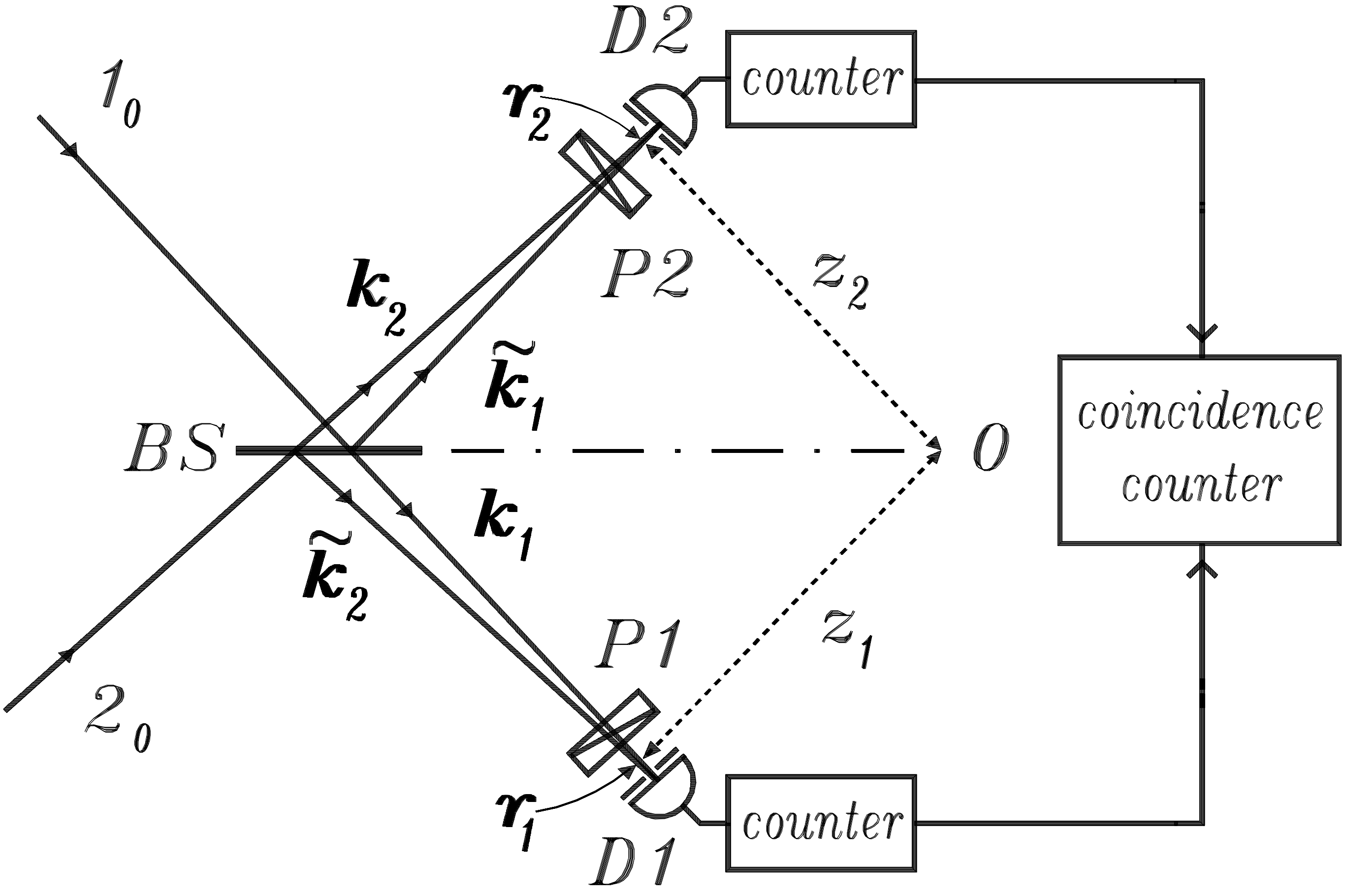

Fig. 1