Constraining $\mathrm{E}_{\mathrm{sym}}$ from Astrophysics of Compact Stars

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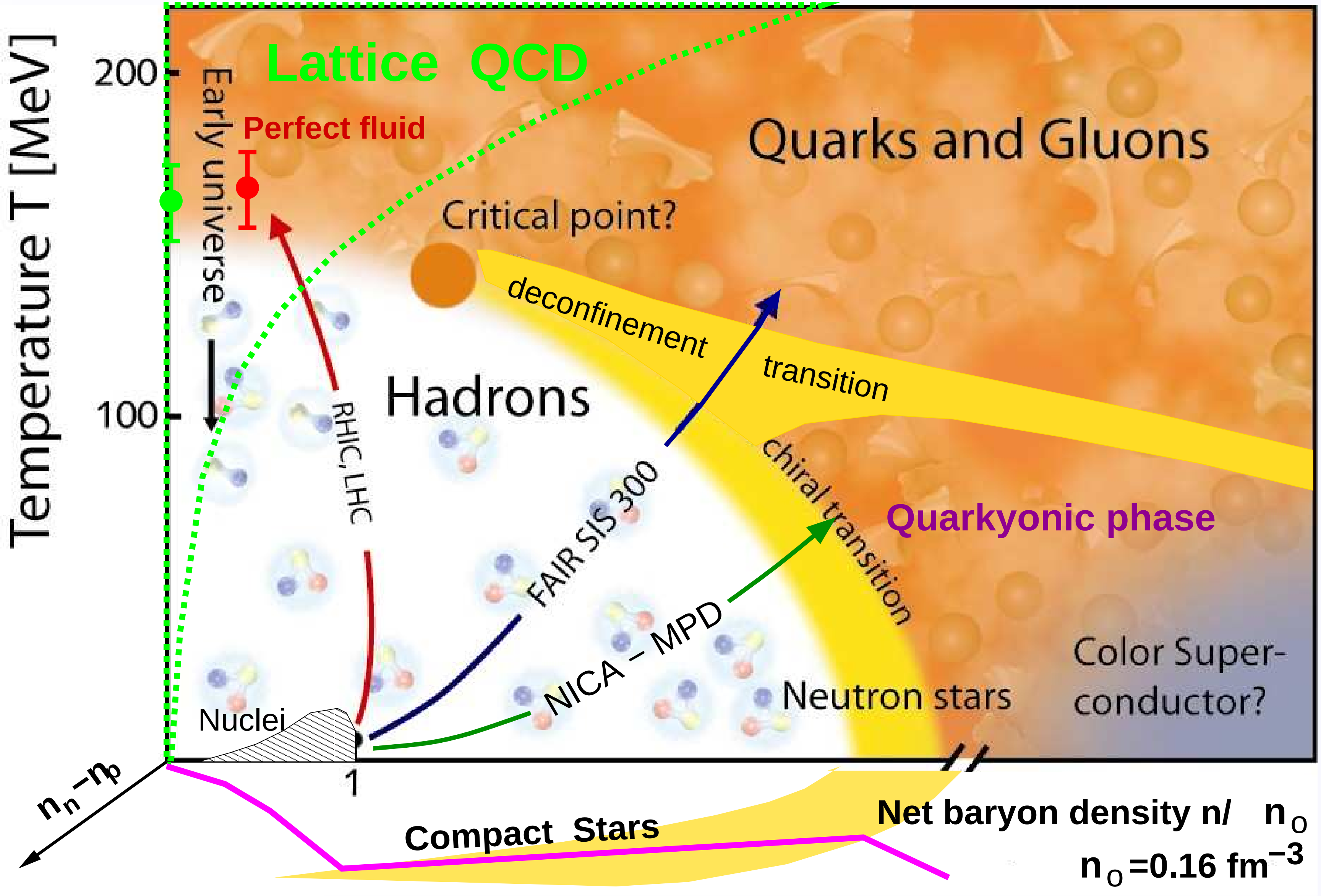
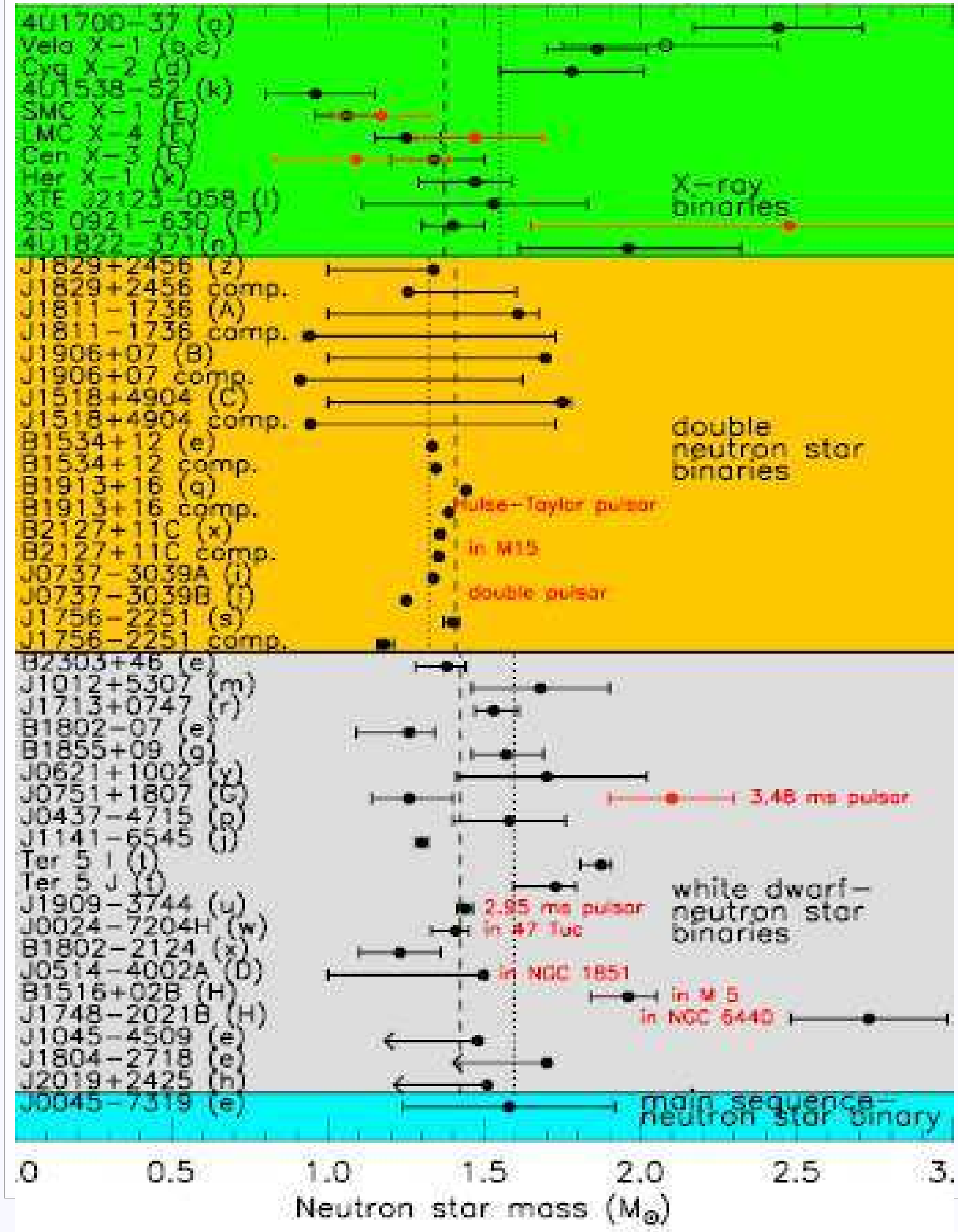
Exploring the Phase Diagram



Outline

- High Density EoS Test Scheme
  - NS Maximum Mass
  - NS Mass-Radius relation
  - NS Gravitational binding
  - Flow in HIC
  - Cooling (direct Urca, Vela mass, logN-logS)
- Nuclear Matter EoS
- Test Scheme vs. Nuclear Matter
- Superconducting Quark Matter and Phase Transition
- Test scheme vs. Quark-Nuclear Matter
- Consequences for the Phase Diagram
- Conclusions

Compact Star Masses (1σ)



binary radio pulsars:

$M_{BRP} = 1.35 \pm 0.04 M_{\odot}$

PSR J1903+0327

(P. Freire et al., arxiv:09... [astro-ph])

$M = 1.67 \pm 0.01 M_{\odot}$

→ constrains minimal **maximum mass**

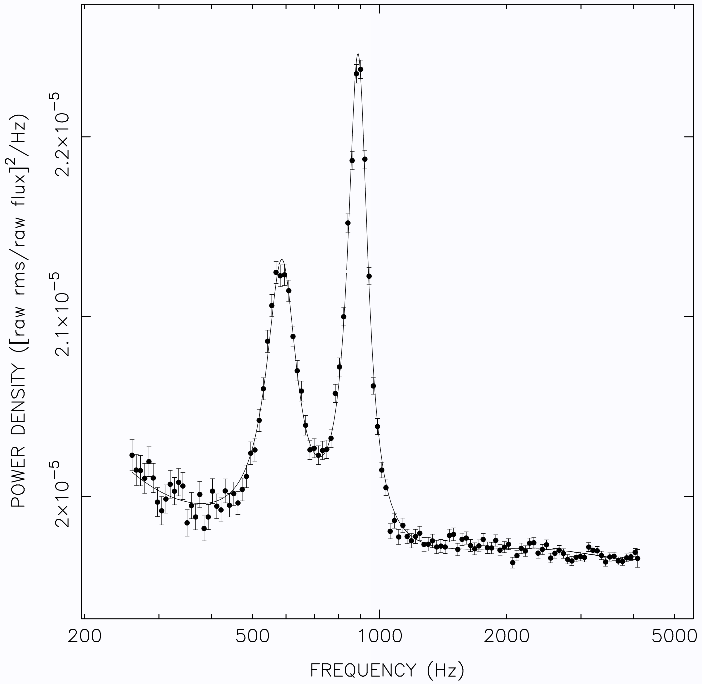
of an EoS model

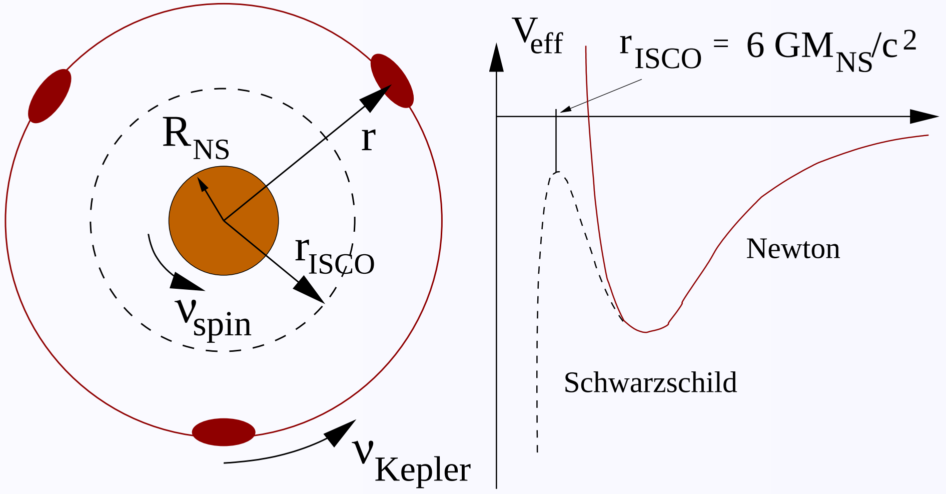
J. M. Lattimer and M. Prakash

Phys. Rev. Lett. 94, 111101 (2005)

Mass-Radius Constraints from QPO’s

<u>Q</u>uasi <u>P</u>eriodic Brightness <u>O</u>scillations





$$\nu_{max} \approx \nu_{orbit} < \nu_{ISCO}$$

Keplerian Orbit r_K

$$R < r_k = (GM/4\pi^2\nu_{max}^2)^{1/3} \to R_{max}(M)$$

$$M < 2.2M_\odot(1000Hz/\nu_{max})(1 + 0.75j) \to M_{max}$$

if(!) $\nu_{max} \approx \nu_{ISCO}$

$$M \approx 2.2M_\odot(1000Hz/\nu_{max})(1 + 0.75j)$$

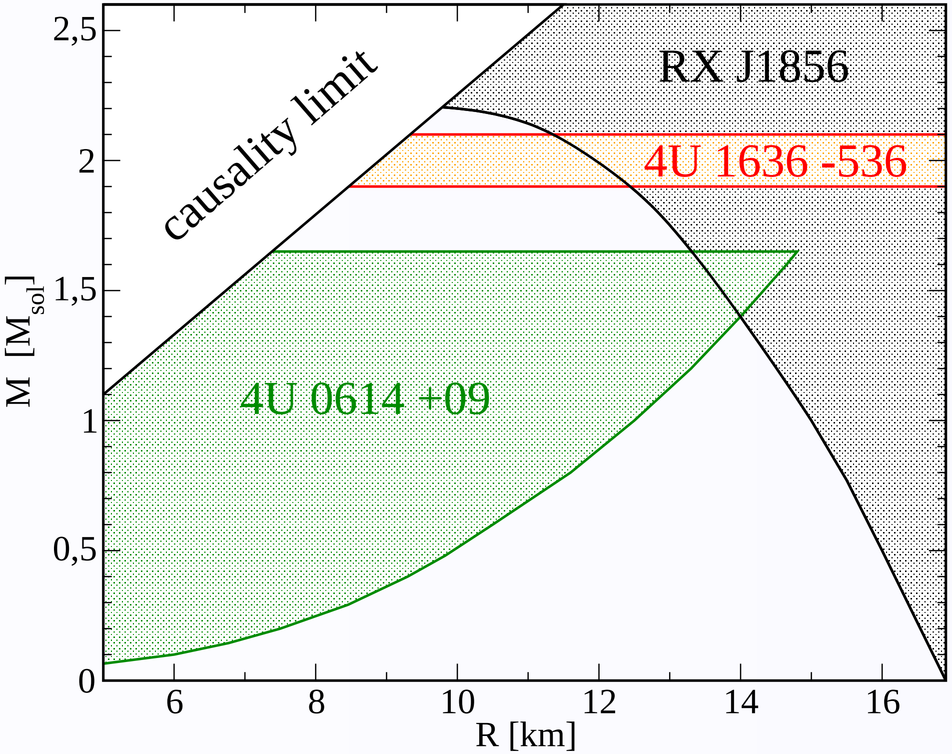
M. van der Klies, ARA&A **38**, 717 (2000)

M-R Constraint from Radio Quiet Isolated NS RXJ1856

RXJ1856 black body spectrum: $T_\infty = 57$ eV

measurement of distance: 60 pc (2002) → 117 pc (2004)

→ photospheric radius: $R_\infty = R(1 - R/R_S)^{-1/2}$ $R_S = 2GM/R$



<u>Mass Radius Constraints</u>

QPO : M-R upper limits

ISCO : max. mass constraint

RXJ1856: M-R lower limits

each region...

→ represents a different object

→ should be touched at least once

J. Trümper et al., Nucl. Phys. Proc. Suppl. 132, 560 (2004)

D. Barret, J.-F. Olive, M.C. Miller, Mon. Not. Roy. Astron. Soc. 361, 855 (2005)

Gravitational Mass ↔ Baryon Number J0737-3039

Double Pulsar System J0737-3039

Pulsar A $P^{(A)}$ = 22.7 ms, $M^{(A)} \approx 1.338 M_\odot$

Pulsar B $P^{(B)}$ = 2.77 s, $M^{(B)} = 1.249 \pm 0.001 M_\odot$ (record!)

Progenitor ONeMg white dwarf, driven hydrodyn. unstable by

e^- captures on Mg & Ne; no mass-loss during collapse

Observational constraint for $M(M_N)$ from PSR J0737-3039:

- observed NSs gravitational mass (remnant star) $M^{(B)} = 1.248 - 1.250 M_\odot$
- critical baryon mass for ONeMg white dwarf $M_N^{(B)} = 1.366 - 1.375 M_\odot$

Theory: $M(M_N)$ characteristic for remnants EoS

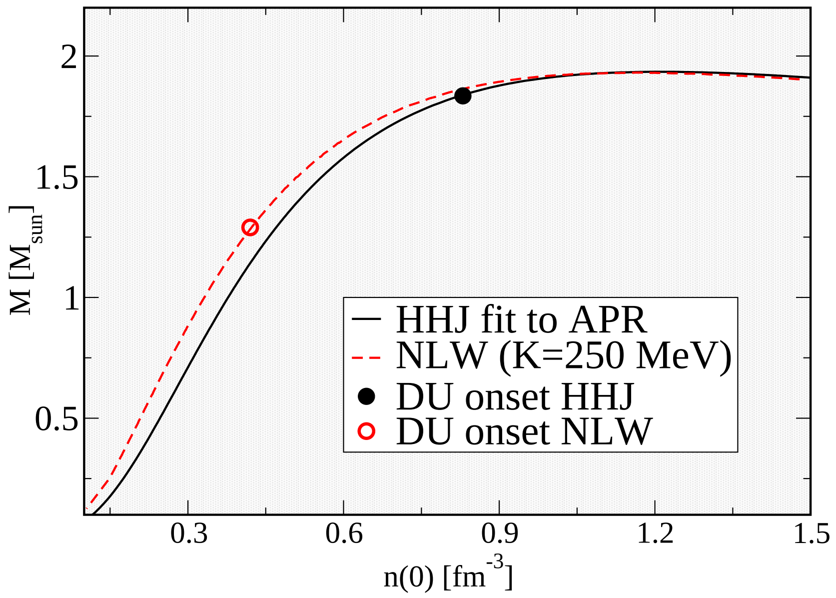
$M = 4\pi \int_0^R \mathrm{d}r r^2 \varepsilon(r)$;

$M_N = u N_B = 4\pi u \int_0^R \mathrm{d}r \frac{r^2 n(r)}{\sqrt{1-2GM(r)/r}}$

(conversion of baryon number to mass by $u = 931.5$ MeV)

P. Podsiadlowski et al., Mon. Not. Roy. Astron. Soc. **361**, 1243 (2005)

Direct Urca Process: $n \longrightarrow p + e^- + \bar{\nu}_e$ (β - decay)

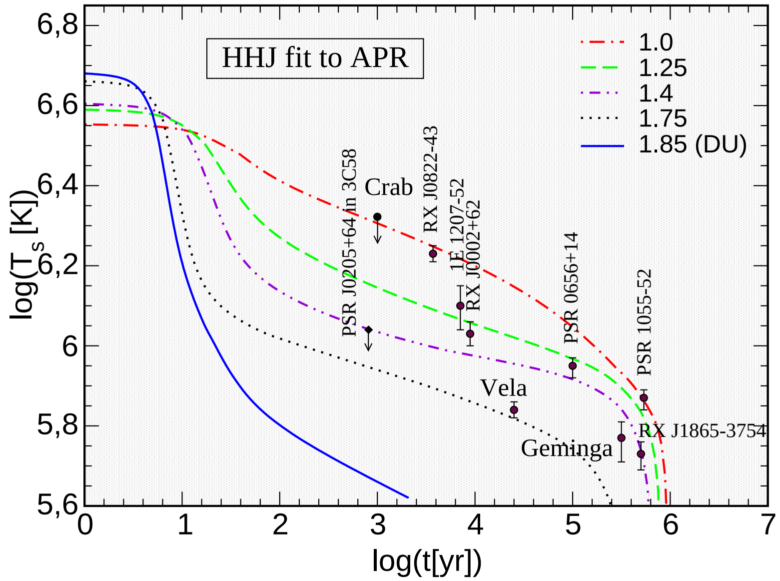


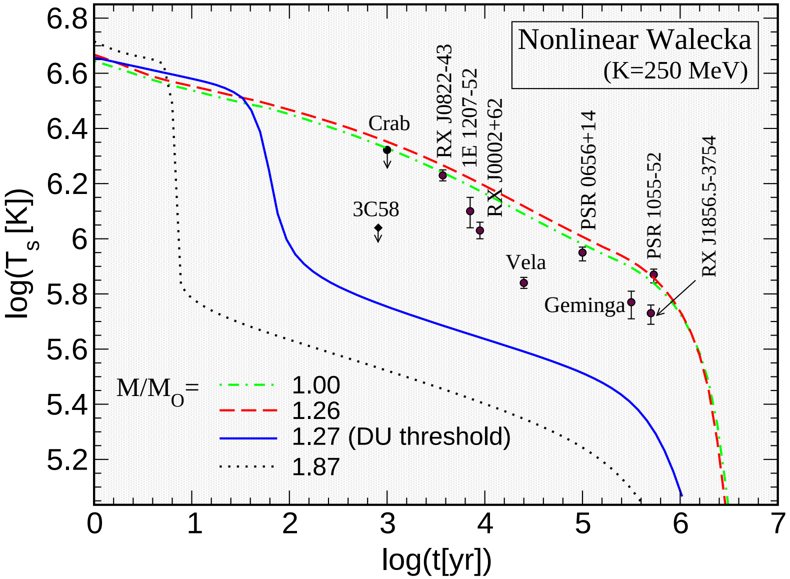
NS cooling – different masses

$\longrightarrow$ **DU cools neutron stars too rapidly**

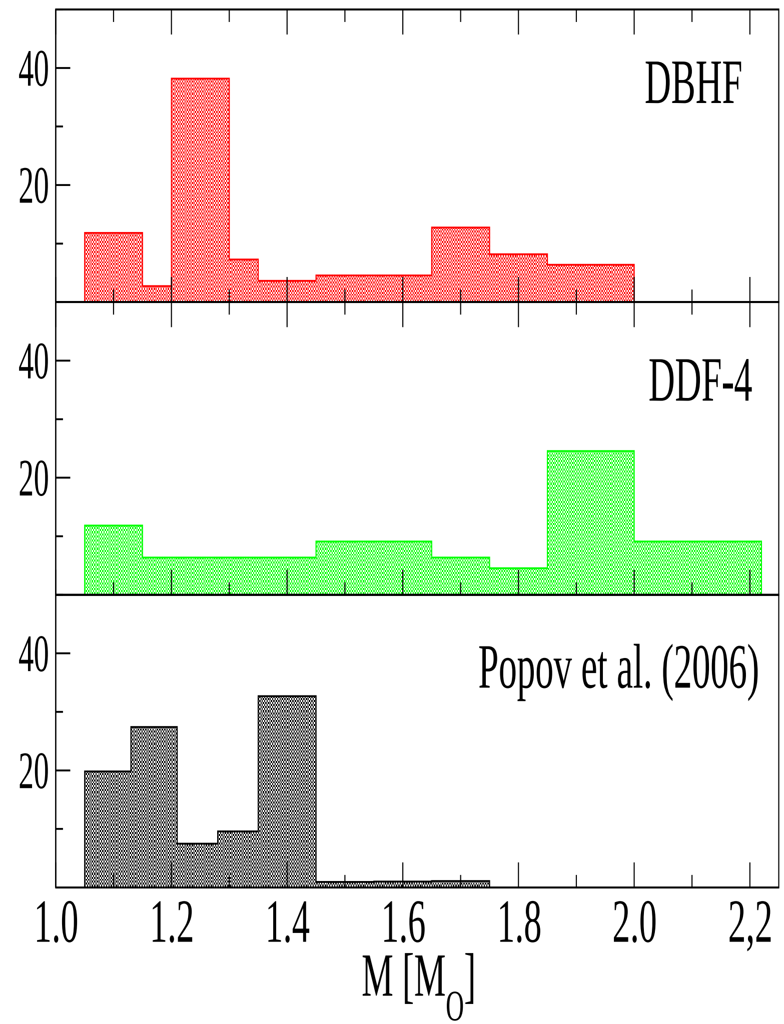
D. Blaschke, H. Grigorian, and D. Voskresensky,

Astron. Astrophys. **424**, 979 (2004)





Direct Urca Process: $n \longrightarrow p + e^- + \bar{\nu}_e$



new test of cooling theory:

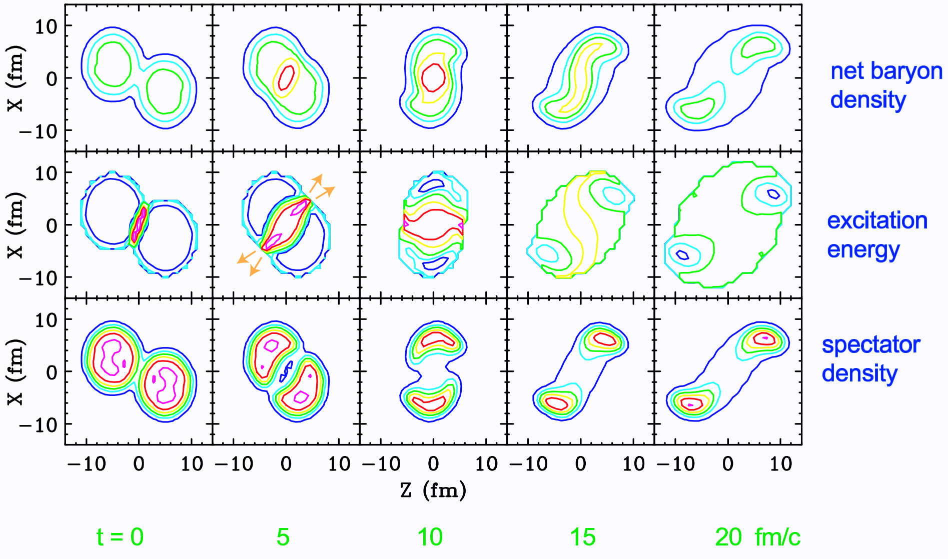
- mass population from cooling
  D.B., Grigorian, PPNP (2007)
  astro-ph/0612092
- NS mass population synthesis
  Popov et al., A&A **448**, 327 (2006)

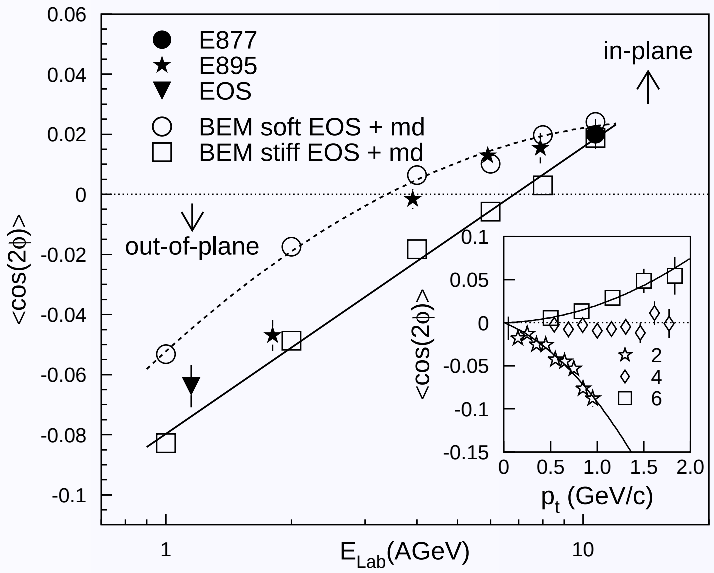
problems with hadronic cooling:

- ”population clustering” at DU onset
- too many ”heavy” stars required
  Vela mass problem

Elliptic Flow in HIC

Heavy Ion Collisions:

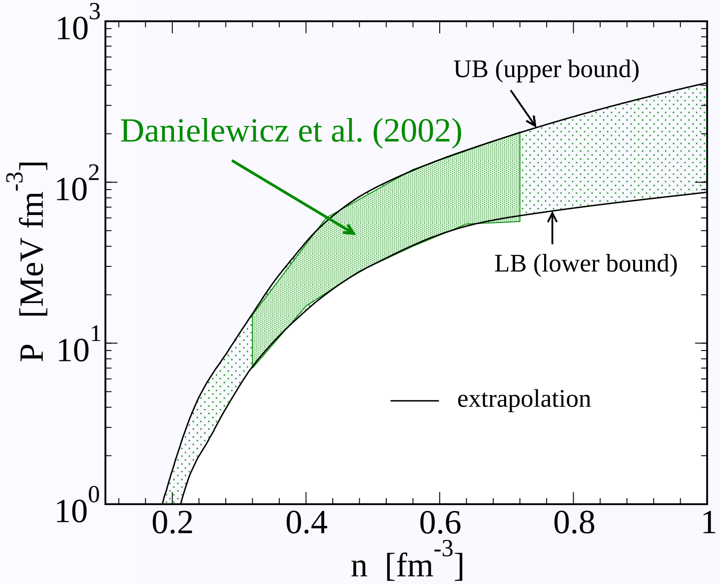




P. Danielewicz et al., Science **298**, 1592 (2002)

Flow data constrain EoS up to $n \approx 4n_0$

$\longrightarrow$ finite range of possible P(n) for given n



Nuclear Matter Equations of State (EoS)

Several approaches to describe dense nuclear matter

- Equations of State at $T = 0$

$$\varepsilon(n_n, n_p, n_e, n_\mu) \rightarrow \varepsilon_h(n_n, n_p) + \sum_{e,\mu} \varepsilon_i(n_i),$$

$$\mu_i = \frac{\mathrm{d}\varepsilon}{\mathrm{d}n_i}, P = \sum_{n,p,e,\mu} \mu_i n_i - \varepsilon_h - \varepsilon_l$$

- expanding binding energy per particle in terms of isospin asymmetry $\beta = \frac{n_n - n_p}{n_n + n_p} = 1 - 2x_p,\ n = n_n + n_p$

$$E(n, \beta) = E_0(n) + \beta^2 E_S(n)$$

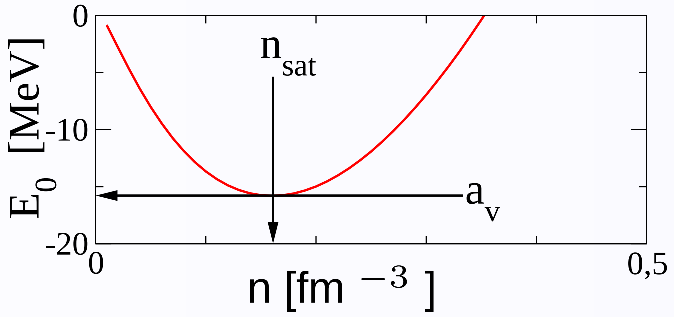
- Thermodynamical Identities hold in SNM and NSM

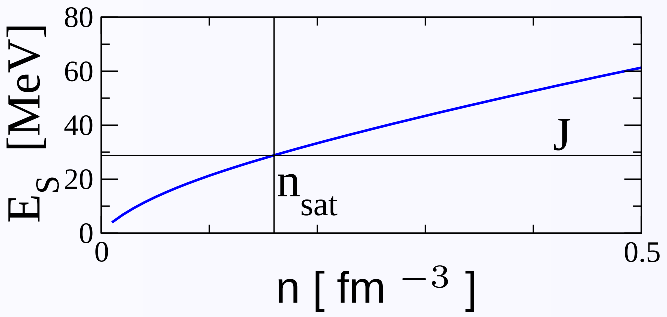
Nuclear Matter Equations of State (EoS)

$$E(n,\beta) = E_0(n) + \beta^2 E_S(n) \approx a_V + \frac{K}{18}\epsilon^2 - \frac{K'}{162}\epsilon^3 + \ldots + \beta^2\left(J + \frac{L}{3}\epsilon + \ldots\right) + \ldots$$

$$\epsilon = (n - n_{sat})/n \qquad \beta = (n_n - n_p)/(n_n + n_p)$$

Model	n_{sat}	a_V	K	K'	J	L	m_D/m
	[fm^{-3}]	[MeV]	[MeV]	[MeV]	[MeV]	[MeV]	
NLρ	0.1459	−16.062	203.3	576.5	30.8	83.1	0.603
NL$\rho\delta$	0.1459	−16.062	203.3	576.5	31.0	92.3	0.603
DBHF	0.1779	−16.160	201.6	507.9	33.7	69.4	0.684
DD	0.1487	−16.021	240.0	−134.6	32.0	56.0	0.565
D^{3}C	0.1510	−15.981	232.5	−716.8	31.9	59.3	0.541
KVR	0.1600	−15.800	250.0	528.8	28.8	55.8	0.800
KVOR	0.1600	−16.000	275.0	422.8	32.9	73.6	0.800
DD-F	0.1469	−16.024	223.1	757.8	31.6	56.0	0.556





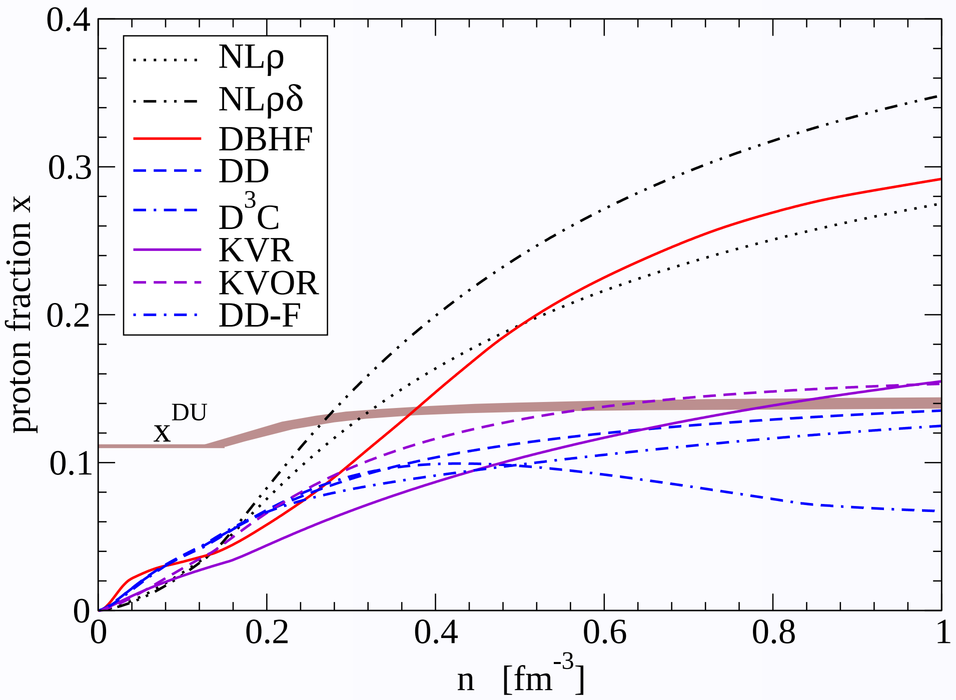
Direct Urca Process

$n \rightarrow p + e + \bar{\nu}_e$ implies $p_n \leq p_p + p_e$, same for muons: $e \leftrightarrow \mu$

charge neutrality: $n_p = n_e + n_\mu$, i.e. $p_p^3 = p_e^3 + p_\mu^3$ results in

$$x_p \geq x_{DU}(x_e) = [1 + (1 + x_e^{1/3})^3]^{-1} \qquad x_e = n_e/(n_e + n_\mu)$$

- no muons: $x_{DU} = 11.1\%$
- relativistic limit ($n_e = n_\mu$): $x_{DU} = 14.8\%$



NLρ, NL$\rho\delta$, DBHF :

DU occurs below $2.5\ n_0$

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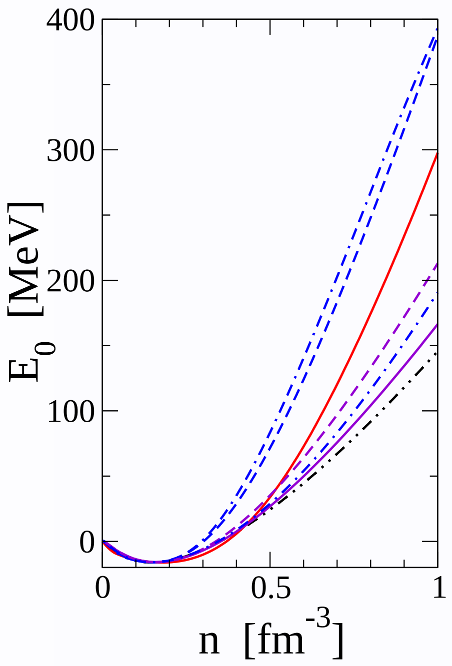
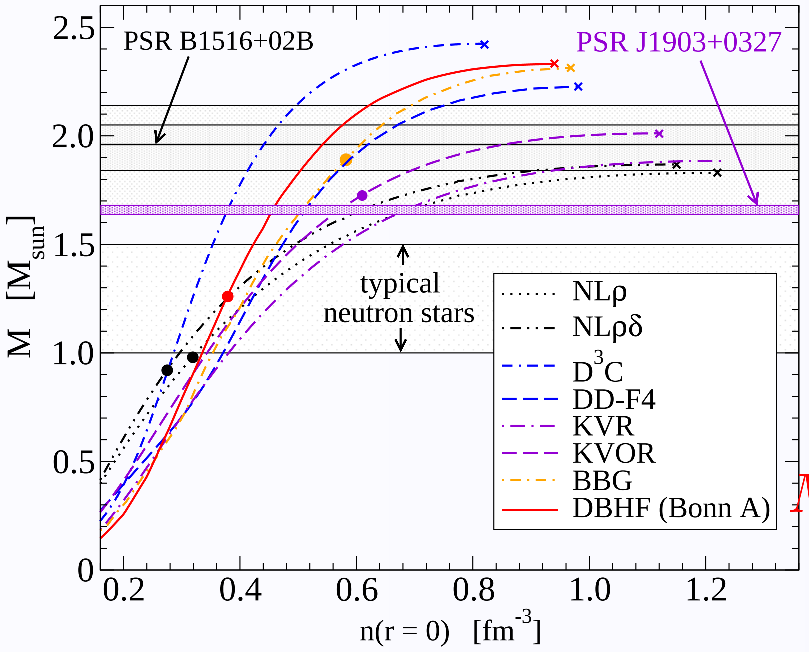




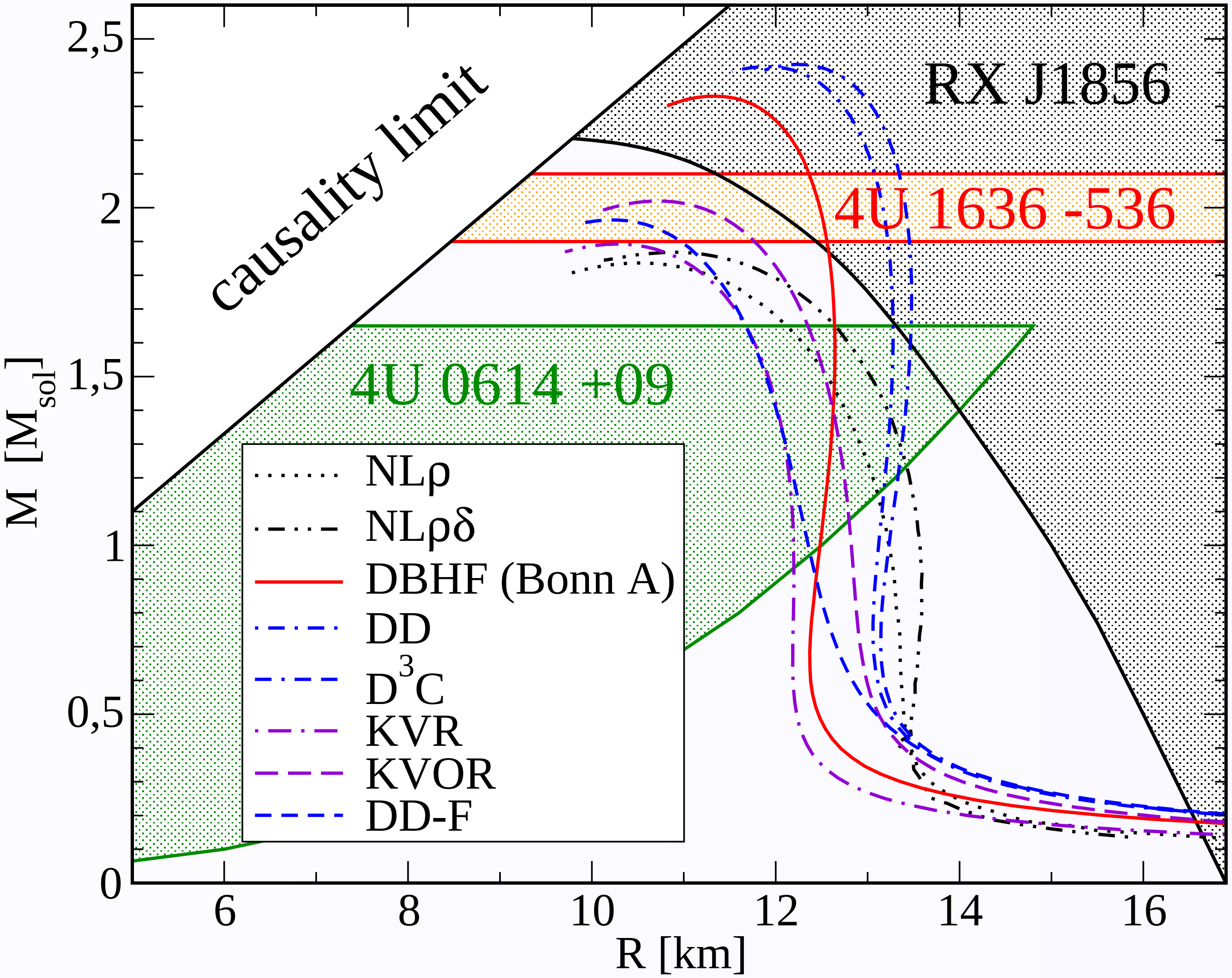
NLρ, NLρδ, DBHF :

DU occurs below $2.5\, n_0$

$M_{DU} \approx 1.0 M_\odot$

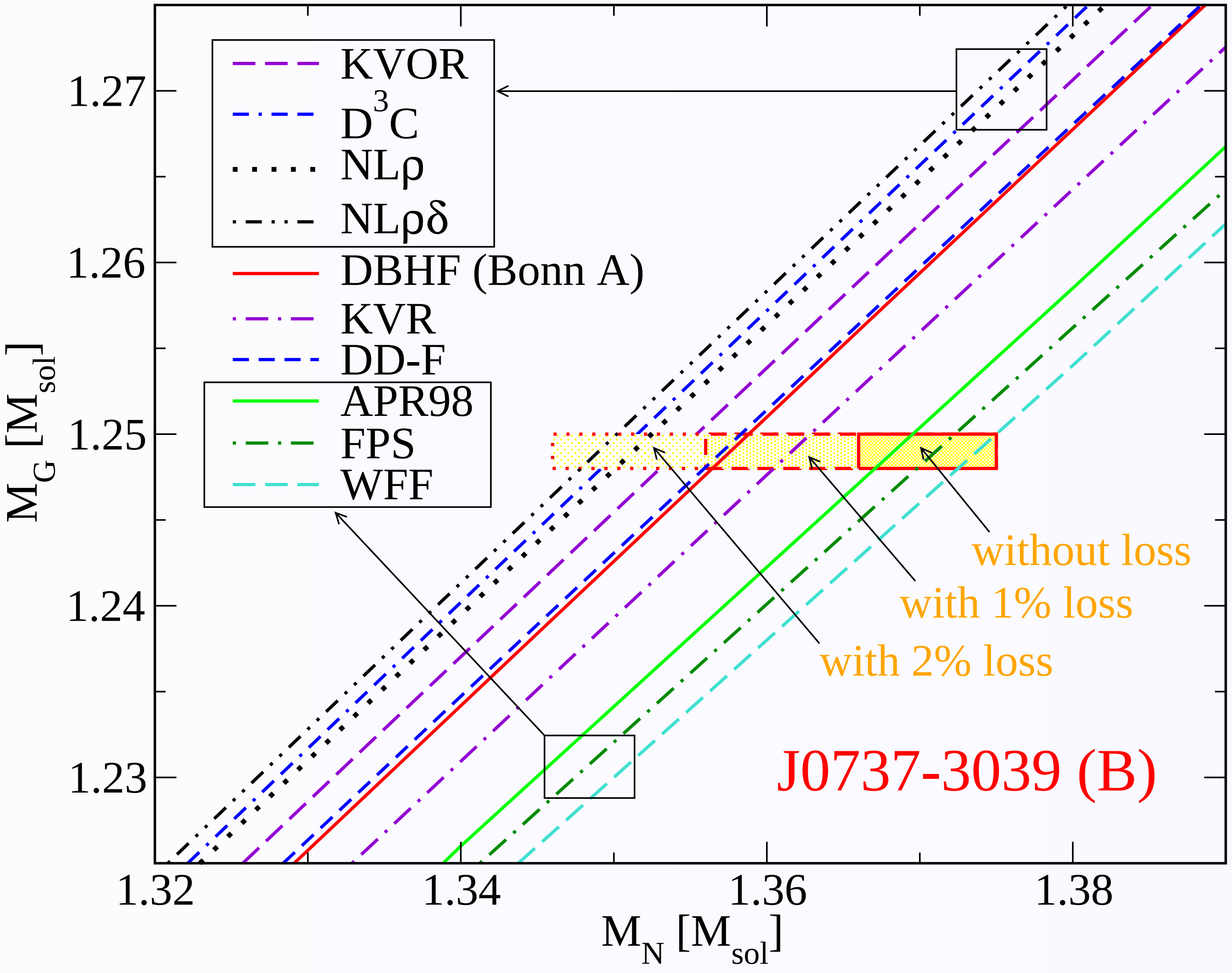
$M(n)$ correlated to $E_0(n)$

Mass Radius Relations



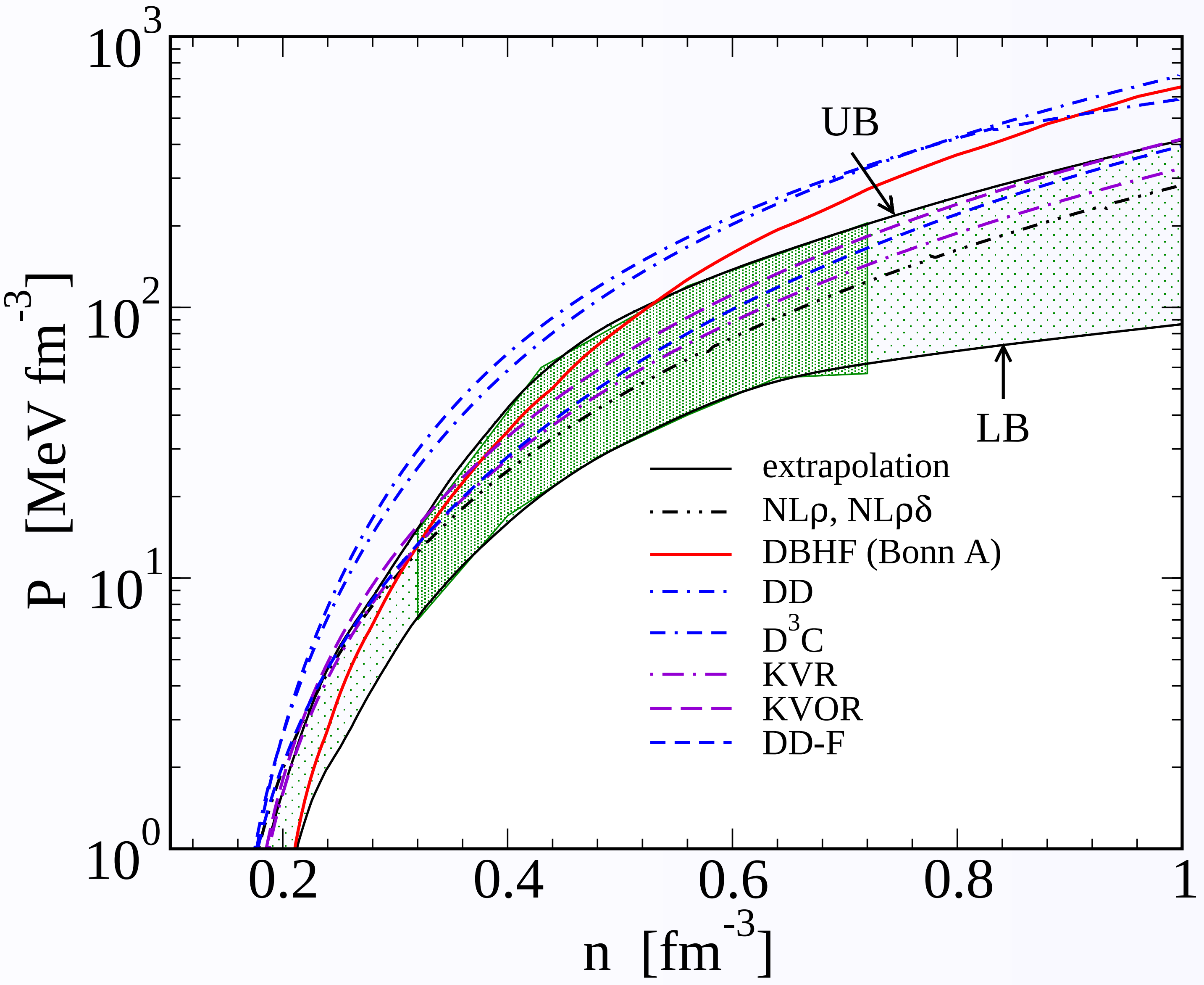
→ agreement with all mass and mass-radius constraints for DBHF, DD, D^{3}C

Gravitational Binding $M(M_N)$ for J0737-3039 (B)



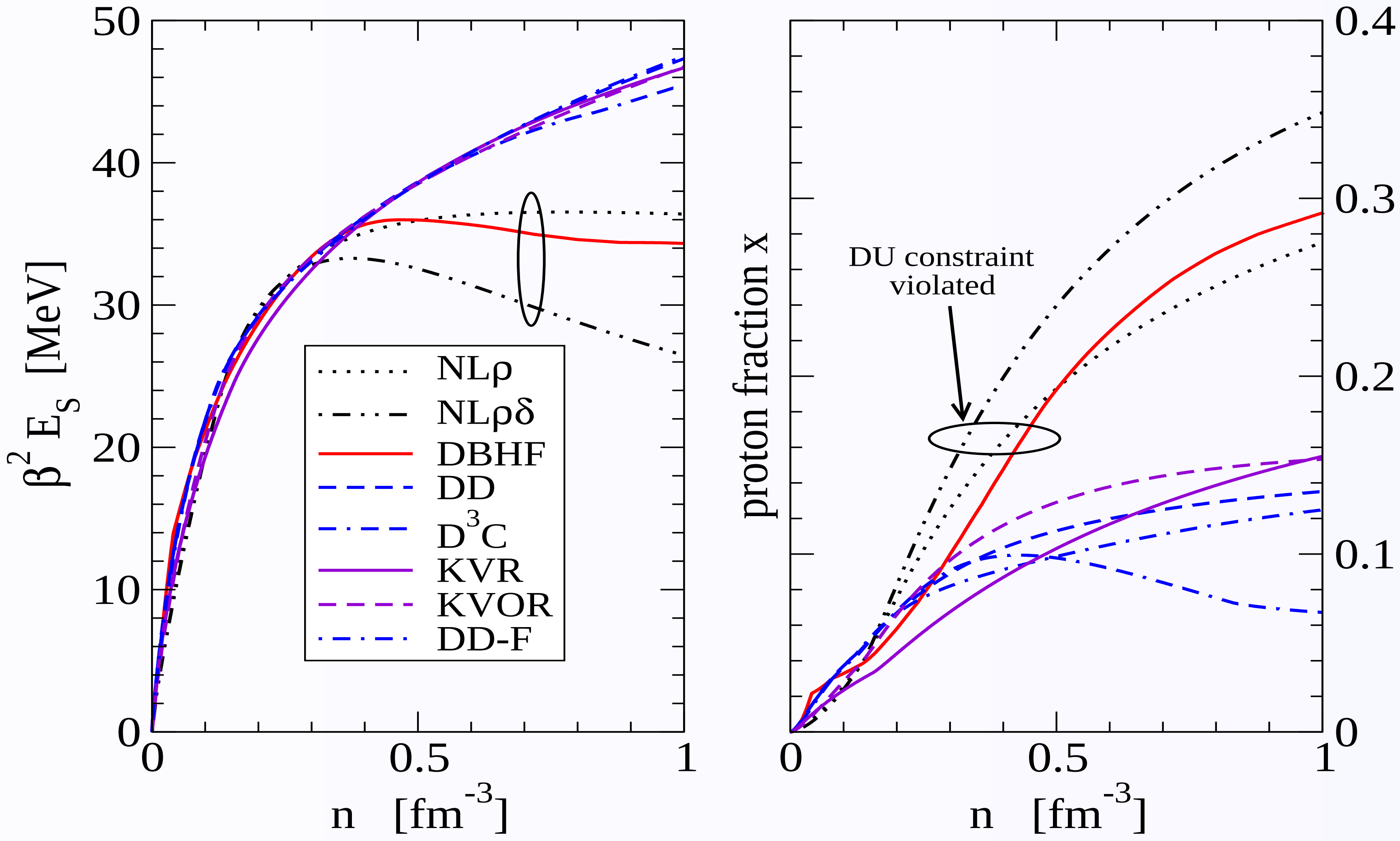
→ applicability depends on level of baryon loss during collapse

Flow Constraint



→ constraint fulfilled for NLρ,NL$\rho\delta$, KVR, KVOR, DD-F; DBHF at low densities

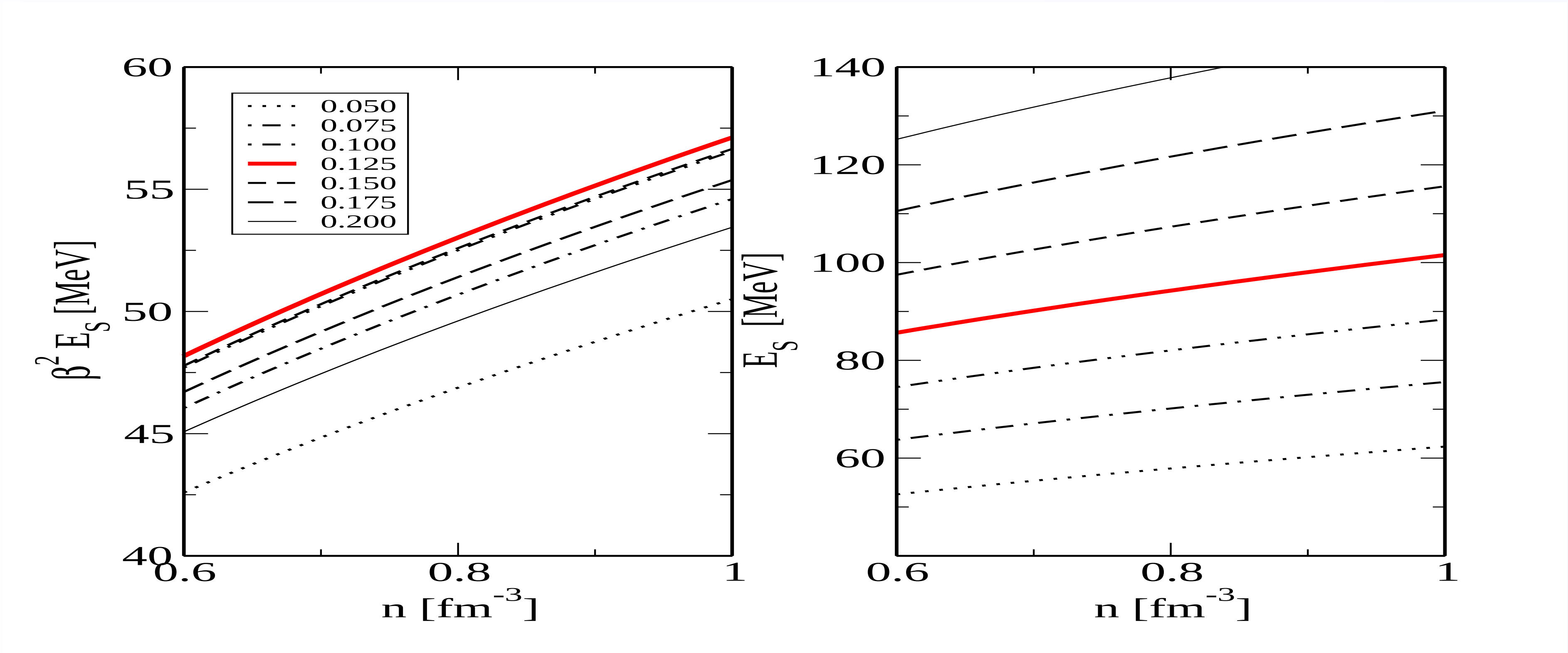
Consequences: Universality conjecture for $\beta^2 E_S(n)$



Exclude NLρ, NL$\rho\delta$, DBHF since DU constraint violated ($M_{DU} < M_{\mathrm{typ}}$)

→ universal $\beta^2 E_S$

Universality conjecture for $\beta^2 E_S(n)$



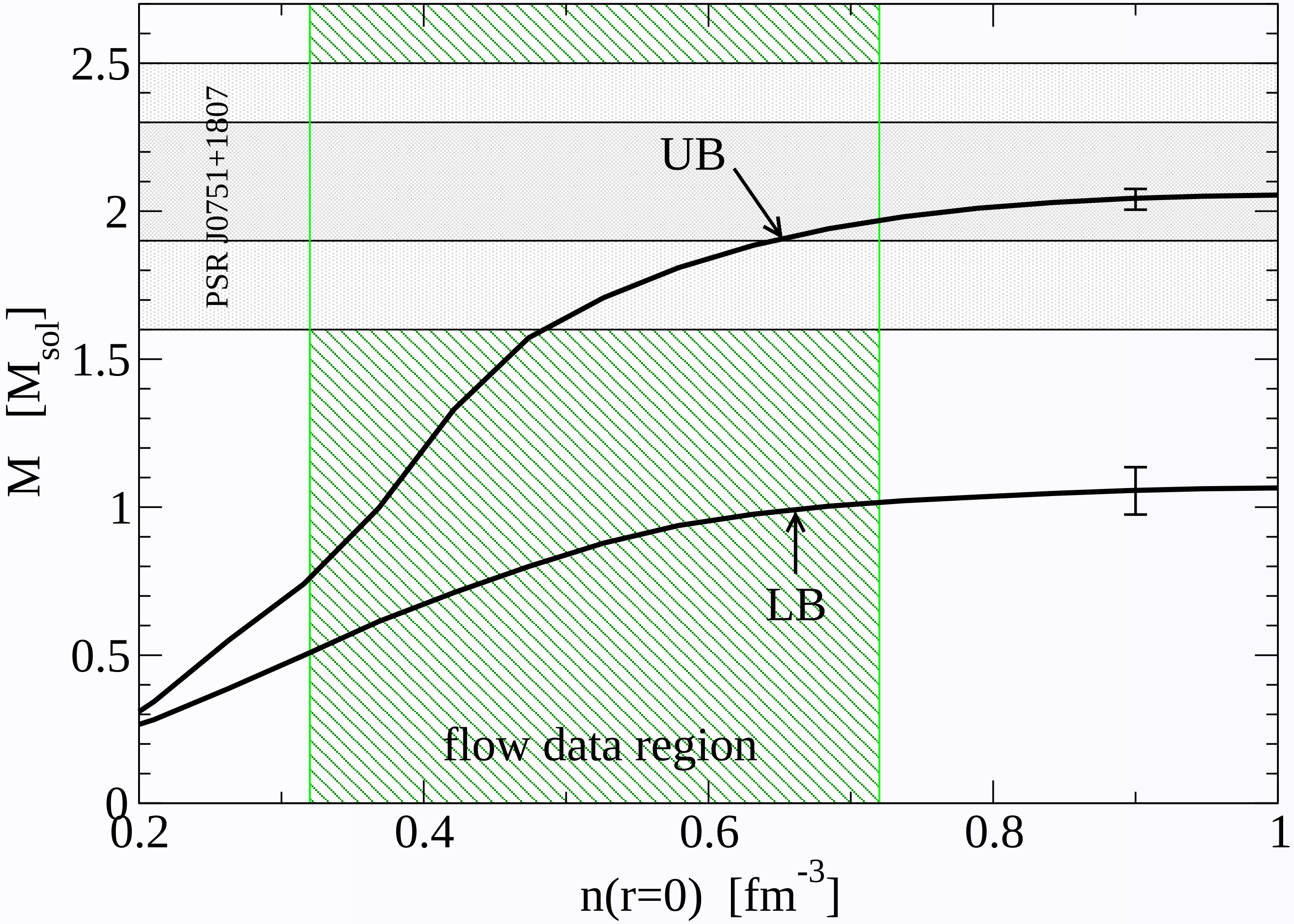
$$\mu_e(n,\beta) = 4\beta E_S(n), \quad xn = \frac{1}{3\pi^2}\mu_e^3 \quad \rightarrow \quad E_S(n,x) = (an)^{1/3}\frac{x^{1/3}}{1-2x}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(1-2x)^2 E_S(n,x)\Big|_{x=x_c} = (an)^{1/3}\left\{\frac{1}{3}x^{-2/3} - \frac{8}{3}x^{1/3}\right\}\Bigg|_{x=x_c} = 0\,, \quad \rightarrow \quad x_c = \frac{1}{8}$$

T. Klähn, D.B., J. Lattimer, in preparation

Consequences: Sharpening the Flow Constraint

How strong is the flow constraint?



LB not reliable ↔ Maximum mass constraint demands stiff EoS

(applied “universal” $\beta^2 E_S$ (error bars!))

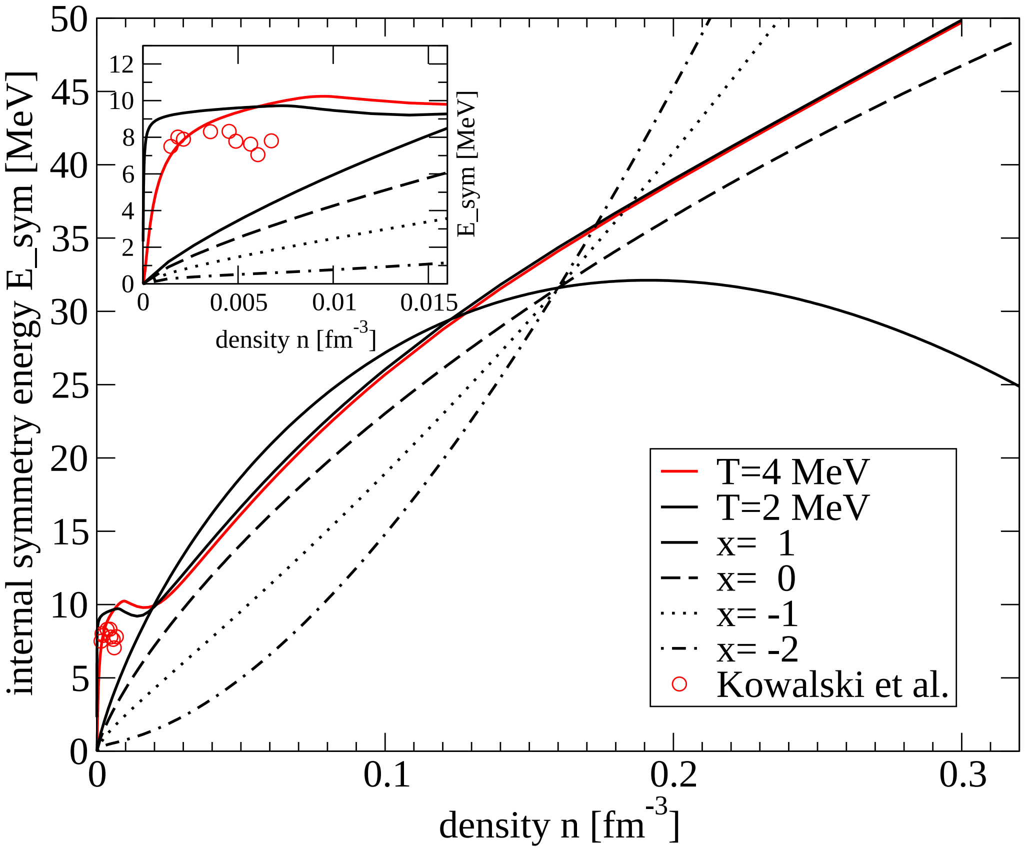
Result

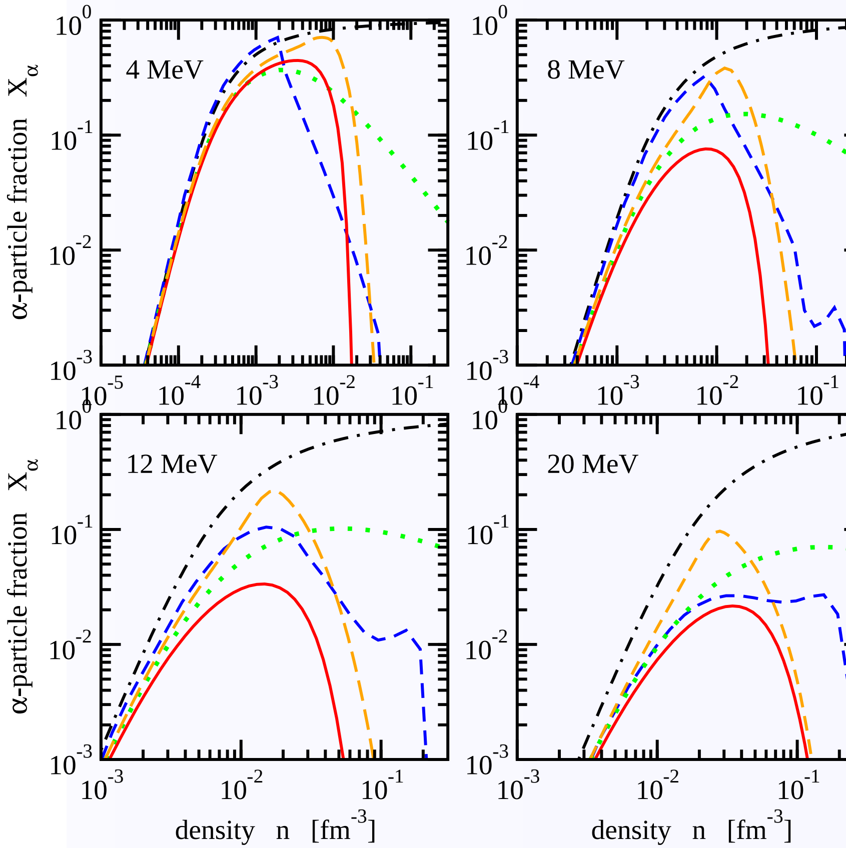
Model	$M_{\mathrm{max}} \geq 1.9\ M_\odot$	$M_{\mathrm{max}} \geq 1.6\ M_\odot$	$M_{\mathrm{DU}} \geq 1.5\ M_\odot$	$M_{\mathrm{DU}} \geq 1.35\ M_\odot$	4U 1636-536 (u)	4U 1636-536 (l)	RX J1856 (A)	RX J1856 (B)	J0737 (no loss)	J0737 (loss 1% $M_\odot$)	SIS+AGS flow constr.	SIS flow+ K^+ constr.	No. of passed strong tests	No. of passed weak tests
NLρ	−	+	−	−	−	−	−	−	−	−	+	+	1	2
NL$\rho\delta$	−	+	−	−	−	−	−	−	−	−	+	+	1	2
DBHF	+	+	−	−	+	+	−	+	−	+	−	+	2	5
DD	+	+	+	+	+	+	−	+	−	−	−	−	3	4
D^{3}C	+	+	+	+	+	+	−	+	−	−	−	−	3	4
KVR	○	+	+	+	−	○	−	−	−	+	+	+	3	5
KVOR	+	+	+	+	−	+	−	−	−	○	+	+	3	5
DD-F	+	+	+	+	−	+	−	−	−	+	+	+	3	5

Complementary scheme with strong (left columns) and weak (right columns) constraints

Favourite EsoS: DBHF, KVR, KVOR, DD-F; None passes all constraints !

Cluster formation in low-density nuclear matter





- RMF and Quantum Statistics (Pauli blocking) combined to describe formation and dissolution of clusters in warm, dilute nuclear matter (→ supernova and HIC applications).
- Important contribution to $E_{\mathrm{sym}}(n)$ at low densities; Prediction of high-density behavior

S. Typel, G. Röpke, T. Klähn, D.B., H. Wolter, arxiv:0908234 [nucl-th]; J. Natowitz et al., in prep.

Quark Matter EoS: NJL-type Model

$$\begin{aligned} S[\bar\psi,\psi] = \quad & \textstyle\sum_p \bar\psi(\not p - \hat m)\psi \\ & +\textstyle\sum_{p,p'} \left[(\bar\psi g(p)\psi)G_S(\bar\psi g(p')\psi) + (\bar\psi i\gamma_0 g(p)\psi)G_V(\bar\psi i\gamma_0 g(p')\psi)\right. \\ & \left.+(\bar\psi i\gamma_5\tau_2\lambda_2 C g(p)\bar\psi^T)G_D(\psi^T C i\gamma_5\tau_2\lambda_2 g(p')\psi)\right], \end{aligned}$$

Bosonization (Hubbard-Stratonovich trick) → Mean-field approximation

$$\Omega_q(\phi,\omega_0,\Delta;\mu_u,\mu_d,T) = \frac{\phi^2}{4G_S} + \frac{|\Delta|^2}{4G_D} + \frac{\omega_0^2}{4G_V} - T\textstyle\sum_n\int\frac{d^3p}{(2\pi)^3}\,\frac{1}{2}\mathrm{Tr}\ln\left(\frac{1}{T}\tilde S^{-1}(i\omega_n,\vec p)\right)$$

Nambu-Gorkov Propagator

$$\tilde S^{-1}(p_0,\vec p) = \begin{pmatrix} \not p - \hat M(p) - \hat\mu\gamma_0 & \Delta\gamma_5\tau_2\lambda_2 g(p) \\ -\Delta^*\gamma_5\tau_2\lambda_2 g(p) & \not p - \hat M(p) + \hat\mu\gamma_0 \end{pmatrix}.$$

Dynamical quark mass matrix (NJL: $g(p) = \Theta(\Lambda - |p|)$)

$$\hat M(p) = \mathrm{diag}(m_u + \phi g(p), m_d + \phi g(p))$$

Renormalized chemical potential matrix

$$\hat\mu = \mathrm{diag}(\mu_u - \omega_0, \mu_d - \omega_0)$$

Nonlocal, Chiral Quark Model (MF)

- chiral gaps (constituent quark mass $m_i = m_i^0 + \phi_i$)

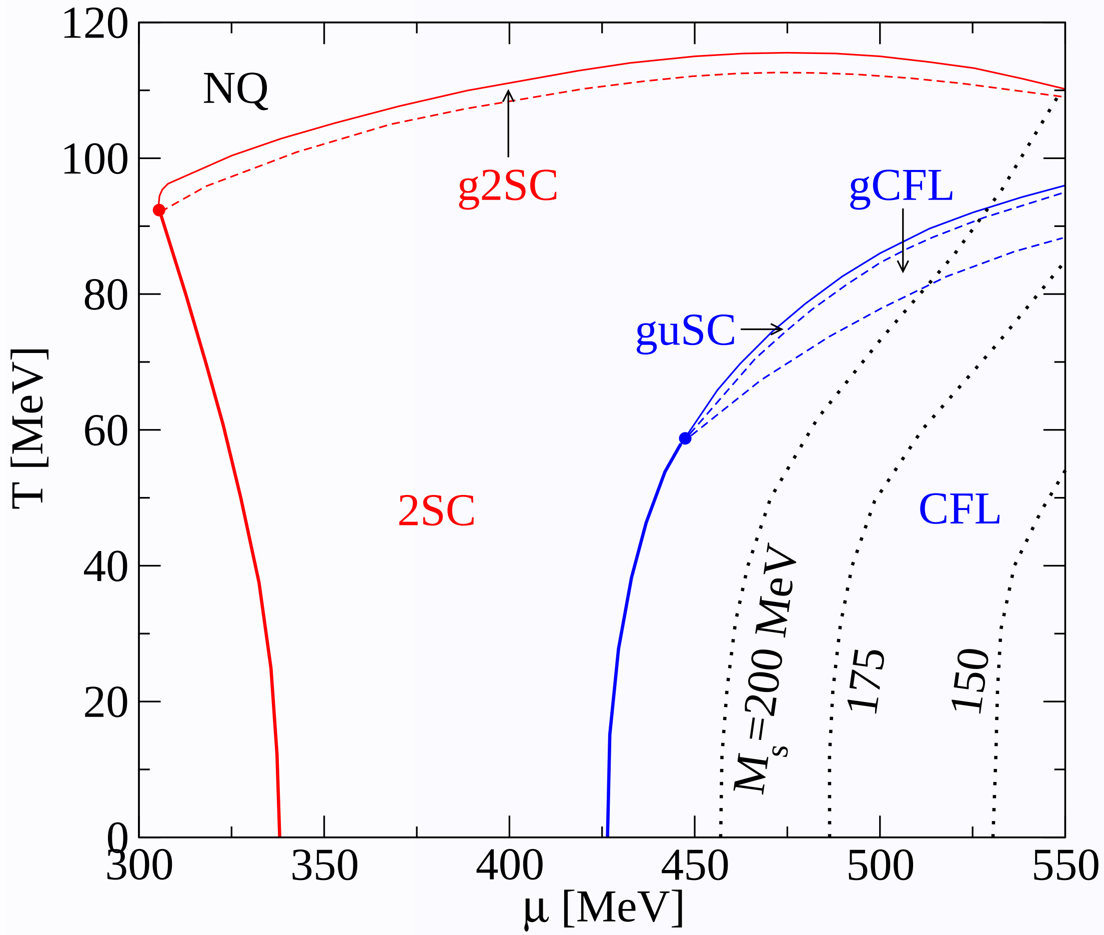
$$\phi_i = -4G_S \langle\langle \bar{q}_i q_i \rangle\rangle$$

- diquark gaps

$$\Delta_{k\gamma} = 2G_D \langle\langle \bar{q}_{i\alpha} i\gamma_5 \epsilon_{\alpha\beta\gamma} \epsilon_{ijk} q^C_{j\beta} \rangle\rangle$$

1. NQ: $\Delta_{ud} = \Delta_{us} = \Delta_{ds} = 0$;
2. NQ-2SC: $\Delta_{ud} \neq 0$, $\Delta_{us} = \Delta_{ds} = 0$ (0<$\chi_{2\mathrm{SC}}$<1);
3. 2SC: $\Delta_{ud} \neq 0$, $\Delta_{us} = \Delta_{ds} = 0$;
4. uSC: $\Delta_{ud} \neq 0$, $\Delta_{us} \neq 0$, $\Delta_{ds} = 0$;
5. CFL: $\Delta_{ud} \neq 0$, $\Delta_{ds} \neq 0$, $\Delta_{us} \neq 0$;

Quark Matter Phase Diagram (NJL case)



Blaschke et al, PRD 72 (2005) 065020

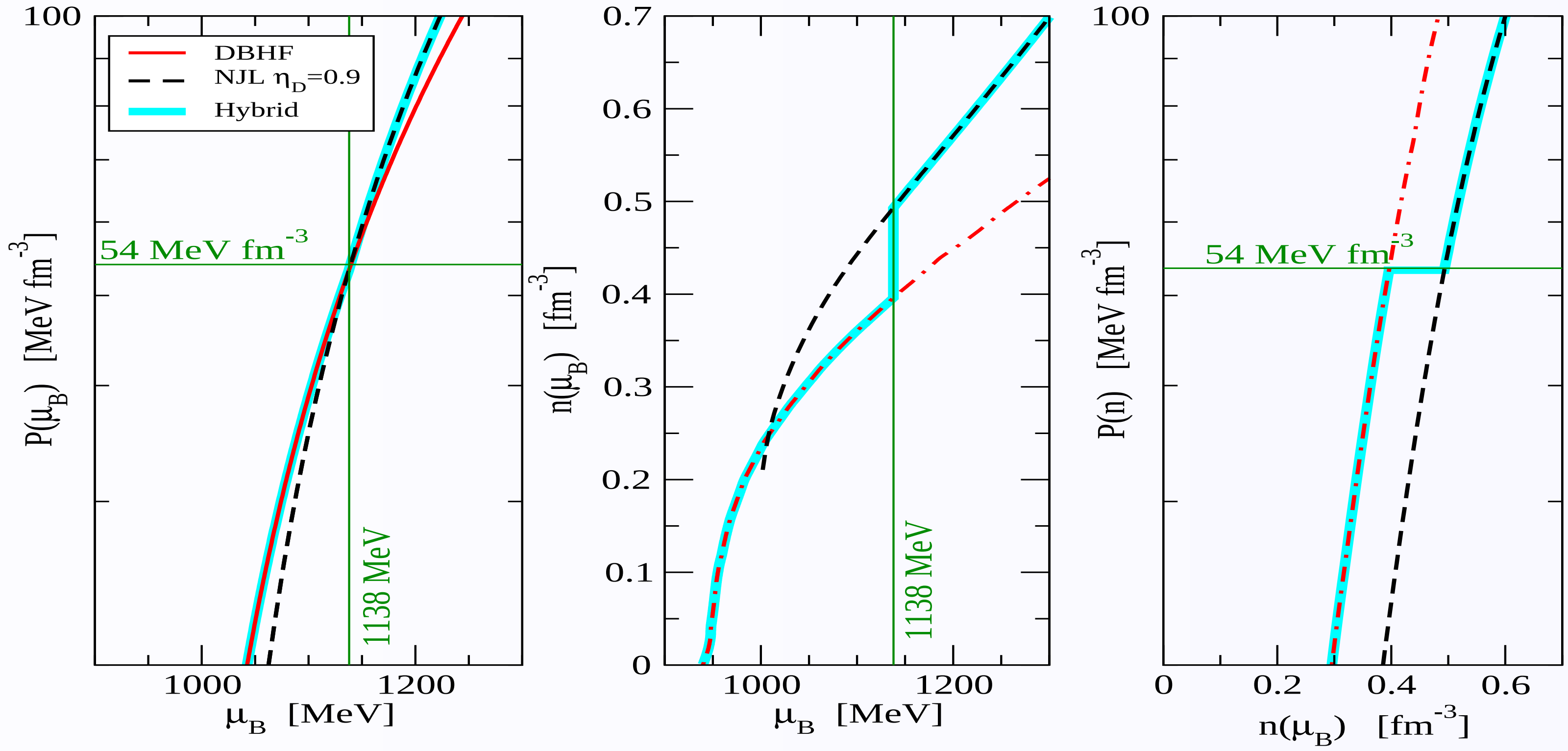
Rüster et al., PRD 72 (2005) 034004

Abuki+Kunihiro, NPA 768 (2006) 118

self-consistent strange quark masses !

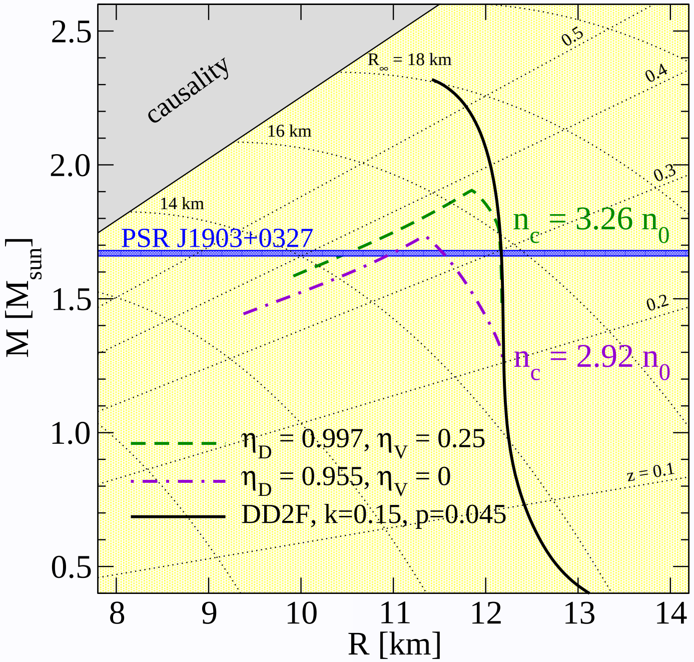
Phase Transition to Quark Matter

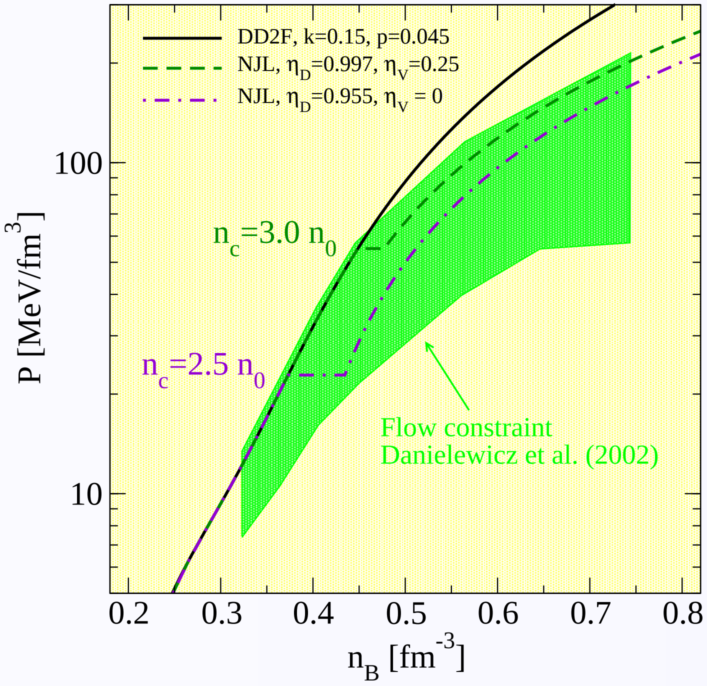
- traditional: two-phase construction



- “masquerade” problem: quark and hadron eos almost identical!
- challenge: hadrons as quark bound states; Beth-Uhlenbeck + Mott-effect

Phase Transition to Quark Matter





- Large Mass $(\sim 2\ M_\odot)$ and radius $(R \geq 12$ km$)$ $\Rightarrow$ stiff quark matter EoS;
  Note: DU problem of DBHF removed by deconfinement! and: CFL core Hybrids unstable!
- Flow in Heavy-Ion Collisions $\Rightarrow$ not too stiff EoS !
  Note: Quark matter removes violation by DBHF at high densities

T. Klähn et al., PLB 654, 170 (2007); [nucl-th/0609067]

Hybrid Star Cooling with 2SC Quark Matter

2SC phase: 1 color (blue) is unpaired

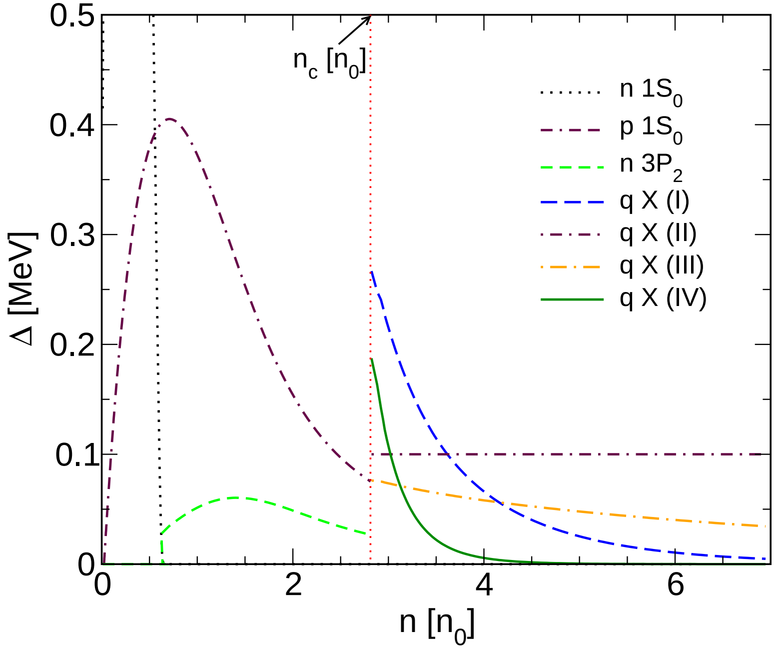
(mixed superconductivity)

Ansatz 2SC + X phase:

$$\Delta_X(\mu) = \Delta_0 \exp[\alpha(1 - \mu/\mu_c)]$$

Model	Δ_0 [MeV]	α
I	1	10
II	0.1	0
III	0.1	2
IV	5	25

Popov, Grigorian, D.B., PRC 74 (2006)

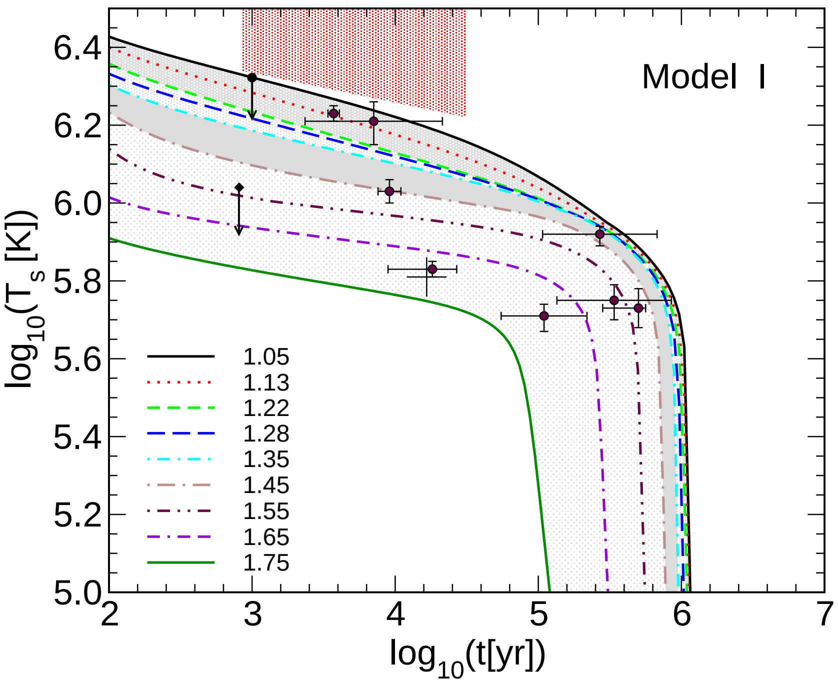


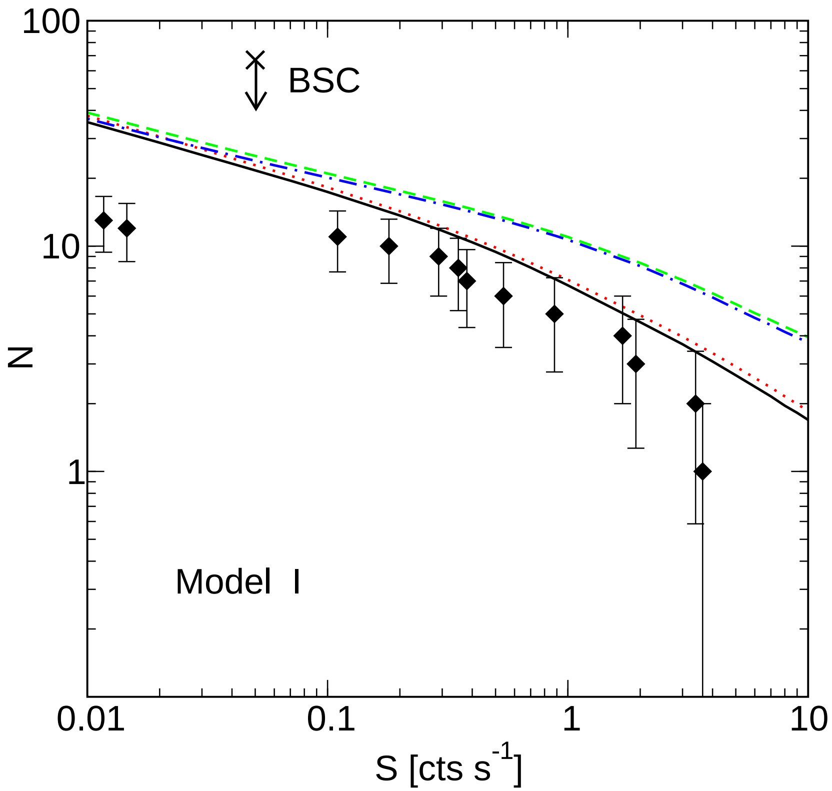
Pairing gaps for hadronic phase

(**Takatsuka, Tamagaki, A&A (2004)**)

and 2SC + X phase

Hybrid Star Cooling with 2SC Quark Matter (II)





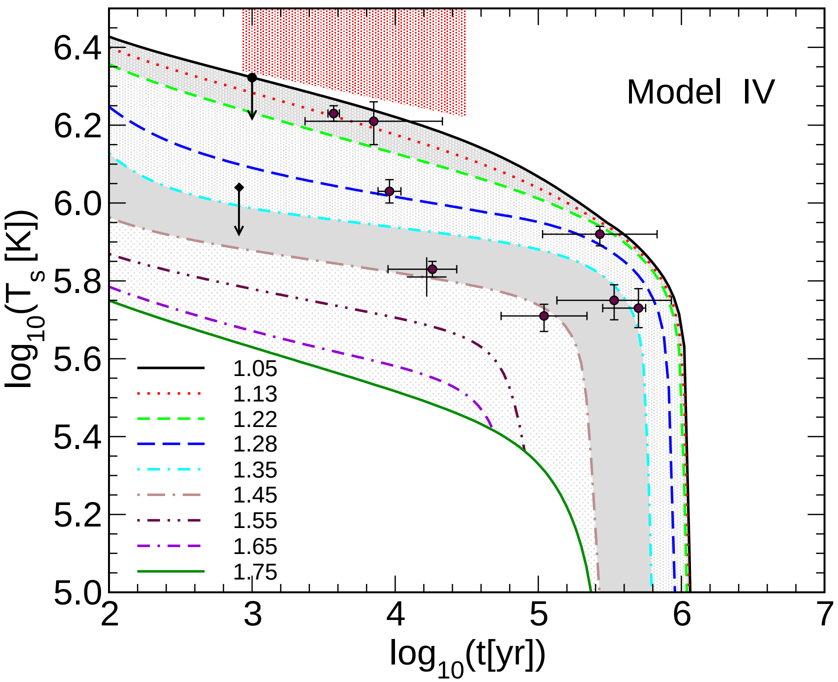
2SC + X phase, $\Delta_0 = 1$ MeV, $\alpha = 10$

Too large mass for Vela required

Popov, Grigorian, D.B., PRC 74 (2006)

Log N - Log S test fails

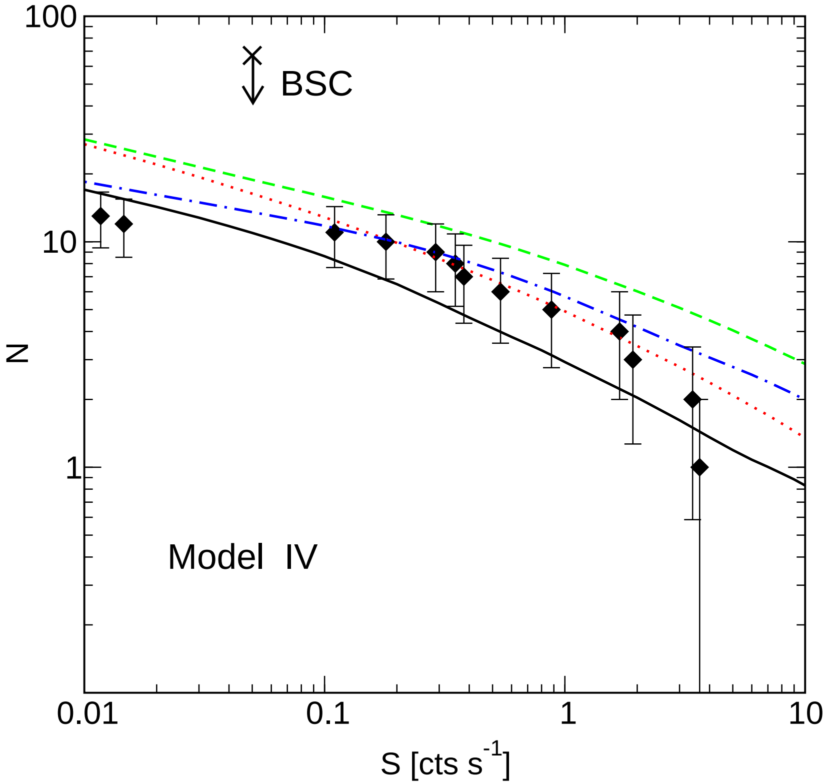
Hybrid Star Cooling with 2SC Quark Matter (III)



2SC + X phase, $\Delta_0 = 5$ MeV, $\alpha = 25$

Temperature-age and Vela mass OK

Popov, Grigorian, D.B., PRC 74 (2006)



Log N - Log S test passed

Hybrid Star Cooling with 2SC Quark Matter (IV)

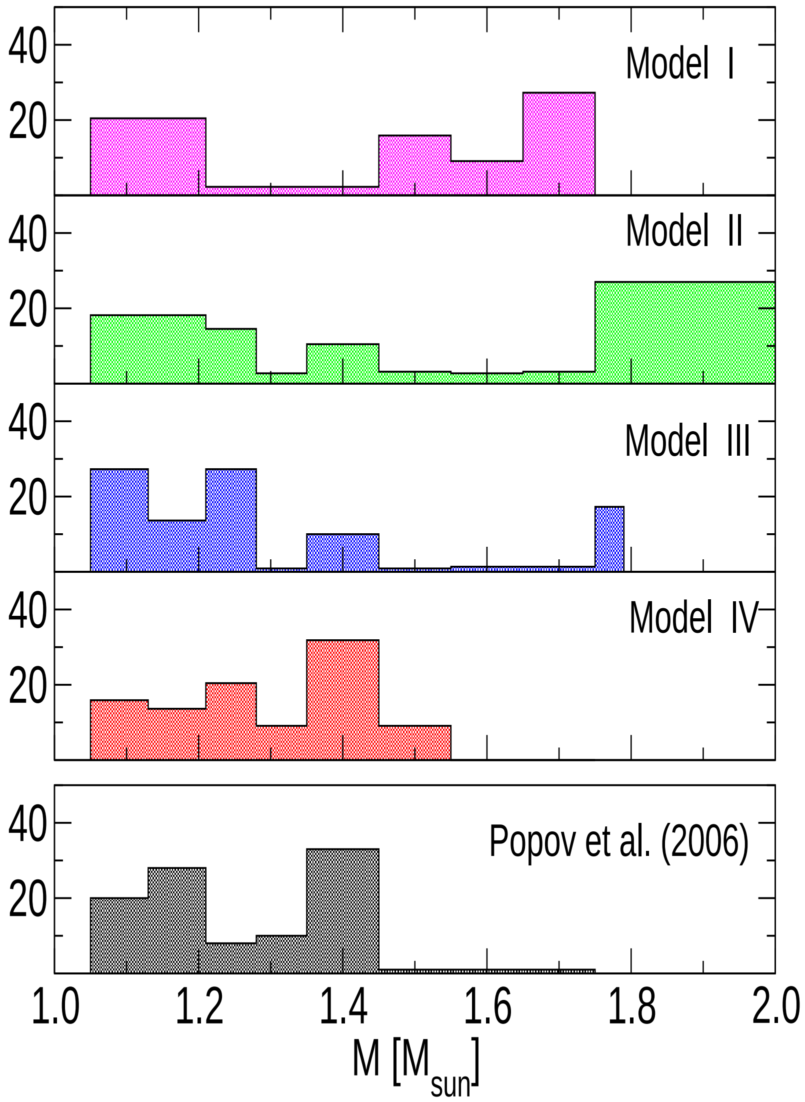
Hybrid star passes all modern cooling tests:

- ⊙ Temperature - age
- ⊙ Log N - Log S
- ⊙ Brightness constraint
- ⊙ Vela mass

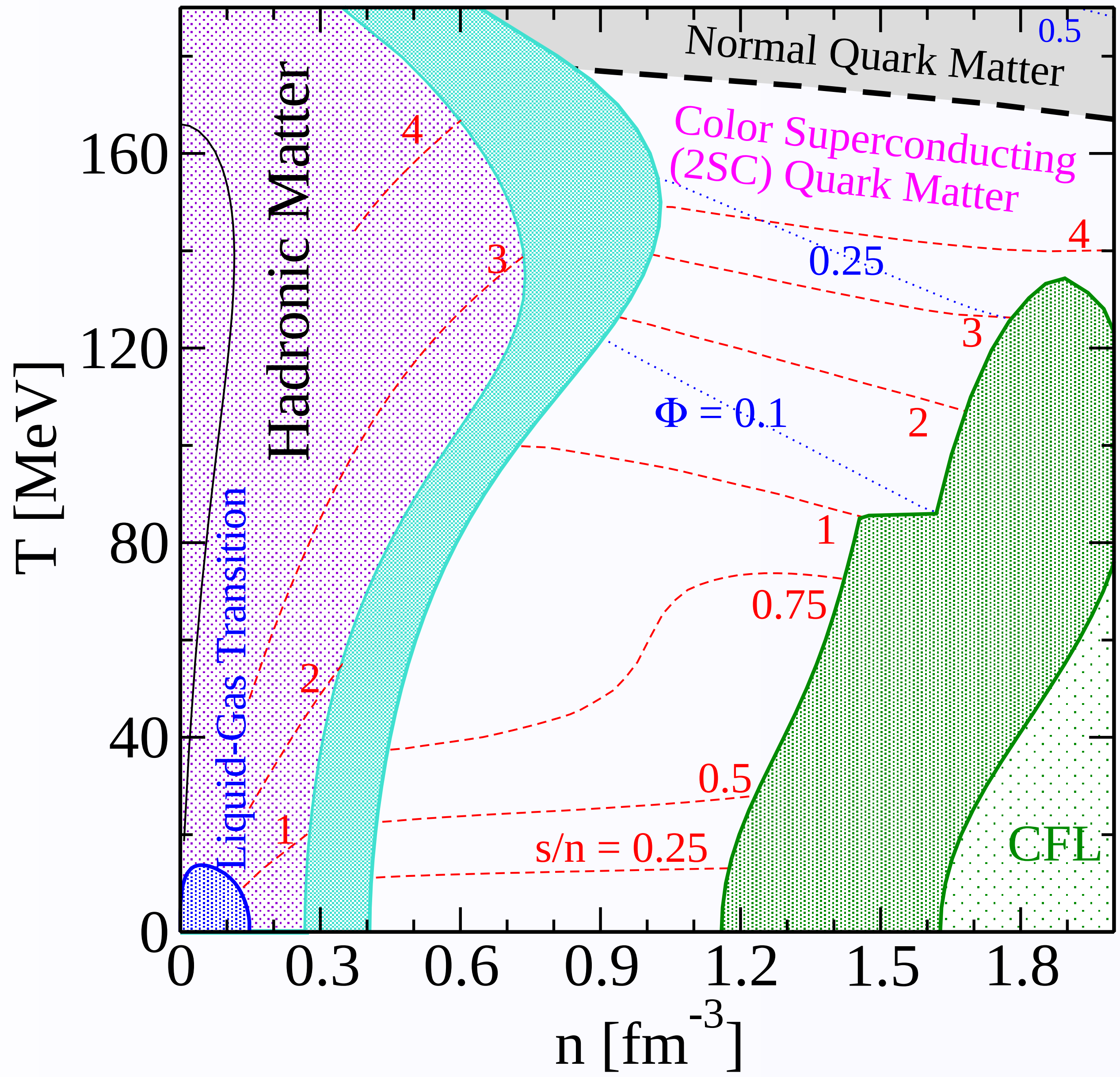
Popov, Grigorian, D.B., PRC 74 (2006)

D.B., H. Grigorian, PPNP (2007)

astro-ph/0612092



Phase diagram, symmetric matter



(T. Klähn et al., in preparation)

Summary

- High density EoS testing scheme
  - set of constraints from HIC flow and new astrophysical observations
  - complementary tests for $E_0(n)$ and $E_S(n)$; cooling !
- Present-day conclusions
  - $E_S(n)$: “soft” (cooling, direct Urca) $\rightarrow \beta^2 E_S(n)$ universal
  - $E_0(n)$: “soft” for $n < n_c$ (flow data); “stiff” for $n > n_c$ (star masses)
  - deconfinement can solve stiffness and DU cooling problems
  - phase diagram for CBM: very weak 1st order transition, early onset!
- Outlook
  - implementation of new astrophysical data (e.g. population statistics)
  - discussion of hyperons and hadronic resonances
  - QM beyond mean-field: hadronic bound and scattering states

unique approach to EoS & phase transition

Collaborators

- *Scheme Development:* H. Grigorian, T. Klähn, G. Röpke
- *Equations of State*

NLρ, NL$\rho\delta$	T. Gaitanos, M. Di Toro, S. Typel, V. Baran, C. Fuchs, V. Greco, H.H. Wolter Nucl. Phys. A**732**, 24-48 (2004)
DBHF	E.N.E. van Dalen, C. Fuchs, A. Faessler Nucl. Phys. A**744**, 227-248 (2004)
DD, D^{3}C, DD-F	S. Typel Phys. Rev. C**71**, 064301 (2005)
KVR, KVOR	E.E Kolomeitsev, D.N.Voskresensky Nucl. Phys. A**759**, 373 (2005)
NJL	F. Sandin Phys. Rev. D**72**, 065020 (2005)

- *Cooling:* H. Grigorian, S. Popov, D. Voskresensky
- *Astrophysical Expertise:* M.C. Miller, J. Trümper, A. Ho, F. Weber

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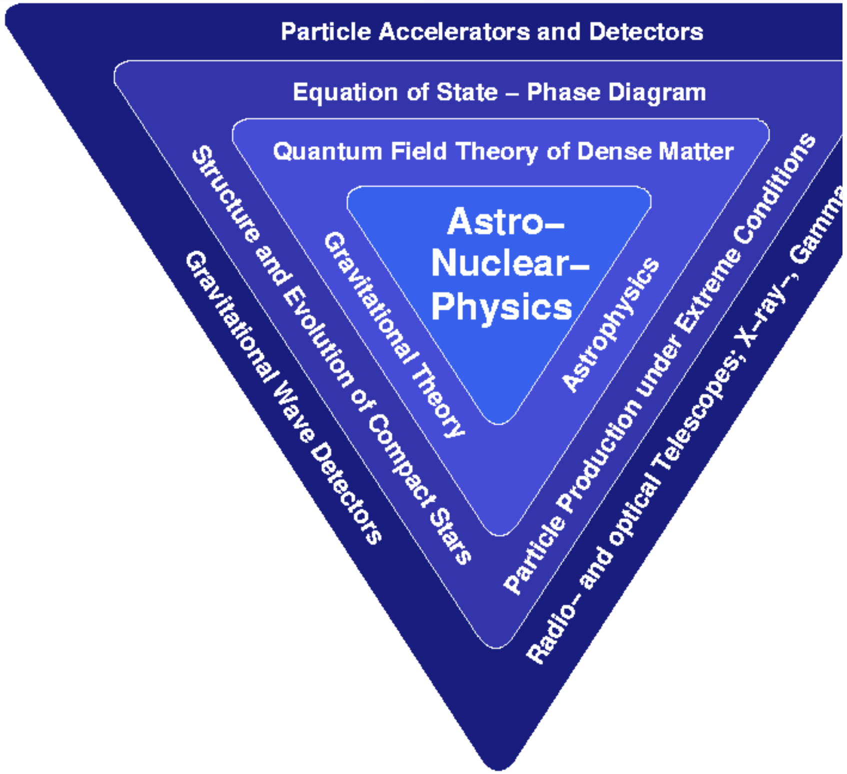
- *Supported by*
  - ⋆ DFG, BMBF, Helmholtz Gemeinschaft VH-VI-041 (Germany)
  - ⋆ US DoE, NSF, Research Corporation, Goddard Space Flight Center (USA)

Advertisement: ESF RNP “CompStar” (2008-2013)

http://compstar-esf.org



Particle Accelerators and Detectors

Equation of State – Phase Diagram

Quantum Field Theory of Dense Matter

Astro–
Nuclear–
Physics

Gravitational Theory

Astrophysics

Structure and Evolution of Compact Stars

Particle Production under Extreme Conditions

Gravitational Wave Detectors

Radio– and optical Telescopes; X-ray–, Gamma