Symmetry Energy in Nuclear Ground State

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ESF Workshop on How to Constrain High-Density Symmetry-Energy

Zagreb, October 16-18, 2009

Outline

Charge Symmetry & Charge Invariance

Charge symmetry: invariance of nuclear interactions under $n \leftrightarrow p$ interchange

An isoscalar quantity F does not change under $n \leftrightarrow p$ interchange. E.g. nuclear energy. Expansion in asymmetry $\eta = (N - Z)/A$, for smooth F, yields even terms only:

$$F(\eta) = F_0 + F_2\,\eta^2 + F_4\,\eta^4 + \ldots$$

An isovector quantity G changes sign. Example: $\rho_{np}(r) = \rho_n(r) - \rho_p(r)$. Expansion with odd terms only:

$$G(\eta) = G_1\,\eta + G_3\,\eta^3 + \ldots$$

Note: $G/\eta = G_1 + G_3\,\eta^2 + \ldots$.

Charge invariance: invariance of nuclear interactions under rotations in n-p space

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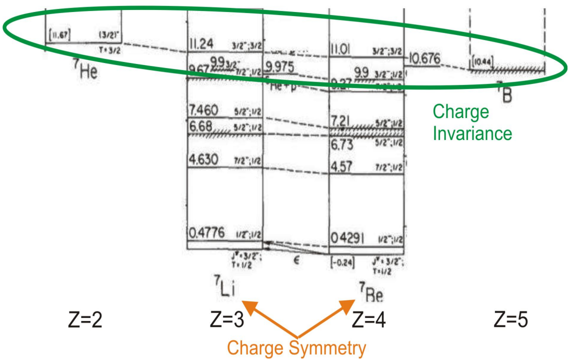
Charge symmetry:
$n \leftrightarrow p$ invariance

Charge invariance:
symmetry under *rotations* in
n-*p* space

Isobars: Nuclei with the same A



Isospin doublets

$$p : (\tau, \tau_z) = (\tfrac{1}{2}, \tfrac{1}{2})$$

$$n : (\tau, \tau_z) = (\tfrac{1}{2}, -\tfrac{1}{2})$$

Net isospin

$$\vec{T} = \sum_{i=1}^{A} \vec{\tau}_i$$

$T = \frac{3}{2}, \ldots$ $T = \frac{1}{2}, \frac{3}{2}, \ldots$ $T = \frac{3}{2}, \ldots$

Nuclear states: (T, T_z), $T \geq |T_z| = \frac{1}{2}|N - Z|$

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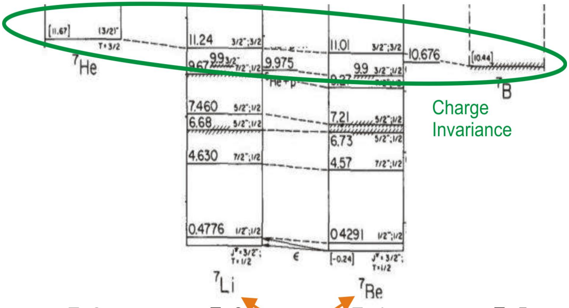

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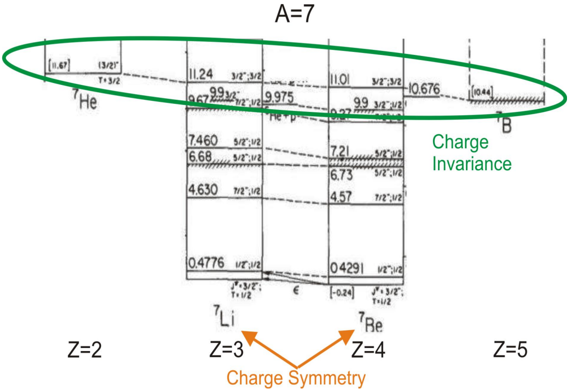

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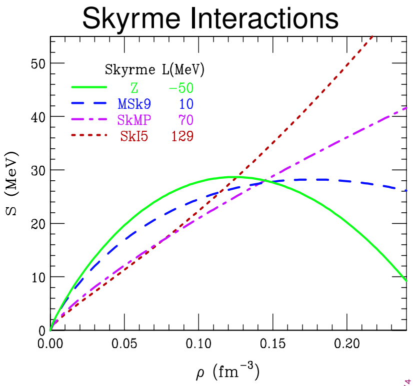
Symmetry Energy: From Finite to ∞ System



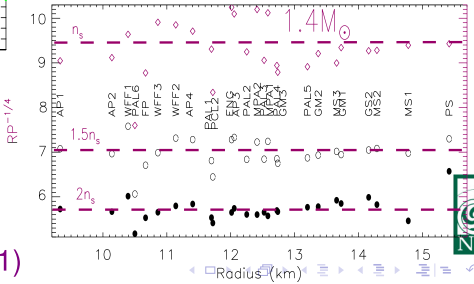
$\eta = (\rho_n - \rho_p)/\rho$-expansion under $n \leftrightarrow p$ symmetry

$$E(\rho_n, \rho_p) = E_0(\rho) + S(\rho) \left(\frac{\rho_n - \rho_p}{\rho} \right)^2$$

$$S(\rho) = a_a^V + \frac{L}{3} \frac{\rho - \rho_0}{\rho_0} + \ldots$$

In neutron matter: $E \simeq E_0 + S$

& $P \simeq \rho^2 \, dS/d\rho \simeq L \, \rho^2/(3\rho_0)$



Empirical correlation for neutron stars

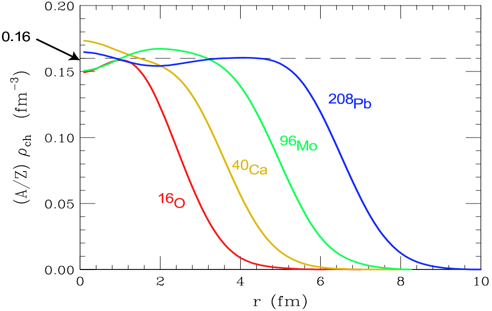
$R \, P^{-1/4} \approx \text{const}$

Lattimer&Prakash ApJ550(01)

Finite System

Interrelation between nucleonic densities…

Net density: $\rho(r) \stackrel{?}{=} \frac{A}{Z}\rho_p(r)$



Bethe-Weizsäcker formula:

$$E = -a_V\,A + a_S\,A^{2/3} + a_C\,\frac{Z^2}{A^{1/3}} + a_a\,\frac{(N-Z)^2}{A} + E_{\text{mic}}$$

$a_V = 16\,\text{MeV}$ $\quad a_S = 18\,\text{MeV}$ $\quad a_a = 21\,\text{MeV}$ $\quad a_C = 0.7\,\text{MeV}$

symmetry term of volume character $\quad a_a^V = a_a = 21\,\text{MeV}$??

mass-dependent $\quad a_a(A)$??

minimally finite system $\quad \Rightarrow$ half-infinite matter

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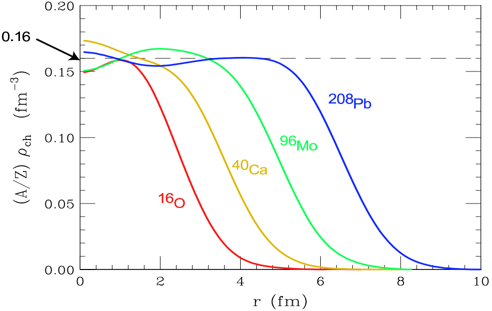

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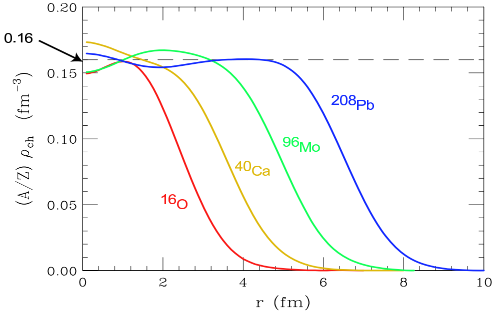

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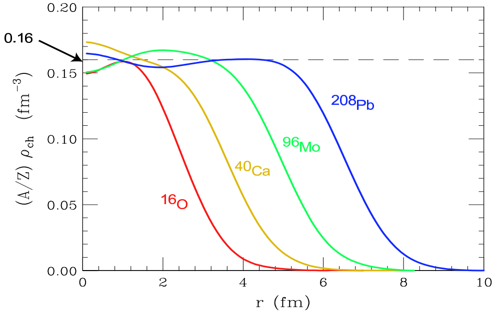

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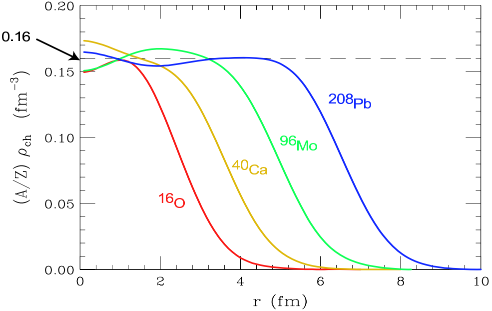

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Nucleus as Capacitor for Asymmetry

$$E = -a_v A + a_s A^{2/3} + \frac{a_a}{A}(N-Z)^2$$
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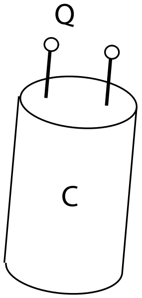

Capacitor analogy

$$E = E_0 + \frac{Q^2}{2C} \Rightarrow \begin{cases} Q \equiv N - Z \\ C \equiv \dfrac{A}{2a_a(A)} \end{cases}$$

Asymmetry chemical potential

$$\mu_a = \frac{\partial E}{\partial(N-Z)} = \frac{1}{2}(\mu_n - \mu_p) = \frac{2a_a(A)}{A}(N-Z)$$

Analogy

$$V = \frac{1}{C}Q \quad \Rightarrow V \equiv \mu_a$$

Note: for connected capacitors, charge (asymmetry) distributes itself *in proportion to capacitance*

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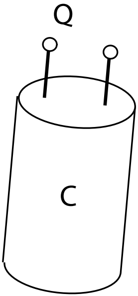

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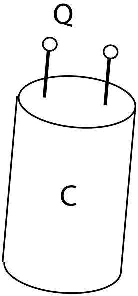

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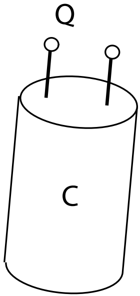

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Invariant Densities

Net density $\rho(r) = \rho_n(r) + \rho_p(r)$ is isoscalar $\Rightarrow$ weakly depends on $(N-Z)$ for given A. [Coulomb suppressed. . .]

$\rho_{np}(r) = \rho_n(r) - \rho_p(r)$ isovector but $A\,\rho_{np}(r)/(N-Z)$ isoscalar! $A/(N-Z)$ normalizing factor global. . . Similar local normalizing factor, in terms of intense quantities, $2a_a^V/\mu_a$, where $a_a^V \equiv S(\rho_0)$

Asymmetric density (formfactor for isovector density) defined:

$$\rho_a(r) = \frac{2a_a^V}{\mu_a}\,[\rho_n(r) - \rho_p(r)]$$

Normal matter: $\rho_a = \rho_0$. Both $\rho(r)$ & $\rho_a(r)$ weakly depend on η!

In any nucleus:

$$\boxed{\rho_{n,p}(r) = \frac{1}{2}\Big[\rho(r) \pm \frac{\mu_a}{2a_a^V}\rho_a(r)\Big]}$$

where $\rho(r)$ & $\rho_a(r)$ have universal features!

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Net isoscalar density ρ usually parameterized w/Fermi function

$$\rho(r) = \frac{\rho_0}{1+\exp\left(\frac{r-R}{d}\right)} \qquad \text{with} \quad R = r_0\, A^{1/3}$$

Isovector density ρ_a?? Related to $a_a(A)$ & to $S(\rho)$!

$$2C \equiv \frac{A}{a_a(A)} = \frac{2(N-Z)}{\mu_a} = 2\int \mathrm{d}r\, \frac{\rho_{np}}{\mu_a} = \frac{1}{a_a^V}\int \mathrm{d}r\, \rho_a(r)$$

In uniform matter

$$\mu_a = \frac{\partial E}{\partial(N-Z)} = \frac{2\,S(\rho)}{\rho}\,\rho_{np} \equiv \frac{\rho_{np}}{c(\rho)}$$

$c(\rho) = \rho/2S(\rho)$
density of capacitance

$$\Rightarrow \quad \rho_a = \frac{2a_a^V}{\mu_a}\,\rho_{np} = \frac{a_a^V\,\rho}{S(\rho)} \propto c(\rho)$$

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$$2C \equiv \frac{A}{a_a(A)} = \frac{2(N-Z)}{\mu_a} = 2\int \mathrm{d}r\, \frac{\rho_{np}}{\mu_a} = \frac{1}{a_a^V}\int \mathrm{d}r\, \rho_a(r)$$

In uniform matter

$$\mu_a = \frac{\partial E}{\partial(N-Z)} = \frac{2\,S(\rho)}{\rho}\,\rho_{np} \equiv \frac{\rho_{np}}{c(\rho)}$$

$c(\rho) = \rho/2S(\rho)$
density of capacitance

$$\Rightarrow \quad \rho_a = \frac{2a_a^V}{\mu_a}\,\rho_{np} = \frac{a_a^V\,\rho}{S(\rho)} \propto c(\rho)$$

asymmetry $\propto$ capacitance

n&p densities carry record of $S(\rho)$! $\Longrightarrow$ Hartree-Fock study of surface

Nuclear Densities

$$\rho_{n,p}(r) = \frac{1}{2}\left[\rho(r) \pm \frac{\mu_a}{2a_a^V}\rho_a(r)\right]$$

Net isoscalar density ρ usually parameterized w/Fermi function

$$\rho(r) = \frac{\rho_0}{1+\exp\left(\frac{r-R}{d}\right)} \qquad \text{with} \quad R = r_0\, A^{1/3}$$

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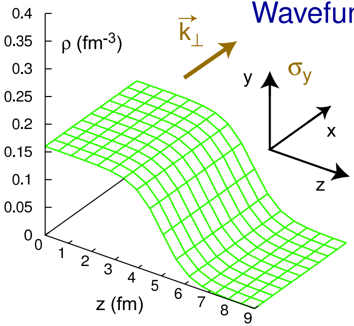
Half-Infinite Matter in Skyrme-Hartree-Fock

To one side infinite uniform matter & vacuum to the other



Wavefunctions: $\Phi(\boldsymbol{r}) = \phi(z)\, e^{i\boldsymbol{k}_\perp \cdot \boldsymbol{r}_\perp}$

matter interior/exterior:

$$\phi(z) \propto \sin\left(k_z\, z + \delta(\boldsymbol{k})\right)$$

$$\phi(z) \propto e^{-\kappa(\boldsymbol{k}) z}$$

Discretization in $\boldsymbol{k}$-space. Set of 1D HF eqs solved until self-consistency for different Skyrme interactions:

$$-\frac{d}{dz}\frac{\hbar^2}{2m^*(z)}\frac{d}{dz}\phi(z) + \left(\frac{\hbar^2\, k_\perp^2}{2m^*(z)} + U(z)\right)\phi(z) = \epsilon(\boldsymbol{k})\,\phi(z)$$

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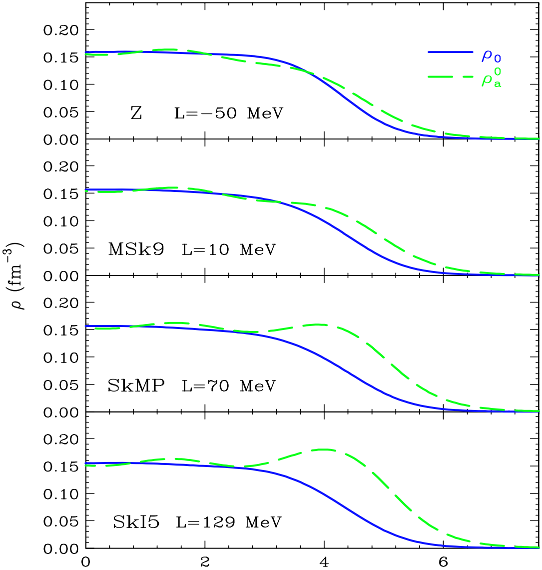
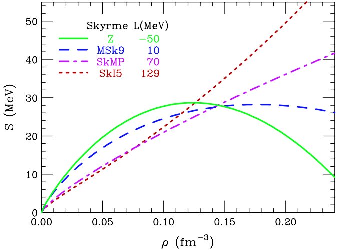
Isoscalar (Net) & Isovector Densities from SHF



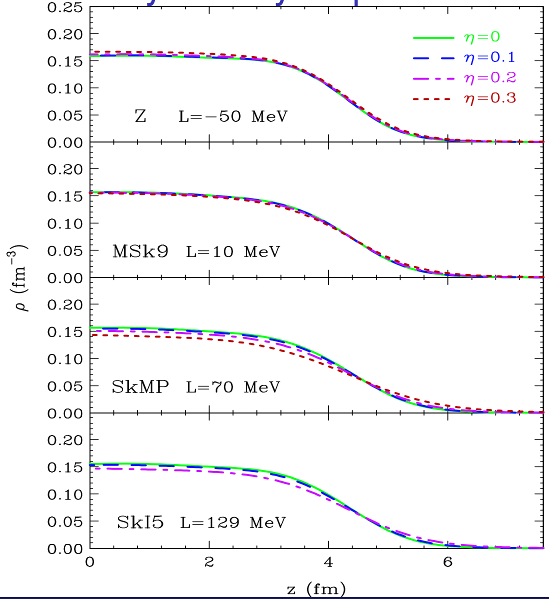
Results for different Skyrme interactions in half-∞ matter.



Net & isovector densities displaced relative to each other

As $S(\rho)$ changes gradually, so does the displacement.

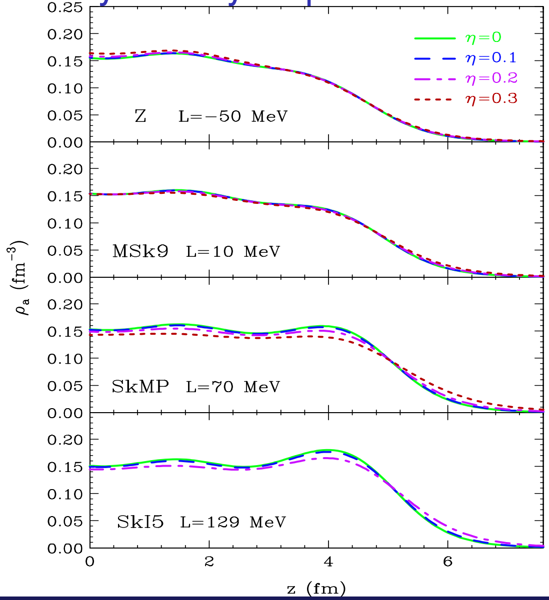
Asymmetry Dependence of Net Density



Results for different asymmetries

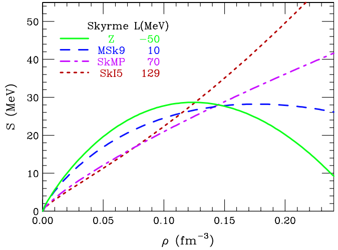
$$\eta = \frac{N-Z}{A}$$

Asymmetry Dependence of Isovector Density



$$\rho_a = \frac{2a_a^V}{\mu_a}(\rho_n - \rho_p)$$

Results for different asymmetries



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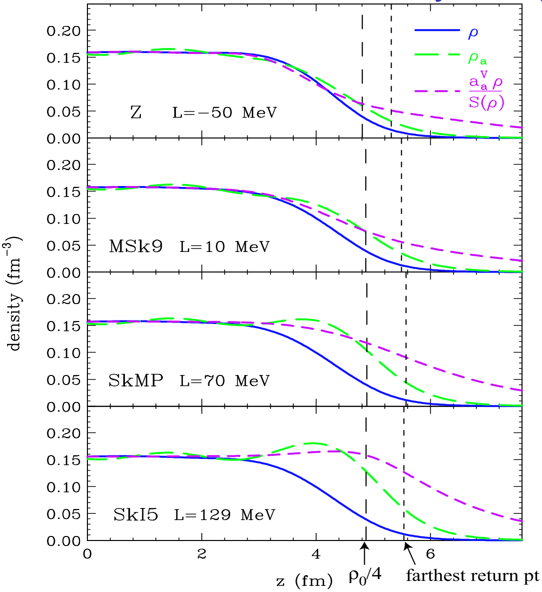
Sensitivity to $S(\rho)$



For weakly nonuniform matter, expected asymmetric density $\rho_a = \rho\, a_a^V / S(\rho)$

Isovector density ρ_a oscillates around the expectation down to $\rho \simeq \rho_0/4$

The higher L, the lower is S in the surface & the higher the surface capacitance ⇔ the more different ρ & ρ_a.

WKB Analysis

Semiclassical wavefunction

$$\phi_{\boldsymbol{k}^\infty}(z) \propto \begin{cases} \sin\Big(\int_z^{z_0} k_z(z')\,\mathrm{d}z'\Big) & \text{allowed} \quad z < z_0 \\ \exp\Big(-\int_{z_0}^{z} \kappa_z(z')\,\mathrm{d}z'\Big) & \text{forbidden} \quad z > z_0 \end{cases}$$

Density from $\rho_\alpha(z) = \int \mathrm{d}\boldsymbol{k}^\infty \, |\phi_{\boldsymbol{k}^\infty \alpha}(z)|^2 \, ,$ classical return pt z_0

Findings:

At $z < z_0$ $\qquad \rho_a(z) \approx \frac{a_a^V \, \rho}{S(\rho)} \Big(1 + \frac{\rho^{2/3}}{S(\rho)} \mathcal{F}\Big)$

where $\mathcal{F}(z) \propto \sin\big(2k_F(z_0 - z)\big)$,
describing Friedel oscillations around ρ/S.

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Skyrme Parameters

Name	a_V	m^*/m	K	a_a^V	L	a_S	a_a^S	ΔR
SkT	-15.40	0.602	333	24.8	28.2	14.2	17.5	0.477
SkT1	-15.98	1.000	236	32.0	56.2	18.2	14.6	0.799
SkT2	-15.94	1.000	235	32.0	56.2	18.0	14.7	0.794
SkT3	-15.94	1.000	235	31.5	55.3	17.7	15.3	0.776
SkT4	-15.95	1.000	235	35.4	94.1	18.1	11.5	0.986
SkT5	-16.00	1.000	201	37.0	98.5	18.1	10.9	1.084
SkM1	-15.77	0.789	216	25.1	-35.3	17.4	59.6	0.180
SkI1	-15.95	0.693	242	37.5	161.0	17.4	11.4	1.126
Gσ	-15.59	0.784	237	31.3	94.0	16.0	10.1	0.929
Rσ	-15.59	0.783	237	30.5	85.7	16.0	10.5	0.888
T	-15.93	1.000	235	28.3	27.2	17.7	22.6	0.587
Z	-15.97	0.842	330	26.8	-49.7	17.7	51.5	0.213
Zσ	-15.88	0.783	233	26.6	-29.3	17.0	46.6	0.233
Z$\sigma\sigma$	-15.96	0.775	234	28.8	-4.5	17.3	29.3	0.406

Symmetry Coefficient

surface region

$$\frac{A}{a_a(A)} = \frac{1}{a_a^V}\int d^3r\,\rho_a(r) = \frac{1}{a_a^V}\int d^3r\,\rho(r) + \frac{1}{a_a^V}\int d^3r\,(\rho_a-\rho)(r)$$

$$\simeq \frac{A}{a_a^V} + \frac{A^{2/3}}{a_a^S}$$

where

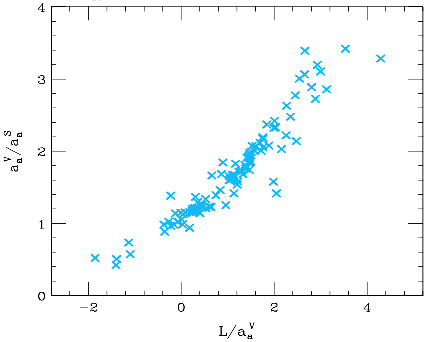
$$\frac{a_a^V}{a_a^S} = 4\pi\, r_0^2 \int dr\,(\rho_a-\rho)(r)$$

$$\simeq \frac{3\,\Delta R}{r_0}$$

and ΔR is displacement of isovector and isoscalar surfaces.

$$\Rightarrow \boxed{\frac{1}{a_a(A)} \simeq \frac{1}{a_a^V} + \frac{A^{-1/3}}{a_a^S}}$$

$a_a^S - L$ Correlation f/SHF



$L \propto$ slope of $S(\rho)$ at ρ_0

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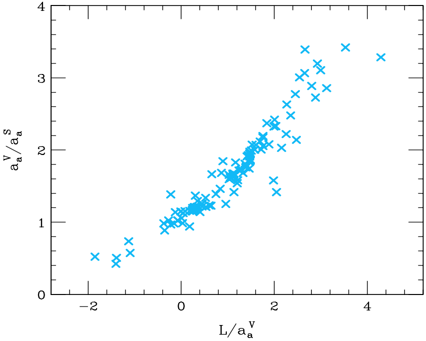

$L \propto$ slope of $S(\rho)$ at ρ_0

Charge Invariance

?$a_a(A)$? Conclusions on sym-energy details, following E-formula fits, interrelated with conclusions on other terms in the formula: asymmetry-dependent Coulomb, Wigner & pairing + asymmetry-independent, due to $(N-Z)/A$ - A correlations along stability line [PD NPA727(03)233]!

Best would be to study the symmetry energy in isolation from the rest of E-formula! Absurd?!

Charge invariance to rescue: lowest nuclear states characterized by different isospin values (T, T_z), $T_z = (Z-N)/2$. Nuclear energy scalar in isospin space:

sym energy $$E_a = a_a(A)\,\frac{(N-Z)^2}{A} = 4\,a_a(A)\,\frac{T_z^2}{A}$$

$$\rightarrow \; E_a = 4\,a_a(A)\,\frac{T^2}{A} = 4\,a_a(A)\,\frac{T(T+1)}{A}$$

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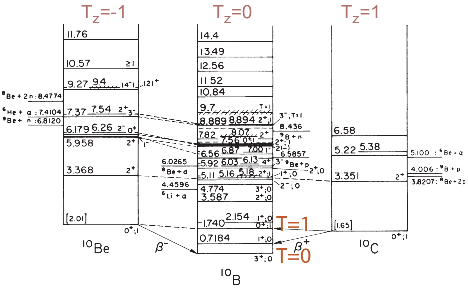
$a_a(A)$ Nucleus-by-Nucleus

$$\rightarrow \; E_a = 4\, a_a(A)\, \frac{T(T+1)}{A}$$

In the ground state T takes on the lowest possible value $T = |T_z| = |N-Z|/2$. Through '+1' most of the Wigner term absorbed.

Formula generalized to the lowest state of a given T (e.g. Jänecke *et al.*, NPA728(03)23). Pairing depends on evenness of T.

?Lowest state of a given T: isobaric analogue state (IAS) of some neighboring nucleus ground-state.



Study of changes in the symmetry term possible nucleus by nucleus

IAS Data Analysis

In the same nucleus:

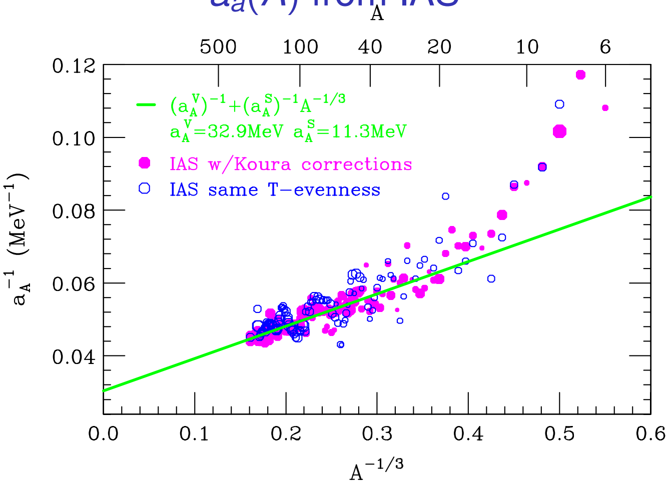
$$E_2(T_2) - E_1(T_1) = \frac{4\,a_a}{A}\{T_2(T_2+1) - T_1(T_1+1)\} + E_{\rm mic}(T_2, T_z) - E_{\rm mic}(T_2, T_z)$$

$$a_a^{-1}(A) = \frac{4\,\Delta T^2}{A\,\Delta E} \overset{?}{=} (a_a^V)^{-1} + (a_a^S)^{-1}\,A^{-1/3}$$

Data: Antony *et al.* ADNDT66(97)1

$E_{\rm mic}$: Koura *et al.*, ProTheoPhys113(05)305
v Groote *et al.*, AtDatNucDatTab17(76)418
Moller *et al.*, AtDatNucDatTab59(95)185

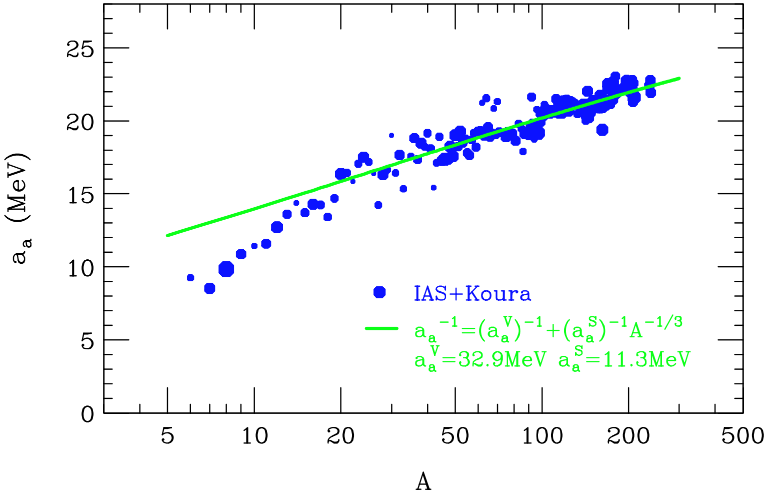
$a_a(A)$ from IAS



Symbol size proportional to relative significance.

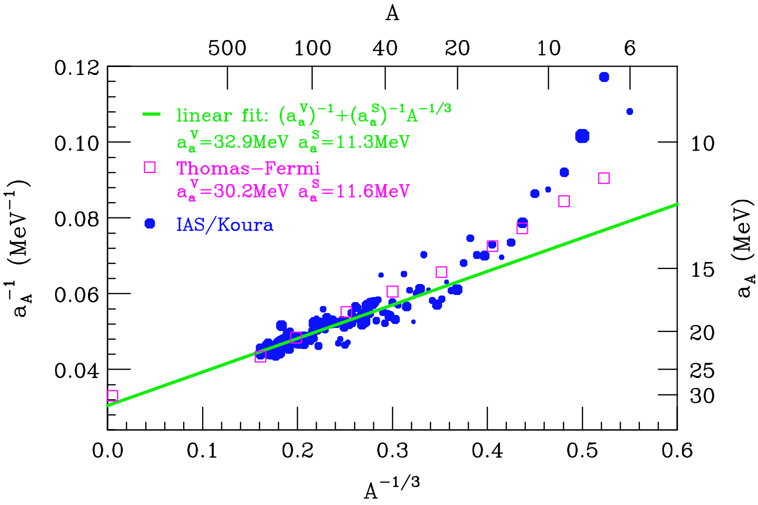
∼Linear dependance from $A \gtrsim 20$ on.

a_a vs A

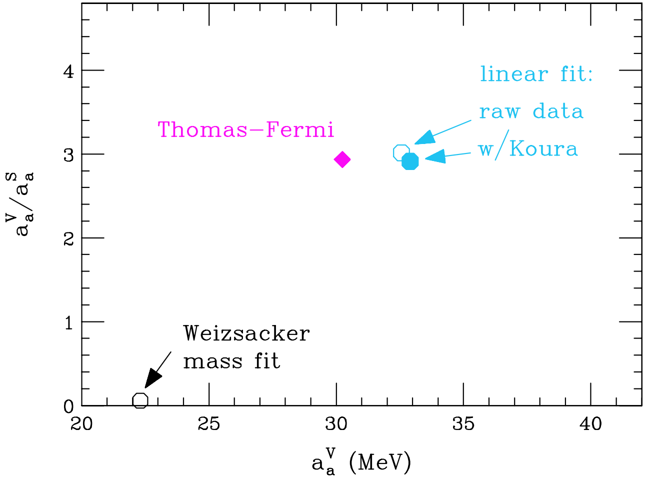


Line: best fit at $A > 20$.

Best $a(A)$-Descriptions



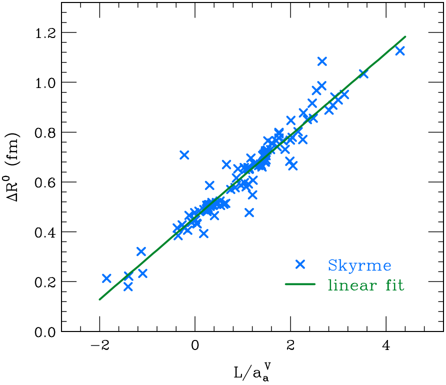
Symmetry-Energy Parameters

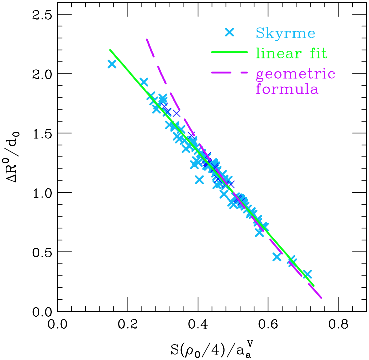


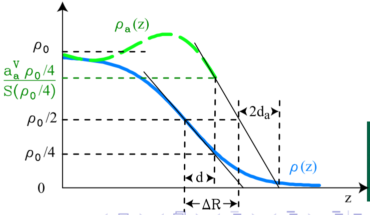
$a_a^V = (31.5 - 33.5)$ MeV, $a_a^S = (9.5 - 12)$ MeV, $L \sim 95$ MeV

Displacement of Isovector Surface

$$\langle r^2 \rangle_n^{1/2} - \langle r^2 \rangle_p^{1/2} \simeq \frac{A\,(N-Z)}{4NZ}\,\frac{\langle r^2 \rangle_a - \langle r^2 \rangle}{\langle r^2 \rangle^{1/2}}$$

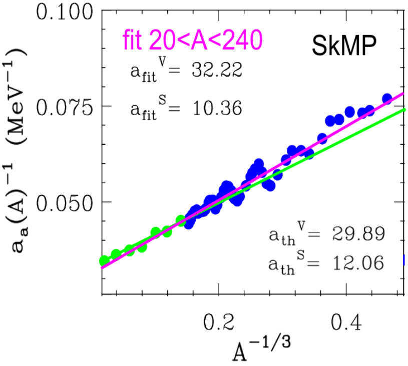


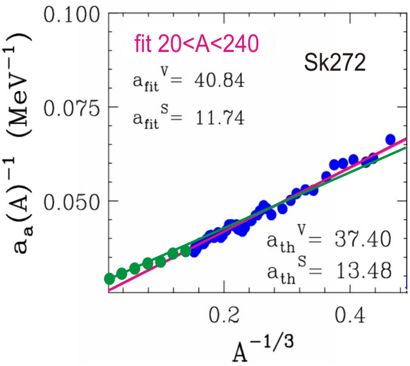




Spherical SHF: Do you get out what you put in?

Coefficient-extraction from spherical SHF results for realistic (blue) and humongous (green) (up to $A \sim 10^6$) nuclei, using P.-G. Reinhard's codes





Nuclei in Nature too small for clean surface/volume separation!

Consequence: a_a^V a bit smaller, a_a^S a bit larger and L a bit lower than from the simple analysis.

Outlier issue.

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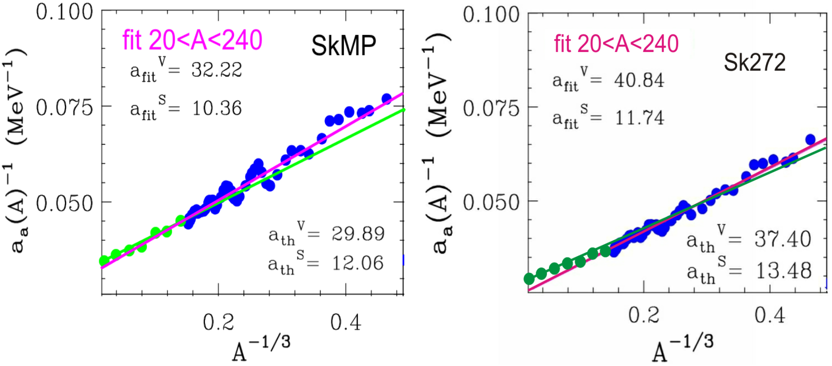

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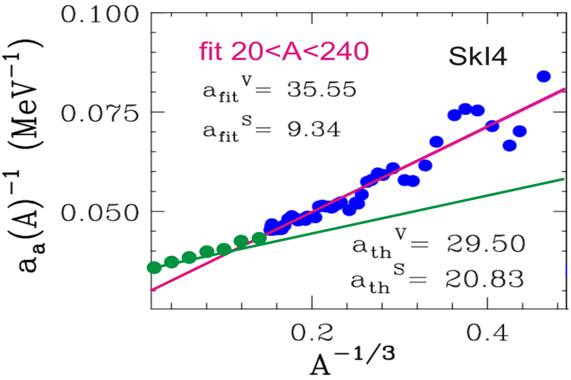
Outlier Skyrme Interactions

For some Skyrme interactions, the deviations between the known symmetry coefficients, a_a^V and a_a^S, and those deduced from $a_a(A)$, for $20 < A < 240$, are dramatic:

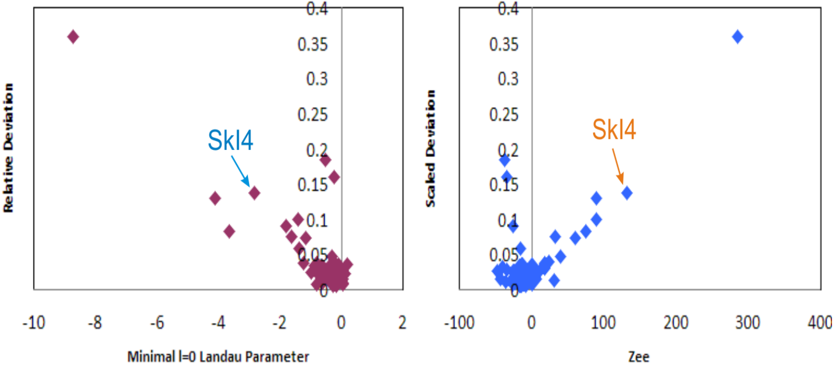


The large deviations appear to be related to instabilities of the interactions and/or excessive nonlocalities.

Deviations vs Skyrme Characteristics

Long-wavelength stability for the Skyrme interactions: Dimensionless Landau parameters must satisfy $L_0 > -1$.

Nonlocality in symmetry energy: $\mathcal{H} = \ldots + \zeta \, (\nabla \rho_{np})^2$.



Discrepancies, between what is put in and what is taken out, appear to be largely related to objectively unphysical properties

Conclusions

- Symmetry-energy term weakens as nuclear mass number decreases: for $A \gtrsim 20$, $a_a(A) \simeq a_a^V/(1 + a_a^V/a_a^S\, A^{1/3})$, where $a_a^V = (31.5 - 33.5)\,\text{MeV}$, $a_a^S = (9.5 - 12)\,\text{MeV}$.
- Weakening of the symmetry term is tied to the asymmetry skins and to the shape of $S(\rho)$.
- Skin sizes in all nuclei quantifiable in terms of single ratio, already known, $a_a^V/a_a^S \lesssim 3.0$. Corresponding $L \lesssim 95\,\text{MeV}$.
- Systematic of proton densities for one A should principally contain as much info as the skins and even more.
- 2 fundamental densities characterize nucleon distributions: isoscalar & isovector. Density surfaces are displaced from each other by $\Delta R \lesssim 0.95\,\text{fm}$ and have different diffusenesses, $d \sim 0.54\,\text{fm}$ and $d_a \sim 0.40\,\text{fm}$.
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- Symmetry-energy term weakens as nuclear mass number decreases: for $A \gtrsim 20$, $a_a(A) \simeq a_a^V/(1 + a_a^V/a_a^S\, A^{1/3})$, where $a_a^V = (31.5 - 33.5)\,\text{MeV}$, $a_a^S = (9.5 - 12)\,\text{MeV}$.
- Weakening of the symmetry term is tied to the asymmetry skins and to the shape of $S(\rho)$.
- Skin sizes in all nuclei quantifiable in terms of single ratio, already known, $a_a^V/a_a^S \lesssim 3.0$. Corresponding $L \lesssim 95\,\text{MeV}$.
- Systematic of proton densities for one A should principally contain as much info as the skins and even more.
- 2 fundamental densities characterize nucleon distributions: isoscalar & isovector. Density surfaces are displaced from each other by $\Delta R \lesssim 0.95\,\text{fm}$ and have different diffusenesses, $d \sim 0.54\,\text{fm}$ and $d_a \sim 0.40\,\text{fm}$.
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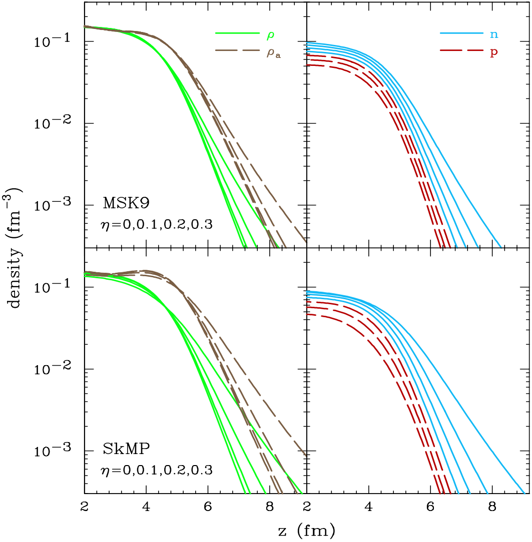
Density Tails



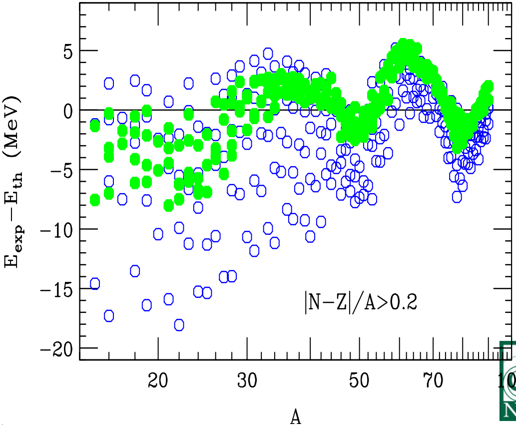
Two Skyrme interactions + different asymmetries

Modified Binding Formula

$$E = -a_V\,A + a_S\,A^{2/3} + a_C\,\frac{Z^2}{A^{1/3}} + \frac{a_a^V}{1 + A^{-1/3}\,a_a^V/a_a^S}\,\frac{(N-Z)^2}{A}$$

Energy Formula Performance:

Fit residuals f/light asymmetric nuclei, either following the Bethe-Weizsäcker formula (open symbols) or the modified formula with $a_a^V/a_a^S = 2.8$ imposed (closed), i.e. the same parameter No.



Droplet Model

Droplet model (Myers & Swiatecki '69)

$$E = \left(-a_1 + J\bar{\delta}^2 - \frac{1}{2}K\bar{\epsilon}^2 + \frac{1}{2}M\bar{\delta}^4\right)A + \left(a_2 + Q\tau^2 + a_3A^{-1/3}\right)A^{2/3} + c_1\frac{Z^2}{A^{1/3}}\left(1 + \frac{1}{2}\tau A^{-1/3}\right) - c_2Z^2A^{1/3} - c_3\frac{Z^2}{A} - c_4\frac{Z^{4/3}}{A^{1/3}}$$

where

$$\bar{\epsilon} = \frac{1}{K}\left(-2a_2A^{-1/3} + L\bar{\delta}^2 + c_1\frac{Z^2}{A^{4/3}}\right), \qquad \tau = \frac{3}{2}\frac{J}{Q}\left(\bar{\delta} + \bar{\delta}_s\right)$$

$$\bar{\delta} = \frac{I + \frac{3}{8}\frac{c_1}{Q}\frac{Z^2}{A^{5/3}}}{1 + \frac{9}{4}\frac{J}{Q}A^{-1/3}}, \qquad \bar{\delta}_s = -\frac{c_1}{12J}\frac{Z}{A^{1/3}}, \qquad I = \frac{N-Z}{N+Z}$$

$Q = H/(1 - \frac{2}{3}P/J)$. Expansion in asymmetry yields results consistent with current, but approach more complex. . .

Liquid Drop Model

The current formula:

$$E = -a_V\,A + a_S\,A^{2/3} + a_C\,\frac{Z^2}{A^{1/3}} + a_A^V\,\frac{(N-Z)^2}{A}\,\frac{1}{1+\frac{a_A^V}{a_A^S}\,A^{-1/3}}$$

Liquid drop model [LDM] (Myers & Swiatecki '66)

$$\begin{aligned} E &= -a_V\left(1-\kappa_V\,I^2\right)A + a_S\left(1-\kappa_S\,I^2\right)A^{2/3} \\ &\quad + a_C\,\frac{Z^2}{A^{1/3}} - a_4\,\frac{Z^2}{A} \end{aligned}$$

with $I = (N-Z)/A$. LDM corresponds to the expansion in the current formula:

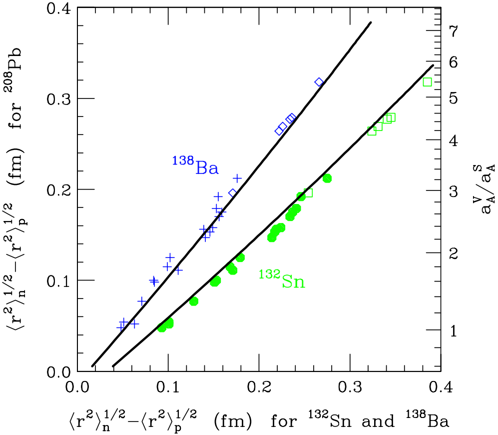
$$\frac{1}{\frac{A}{a_A^V}+\frac{A^{2/3}}{a_A^S}} \simeq \frac{a_A^V}{A}\left(1-\frac{a_A^V}{a_A^S}\,A^{-1/3}\right)$$

But that expansion only accurate for $A \gtrsim 1000$, i.e. never!

Analytic Expression for Skin Size

symmetry energy only — Coulomb correction

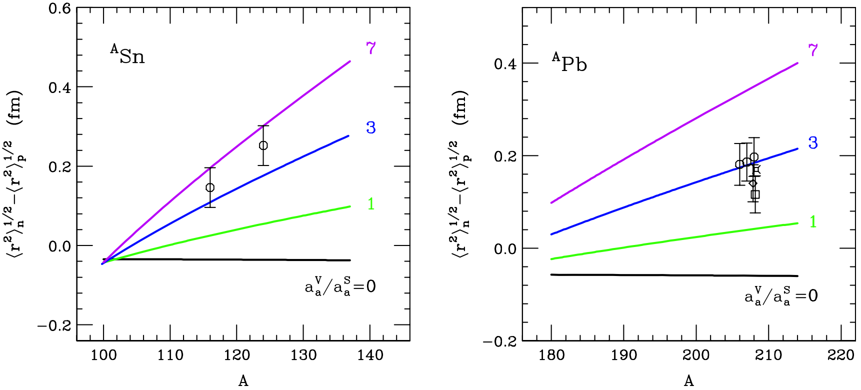
$$\frac{\langle r^2\rangle_n^{1/2} - \langle r^2\rangle_p^{1/2}}{\langle r^2\rangle^{1/2}} = \frac{A}{6NZ}\,\frac{N-Z}{1+A^{1/3}\,a_a^S/a_a^V} - \frac{a_C}{168 a_a^V}\,\frac{A^{5/3}}{N}\,\frac{\frac{10}{3}+A^{1/3}\,a_a^S/a_a^V}{1+A^{1/3}\,a_a^S/a_a^V}$$



Formula (lines) vs Typel & Brown results (symbols) from nonrelativistic & relativistic mean-field calculations, PRC64(01)027302

Skin Sizes for Sn & Pb Isotopes

Lines - formula predictions, PD NPA727(03)233



Favored ratio $a_a^V/a_a^S \simeq 32.5/10.8 \simeq 3.0$

Comparison with Droplet-Model Values